Modern Physics (PHY 3305) Lecture Notes

HomeworkSolutions010

SteveSekula, 5 May 2010 (created 3 May 2010)

Point Distributions

no tags

Points were distributed as follows for each problem:

Problem	Total	Point Distribution
CH12-10	10	Used the uncertainty principle (7 Points) to solve for the range of the weak interaction (3 Points)
CH12-12	10	Used relativistic conservation of energy (7 Points) to solve for the photon energy and wavelength (3 Points)
CH12-18	10	Used matter wave relationships (3 Points) and relativistic energy relationships (4 Points) to solve for the electron energy (3 Points)
CH12-20	10	Part a: used the work-force relationship (2 Points), recognizing that the work is equal to the energy needed to make a pion (2 Points), to solve for the force (1 Point) Part b: used the Coulomb Force equation (3 Points) to solve for the electrostatic force (1 Point) and compared to the strong force (1 Point)
CH12-27	20	Part a: Determined relativistic quantities (10 Points) and used the force balancing to solve for the magnetic field (5 Points) Part b: inverted the equation to solve for radius using a fixed magnetic field (5 Points)
SS-13	40	Part 1: provided an argument as to how to assign uncertainty to the mass of the top quark (10 Points) and determined the lifetime from the uncertainty principle (5 Points) Part 2: chose a frame of reference (10 Points), taking

into account relativistic effects on the lifetime of the quark (5 Points), and argued as to the chance of top quarks forming bound states using gluon and quark kinetics (10 Points)

Deductions outside of the above:

- 1 point is deducted for failing to box the numerical answer
- 1 point is deducted for incorrectly applying significant figures
- other points are deducted as outlined in the homework policy

---

HARRIS *CH12-10* (10 Points)

SOLUTION

The average mass of the force particles in question is about $85\ \mathrm{GeV/c^2}$. In order to determine the range of the force, we need to use the uncertainty principle, $\Delta E \Delta t \geq \hbar/2$. Using the same argument as for the nuclear force,

$$\Delta x \approx \hbar/(cm) = 2 \times 10^{-18}\ \mathrm{m} \approx 10^{-3}\ \mathrm{fm}.$$

HARRIS *CH12-12* (10 Points)

SOLUTION

If we assume that the proton-antiproton pair is initially at rest and just in contac when they both annihilate, their initial energy is $(E_p + E_{\bar{p}})^2 = (2m_pc^2)^2$. There is ZERO momentum in the initial state. The photons that result from the annihilation each have the same mass (zero mass), and so by conservation of energy share equally in the total energy and by conservation of momentum must be traveling back-to-back, to preserve the zero-momentum of the initial state. Thus the energy of either photon can be written as $E = \frac{1}{2}2m_pc^2 = m_pc^2$. Writing the proton mass as $938.3\ \mathrm{MeV/c^2}$, the single-photon energy must be $938.3\ \mathrm{MeV}$. This corresponds to a wavelength given by $E = hf = hc/\lambda \rightarrow \lambda = hc/E = 2\pi\hbar c/E = (197.7\ \mathrm{MeV \cdot fm})/(149.34\ \mathrm{MeV})$ (it's a "useful trick" to remember that $\hbar c = 197.7 \text{ MeV} \cdot \text{fm} \approx 200 \text{ MeV} \cdot \text{fm}$). This yields $\lambda = 1.3 \text{ fm}$.

HARRIS *CH12-18* (10 Points)

SOLUTION

In order to probe forces with ranges as short as the weak force ($\sim 10^{-3} \text{ fm}$), the wavelength of the electron would need to be this small or smaller to resolve the effects. We can then compute the energy of such an electron, using this as the wavelength of its matter wave. For matter waves, recall that $p = h/\lambda$ and $E = hf$, but since $\lambda f \neq c$ we cannot merely substitute into E and write $E = hc/\lambda$. Instead, let's compute the momentum and use the relativstic momentum and mass-energy relationships, $E^2 = (mc^2)^2 + (pc)^2$.

$$p = h/\lambda = 6.63 \times 10^{-16} \text{ kg} \cdot \text{m/s}.$$

We can then solve for the energy of the matter wave: $E^2 = (0.511 MeV/c^2 \cdot c^2)^2 + (6.63 \times 10^{-16} \text{ kg} \cdot \text{m/s} \times c)^2 = 3.95 \times 10^{-14} \text{ J}^2$. Finally, we find that $E = 2.0 \times 10^{-7} \text{ J} = 1.2 \text{ TeV}$.

