Modern Physics (PHY 3305) Lecture Notes

Matter as Waves (Ch. 3.6,4.1-4.2)

SteveSekula, 4 February 2010 (created 13 December 2009)

tags: lecture

Review of Last Lecture

- We discussed massless objects in relativity, and found that relativity predicts light to be massless (whatever it is)
- We discussed the properties of waves and particles
- We tried to apply the wave hypothesis of light to many phenomena that were explored in the late 1800s and early 1900s
    - blackbody problem
    - photoelectric effect
    - others (compton scattering)
- For many experiments, light's behavior was explicable as particle-like

So which is it?

What is the "truth" about light? Is it a wave or is it a particle?

Let's start an experiment

Begin the quantum interference demonstration now, to give it time to build up intensity on the screen.

http://phet.colorado.edu/simulations/sims.php?sim=Quantum_Wave_Interference
http://phet.colorado.edu/sims/quantum-wave-interference/quantum-wave-interference_en.jnlp

Let's start an experiment. We design a device that shoots light through two holes (slits) in a barrier wall. There is a camera on the other side that can detect when a particle of light strikes it, and when that happens the pixel lights up and gets more intensely lit as more photons hit it.

We see the gun shooting light at very LOW INTENSITY (few particles per second, in this case about 1 every 3-4 seconds). Let's return to this experiment after talking more about light and its nature.

It takes about 5-10 minutes to build up a good exposure

The Nature of Light

Simple answer: "The situation determines its nature" - it has no predetermined waveness or particleness. Harris refers to light simply as a "phenomenon" that we observe as exhibiting wave-like and particle-like properties.

More complicated answer: "The behavior of a phenomenon like light depends on the comparison between the wavelength of the phenomenon, λ, and the *relevant dimensions* of the experimental apparatus, which we'll abbreviate as D.

- What does this mean - relevant dimensions?
  - consider light scattering off metal, as in the Compton scattering experiment - when the wavelength was "big" compared to the size of the effects due to particle behavior, they are hard to see. When they are "small" (e.g. x-rays) compared to the size of the particle-behavior effects, you notice them. The relevant dimensions here are the sizes of atoms in metal.

The light phenomenon can be described by an object called a "Wave Function" - just like in homework 001, this tells you about the properties of the light as a function of time and space.

- When $\lambda << D$: light behaves more like a particle
- When $\lambda >> D$: light behaves more like a wave

Boats on the Water

Discuss blind-folded boat inhabitants subjected to different water phenomena. How will they describe the events?

Single-slit diffraction

To see the diffraction effect, the size of the slit must be comparable to the wavelength of light. Otherwise the wave passes through undisturbed by the presence of the slit.

Dual Nature

The fact that light or radiation phenomena exhibit both wave-like and particle-like properties often leads to the statement that they have a "dual nature.

That duality is not particle vs. wave, but rather complimentary between the two. The simple fact is that the true nature of radiation, perhaps, does not have words, but exploration of the phenomenon has revealed two different sides of the phenomenon.

In the same way that Einstein's discovery that the energy of stationary body and the mass of a stationary body are intertwined, via

$$E = mc^2,$$

the relationship discovered between wavelength + frequency and energy + momentum reflects the duality of the phenomenon:

$$(\mathrm{particle}) E = hf (\mathrm{wave})$$

$$(\mathrm{particle}) p = h/\lambda (\mathrm{wave})$$

A final piece

There is one more property of radiation that will help us to connect the two natures of the phenomenon: probability. This relationship, among the most fundamental we know, is revealed by the "*Double-Slit* Experiment"

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We can return to our experiment.

What do we learn from this experiment?

- the probability of detecting a particle in a given location on the screen must be related to the number of particles detected in that location
    - more particles in a given place on the screen = higher chance of detecting particle at that place
- the density of spots (detected particles of light) follows exactly the prediction for the intensity of *light waves* on the screen
    - Light intensity is given mathematically by the SQUARE OF THE WAVE AMPLITUDE

Although the light is being detected "on particle at a time," its wave nature is apparent!

DISCUSSION

Conclusions about the wave and particle natures of radiation

When a phenomenon is detected *as particles* we cannot say for certain where it will be detected. The most we can determine is the probability of finding it in a certain location. That probability (P) is proportional to the square of the amplitude (A) of the associated wave in that region:

$$P \propto A^2$$

To clarify the nomenclature:

- if the wave in question is the electromagnetic field, its "associated particle" is the photon.
- equivalently, if the particle in question is the photon, its "associated wave" is the electromagnetic field
- In electricity and magnetism, we learn that the EM waves exert forces on electric charges, and we now claim that these waves also measure the probability of finding the photon.

What's coming:

- things we think of as "solid matter," as opposed to radiation traveling from one place to another, also has dual particle and wave natures (why would light be special?).
- the wave associated with matter also is a measure of the probability of

finding the particle, but that wave has no other side to its personality (that is, we don't see an equivalent of "EM waves exert forces on charges" for matter.

A question: which slit did the 17th photon pass through?

