

Modern Physics (PHY 3305) Lecture Notes

The Discovery of Spin (Ch. 8.1-8.3)

Steve Sekula, 4 March 2010 (created 4 March 2010)

Magnetic Moment

no tags

For a loop of current,

$$\mu = IA = \frac{e}{T} \cdot \pi r^2 = \frac{e}{2\pi r/\nu} \pi r^2 = \frac{e}{2} \nu r = \frac{e}{2m_e} m_e \nu r = \frac{e}{2m_e} L$$

Imagine, now, immersing the loop of current in a non-uniform magnetic field - that is, one which is changing as a function of z . In that case, instead of B we have

$$\frac{\partial B_z}{\partial z} \hat{z}$$

You can then work out the force on such a loop, and substituting $L_z = m_\ell \hbar$:

$$\vec{F} = \left(-\frac{e}{2m_e} m_\ell \hbar \frac{\partial B_z}{\partial z} \hat{z} \right)$$

QUESTION: What happens if we send in Hydrogen in the ground state? What is ℓ ?

ANSWER: in that case, $\ell = 0$. We expect NO force.

(see slides for a comparison of classical and quantum (so far) expectations, and then the reality).

Spin

The *Stern-Gerlach* experiment suggested that there was some INTRINSIC

angular momentum in the "current loop", even though the orbital angular momentum ℓ was zero in the ground state. Understanding this effect took a long time. Originally, it was supposed that the electron was "spinning," and thus carried with it some intrinsic angular momentum. However, if that were a real effect the INTRINSIC ENERGY of the electron would be far greater than it actually is.

"Spin" remains as a name, but means instead simple an irreducible, intrinsic angular-momentum that is one more identifying mark of a fundamental particle. This means that each particle with spin also has an intrinsic magnetic moment, something that makes it respond under the influence of a magnetic field. Like orbital angular momentum,

$$S_z = m_s \hbar \quad (m_s = -s, -s + 1, \dots, s - 1, s)$$

Spin is a new QUANTUM NUMBER, bringing the total for an atomic system to:

$$(n, \ell, m_\ell, m_s)$$

Like total orbital angular momentum,

$$S = \sqrt{s(s+1)} \hbar$$

The spin of many particles has been measured. The electron, proton, and neutron all carry $s = \frac{1}{2}$. The photon, in contrast, is $s = 1$. Spin explain the *Stern-Gerlach* experiment, since each current loop carries two "spin states": $s = \frac{1}{2}, -\frac{1}{2}$. This explains the bifurcated image of the atoms on the other side of the experiment.

Spin is completely NON-CLASSICAL. It truly represents a complete departure from the everyday - there is simply nothing like it (except bad analogy) at the human scale.

Wave Functions in Light of Spin

We have to adjust our wave functions for electrons in atoms:

$$\psi_{n,\ell,m_\ell,+\frac{1}{2}}(r,\theta,\phi)$$

$$\psi_{n,\ell,m_\ell,-\frac{1}{2}}(r,\theta,\phi)$$

Identical Particles

Particles are defined by their quantum numbers, their mass, and their coordinates (space and time). "Identical Particles" are those for whom all quantum numbers and mass are THE SAME; the only thing that might distinguish them is their place in space, but if they occupied the same space (e.g. were both in a box) and you were asked to answer "which is which?" you would not be able to since they otherwise are indistinguishable.

Let's consider a situation where two identical particles are involved. To describe the system, we need a *TWO-PARTICLE* WAVE FUNCTION. In the language of quantum mechanics, we don't have to think of these as distinct things - instead, we can describe both particles using a single wave function:

$$\psi(x) = \psi(x_1, x_2)$$

where subscripts label the two particles. The trick with indistinguishable particles is that if you were to turn your back and I were to swap them in the box, you would have NO WAY OF KNOWING I DID IT. Not only that, but the probability density cannot be changed by swapping them - the outcomes and their probabilities can't be different just because I swap the particles. They are truly indistinguishable. This, we can express this mathematically:

$$|\psi(x_1, x_2)|^2 = |\psi(x_2, x_1)|^2$$

QUESTION: Is the following choice for the two-particle wave function unchanged under swap of particles 1 and 2?

$$\psi(x) = \psi_n(x_1)\psi_{n'}(x_2)$$

- Consider what happens if you have an electron in state n_1 and another in a different state n_2 . If you swap them in space, the probability densities are not necessarily the same any more:

(sketch this picture for an infinite square well)

There are, then, two kinds of wave functions that can satisfy this requirement: SYMMETRIC and ANTISYMMETRIC wave functions:

SYMMETRIC:

$$\psi(x_1, x_2) \equiv \psi_{n_1}(x_1)\psi_{n_2}(x_2) + \psi_{n_2}(x_1)\psi_{n_1}(x_2)$$

ANTI-SYMMETRIC:

$$\psi(x_1, x_2) \equiv \psi_{n_1}(x_1)\psi_{n_2}(x_2) - \psi_{n_2}(x_1)\psi_{n_1}(x_2)$$

Spin and the Total Wave Function

It is not sufficient to only consider space - we need to include SPIN states, too. Why is spin SO crucial to the behavior of multi-particle systems?

- whether or not the TOTAL WAVE FUNCTION of a system of particles is symmetric or anti-symmetric depends ENTIRELY on the spin of the particles.

Theory has predicted, and experiments have confirmed, that:

- **BOSONS**: particles for which $s = 0, 1, 2, \dots$ manifest a symmetric multiparticle state

"FERMIONS" : particles for which $s = \text{onehalf}, \frac{3}{2}, \dots$ manifest an anti-symmetric multiparticle state

All building blocks of nature - electrons, protons, neutrons, quarks, etc. - are FERMIONS. All particles that transmit forces, like photons, are BOSONS.

Consequences of Fermion *Anti-Symmetric* Multiparticle Wave Functions

Consider two fermions that are forced to occupy the same state. Then:

$$\psi_n(1)\psi_n(2) - \psi_n(1)\psi_n(2) = 0$$

Nature abhors two fermions occupying the same space - the probability density for such a thing to happen is ZERO exactly. This is known as the EXCLUSION PRINCIPLE:

- No two indistinguishable fermions may occupy the same individual particle state.

This principle was articulated by Wolfgang Pauli in 1924 and earned him the 1945 Nobel Prize.

The exclusion principle does not apply to bosons. Indistinguishable bosons can ALL OCCUPY THE SAME STATE.

Next Time

The spin of particles has obviously deep implications for the behavior of large ensembles of particles. That's what's coming in chapter 9, where we will begin to discuss STATISTICAL MECHANICS, the study of large numbers of particles.

Superconductivity is one phenomenon that emerges from spin consideration: when two electrons pair in a metal, they go from being two fermions to acting like a single- integer-spin state - a BOSON. Since bosons can occupy the same space, there is no resistance to motion through the metal. Zero resistance, zero viscosity, monolithic behavior of a huge collection of particles - these are all manifest behaviors of bosonic states.