HARRIS *CH12-20* (10 Points)

SOLUTION

Part (a):

To solve this problem, let's fall back on the classical relationship between force and energy - the work equation: $W = F \times \Delta x$. We need to solve for the work required to create the π^0. At minimum, this is the energy required to create the π^0 at rest - the mass-energy of the pion. The mass-energy of the pion is 134.9 MeV/c^2. The distance the quarks have been displaced to create this pion is 1 fm. Thus the force required to do this is

$$F = (134.9 \times 10^6 \text{ MeV}) \times (1.6 \times 10^{-19} \text{ J/eV})/(1 \times 10^{-15} \text{ m}) = 2.2 \times 10^4 \text{ N}.$$

Part (b):

The ratio of the electrostatic force to the strong force in table 12.1 is about 1/100. For a separation of 1 fm between two fundamental charges (1e each), we would expect a force of $F = ke^2/r^2 = 230$ N. That's a ratio of about 1/100 of the strong force, so this seems sensible.

HARRIS *CH12-27* (20 Points)

SOLUTION

Part (a):

In order to maintain a 1 *TeV* proton in an orbital radius of 1 km, we have to balance the magnetic and centripetal forces:

$$q\nu B = \gamma_\nu m\nu^2/r$$

$$B = (\gamma_\nu \nu)^2(m/qr)$$

Recall that $\gamma_\nu \nu = c\sqrt{\gamma_\nu^2 - 1}$. If we can solve for the gamma factor, we're home-free. To get the gamma factor, let's use the fact that the protons have energy $E = 1\ \text{TeV} = \gamma_\nu mc^2$. Solving for γ_ν we obtain $\gamma_\nu = 1065.8$. We can now use this to solve for the magnetic field:

$$B = c\sqrt{\gamma_\nu^2 - 1}m/qr = 3.3\ \text{T}.$$

Part (b):

We can work the same exercise with 20 *TeV* protons and solve for the radius, using the same strength magnetic field. The gamma factor for 20 *TeV* protons is 2.0×10^{10}. Solving for the radius we find:

$$r = c\sqrt{\gamma_\nu^2 - 1}m/qB = 20km.$$

This can be quickly seen, since the gamma factor scales LINEARLY with energy and the energy is 20-times bigger, so the radius must also be 20-times bigger.

Problem *SS-13*: Top Mesons

A "resonance" occurs when two (or more) fundamental particles exchange force carriers and compose a bound state. Resonances can be long or short-lived, depending on the nature of the interactions. The top quark has a mass of $173.1\ \mathrm{GeV/c^2}$.

1. What is the approximate lifetime of the top quark?
2. Imagine you have an experiment capable of producing a top quark and an anti-matter top-quark ($t\bar{t}$) in such a way that once produced they are moving back-to-back, away from one another, each with a speed of 0.86c (roughly speaking, this is the case in the Tevatron at Fermilab). Given the lifetime you computed in Part 1, and given their relative motion, is it possible for the $t\bar{t}$ system to form a bound state (a top meson)? To answer this, let us define the minimum condition for a bound state to form to be that a gluon, emitted from one top quark at the speed of light, reaches the other top quark before either of them decays.

SOLUTION

Part 1:

In order to solve for the approximate expected lifetime of the top quark, you should apply the energy/time uncertainty principle:

$$\Delta E \Delta t \geq \hbar/2.$$

Given the mass of the top quark, $173.1\ \mathrm{GeV/c^2}$, you can estimate the uncertainty on the mass-energy in one of several ways (the width of the top quark mass - that is, the real uncertainty on its mass - has not yet been measured). You can assume it is 100% uncertain, in which case $173.1\ \mathrm{GeV/c^2} \Delta m \geq \hbar/(2\Delta t)$. Solving for the lifetime, here you would find $2.7 \times 10^{-29}\ \mathrm{s}$. If instead you assumed the uncertainty in the mass was in the last decimal place (this is a safe assumption, when numbers have been quoted without uncertainty, although it's not always accurate), then you don't know if the "true mass" is 173.9 or 173.0. This gives a range of about $\pm 1.0\ \mathrm{GeV/c^2}$ for the uncertainty. In this case, the lifetime comes out to be

4.7×10^{-27} s.

Let's call either of these two times τ_t, the lifetime of the top quark.

Part 2:

Let's begin by assuming that the two top quarks start their lives at the same point in space, and travel away from each other. Each has a speed of 0.86c. You could argue this two ways. First, from the perspective of an observer who is in a reference frame where both top quarks move with equal and opposite velocity, the distance between them appears to be increasing at a rate of $2 \times 0.86c = 1.72c$.

There are potentially many ways to attack this problem. These solutions demonstrate a few; depending on how you attacked the problem (your approach may differ from the ones I can consider, but still be valid), you'll receive points.