Photons are particle-aspects of the light phenomenon, so it seems natural to ask: which slit did the 17th photon pass through in the double-slit experiment. In order to observe the photon, you have to detect it - that in and of itself is an experiment, requiring some interaction with the light.

Detecting the photons en route to the slits or the screen interferes with the aforementioned behavior. Interference requires two coherent waves - in our case, the single photons have to be thought of as passing through both slits at the same time - that is, the wave-like aspects of light's behavior can only be seen by letting light behave as a wave, as pass through both slits simultaneously. Changing the experiment forces the photon to interact with something, exposing its particle nature and changing the outcome of the experiment.

If Light is both a wave and a particle, what is matter?

Matter and light are thought of as distinct things. Light is a form of radiation, emitted by moving electric charges and transmitted from one place to another. Matter has mass, light is massless. Matter is particles, light is a wave.

But if light can be exhibit both wave and particle properties, what about matter?

To expose the particle nature of light, we had to expose it to an apparatus with comparable dimensions to its wavelength.

- Sodium street-light, with wavelength $0.6\mu\mathrm{m}$, passes easily through a window or doorway without diffracting but readily diffracts when passed through a $1\mu\mathrm{m}$ slit.

The wave nature of matter would seem less foreign an idea if there was a comparable and common-place apparatus that does the same thing to matter - but there is not. It's not easy to reveal the wave-nature of matter, because as a general rule the wavelengths of matter-waves are MUCH

SMALLER than those of light.

Atoms Themselves

Actually, there is a useful pre-example of how the wave nature of matter is manifest all around us: atoms themselves. It's known that the energies of electrons in atoms can only attain certain discrete energies. Standing waves in a confined space - like a guitar string - can also only attain certain frequencies (which for wave-particle situations, means discrete energies). The discrete behavior of atomic energy levels can be described as what happens to electrons when they are confined to something about the size of an atom (less than 1nm) - their wave nature becomes apparent and discretizes their allowed energies.

Again: Particles and Waves

We tend to ask different questions of particle and wave behavior:

- Particle:
  - where is it going?
  - when will it get there?
- Wave
  - What is its amplitude?
  - What is its wavelength?
  - How spread out is it?
  - Where is it zero?

The *Double-Slit* Experiment for Electrons

Perform the same double-slit demonstration in *PhET*, but use electrons instead of light. Emphasize that if the wave-particle duality exists for electrons, you expect the pattern of intensities on the screen to be connected to the wave properties of the electrons.

DISCUSSION

Question:

- in light, we speak of the E/M fields as oscillating, and that is the wave behavior
- in matter, what is oscillating?

Bragg Scattering: evidence for the wave nature of electrons

Send electrons into a metal, which is a closely spaced arrangement to atoms in a "crystal lattice". The lattice spacing is like the diffraction slit (VERY TINY!) and one expects to see a wave diffraction pattern of the scattered electrons if they have wave nature. This is observed.

In fact, the wave nature of electrons is now routinely used to tell us about the crystal properties of new materials.

Wave Behavior of Matter

Light is easy to think about in terms of waves, since we learn about how the oscillation of electric and magnetic fields in space gives rise to waves of electromagnetism, which are the same as light.

Next time, we'll begin to address the question of just what is oscillating in matter

Today, we will simply finish by writing down the conjecture for the wave properties of matter. Since we're used to thinking about the particle properties for matter, we want to use them to determine the wave properties. Prince Louis de Broglie was the first to pursue this hypothesis - that matter has wave properties.

He made the conjecture that:

$$\lambda_{matter} = h/p$$

and thus that

$$f_{matter} = E/h$$

Electrons passing through atomic spacing, or electrons in atoms, are confined to small spaces. What about big things? What are the wave properties of terrestrial things?

- What are the wave properties of your professor during his commute to work? Assume he drives a 1.5 ton car at 65 mph.

  - We first need the momentum of the vehicle, and we can use the non-relativistic form of momentum since the speeds are extremely small compared to light. Thus $p = mu = (29.1 m/s) \times (1360 kg) = 39.6 \times 10^3 \mathrm{kgm/s}$

    We can then solve for the de Broglie wavelength of the car:

    $$\lambda_{matter} = h/p = (6.63 \times 10^{-34} J \cdot s)/(39.6 \times 10^3 J \cdot s/m) = 1.68 \times 10^{-38} \mathrm{m}.$$

    It's no wonder nobody noticed my wave properties.
- When I drive through a toll-tag plaza, do I diffract?
  - Even if I diffracted while passing through a toll-tag plaza on the Dallas North Tollway, it would be impossible to notice the effect. The dimensions of the toll-tag booth (D) and vastly larger (about 38 orders of magnitude) than the wavelength of my car, and the wave properties are not important. Recall that diffraction intensities on the other side of the toll booth are given by

    $$I(\theta) = I_0 sinc^2(x)$$

    where

    $$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

    and

    $$x = D\sin(\theta)/\lambda$$

    and the spacing between diffraction maxima is NEGLIGIBLY SMALL.