The *Center-of-Mass* Rest Frame Solution

Let us attack the problem in the frame of reference where both top quarks travel with equal speeds in opposite directions (along the x-axis, for instance). We have several things to consider in this frame:

- the distance between the quarks is increasing faster than the speed of light, but there is a chance that if the gluon is emitted very early it could reach the other top quark before it decays
- the lifetime given in Part 1 is defined in the REST FRAME of the top quark, not in this frame; in this frame, the top quarks' clocks appear to run slower, and we'll have to recompute the lifetime based on their gamma factor

Let's begin by recomputing the lifetime. It will be longer in this frame, since the top quarks are in motion. Thus: $\tau_t' = \tau_t \gamma_\nu$. The gamma factor is $\gamma_\nu = 1.96$, based on the speed of each top quark being 0.86c. Thus the lifetime in this frame is either 5.3×10^{-29} s or 9.2×10^{-27} s, depending on the approach from part 1.

Let's think of the problem as follows. At time t=0, the top quarks appear; one is traveling in the -x direction at 0.86c, the other is traveling in the +x direction at 0.86c. The left-going top quark is the one I will treat as emitting the gluon. I will treat the gluon as massless and moving at the speed of light. Harris quotes only a LIMIT on the mass of the gluon (in physics, we treat it as massless), and if you used that instead I will accept your results. Let's assume the gluon is also "soft" enough (low enough in energy) that it doesn't affect too much the speed of the left-going top quark.

Given that picture, we can ask the question: how far apart can the top quarks get (or how much time can pass since they appear) before a gluon can NO LONGER reach the other quark?

This becomes an exercise in algebra. Let us define the maximum distance either quark can travel from x=0 as $d = \tau_t' \nu$, where $\nu = 0.86c$. If the gluon is emitted at a time $t < \tau_t'$, then the distance traveled from x=0 by the quark is $x = \nu t$, and the time the gluon has left in order to reach the other quark is $\tau_t' - t$.

Let us now put this all together and ask: how much time can pass before the left quark emits a gluon, and still have that gluon reach the other quark? If the gluon is emitted at time t, then the distance that the gluon has to maximally travel to reach the other quark before it decays is:

$$d + x = d + \nu t$$

That distance must be equal to the distance the gluon can travel in the time remaining to it in order for the gluon to reach the other quark. Thus:

$$d + \nu t = c(\tau_t' - t)$$

Rearranging to solve for t, the maximum time at which the left quark can emit a gluon and still have it reach the right quark, we find:

$$t = \frac{c\tau_t' - d}{c + \nu} = \frac{(c - \nu)}{(c + \nu)}\tau_t'$$

What is the value of this factor multiplying the lifetime? In our case, for $\nu = 0.86c$, it is 0.075 - that is, the maximum time after the creation of the quarks that a gluon can be emitted and STILL reach the other gluon is up to 7.5% of the lifetime. This is a small fraction of the total time that the quarks have to live; in other words, unless that gluon is emitted very soon after the quarks appear, it's impossible for a bound state to form. Thus, it is UNLIKELY that bound states of top quarks are possible.

The Top Quark Rest Frame Approach

Instead, we could attack the problem in the rest frame of one of the top quarks. Let us choose the left-going top quark, since the other quark will then be moving along the +x direction away from it. In that case, we need to transform the velocity of the second quark into this rest frame. That gives us:

$$\nu' = \frac{u+\nu}{1+\frac{u\nu}{c^2}} = 0.989c$$

This makes sense. We expect in this frame the speed of the other quark to be even greater than in the center-of-mass rest frame.

We can then ask the same question as in the other solution: how much time can maximally lapse before a gluon is emitted, and still reach the other quark before the quark at rest decays? Remember, in this frame the lifetime of the resting quark is as short as it can be, and is shorter than that of the other quark. Therefore, the decay of quark at rest defines the maximum time the gluon has to make its journey.

If the gluon is emitted at some time t after the quarks appear, then the distance the other quark has traveled is $\nu' t$. In the time remaining before the resting quark decays, that gluon will travel $c(\tau_t - t)$. The LATEST that gluon can be emitted and still reach the other quark before the resting quark decays is then defined by:

$$c(\tau_t - t) = \nu' \tau_t$$

where $\nu' \tau_t$ is the maximum distance the gluon will have to travel before the resting quark decays. Solving for t:

$$t = (1 - \nu'/c)\tau_t = 0.021\tau_t$$

In this frame, the maximum time after t=0 that the gluon can be emitted and still reach the other quark before the first one decays is just 2.1% of the lifetime of the top quark. Again, in this frame we conclude it's unlikely to form a bound state unless the gluon is emitted very promptly.