Modern Physics (PHY 3305) Lecture Notes

Nuclear Physics: Nucleons and Binding (11.1-11.2)

SteveSekula, 6 April 2010 (created 6 April 2010)

Review

no tags

Last time, we closed our discussion of solid-state physics with the topic of Superconductivity. Let's pause for a moment and take stock of what we've done so far in the course. This will help to set the stage for the two topics left: nuclear and subatomic particle physics.

- We have studied the theory of relativity, which is a more general description of space and time and allows us to understand phenomena at all velocities.
- We have studied the quantum theory, which is also a more general description of nature from terrestrial to subatomic distances. In the quantum picture, all particles can be described by waves of probability. When confined to small spaces, bound states arise. When confronted with boundaries, waves can tunnel through them and travel to places a classical point particle cannot go.
- We have studied statistical mechanics, which gives us the tools to study large distributions of distinguishable or indistinguishable particles
- We have applied these ingredients, primarily quantum physics and statistical mechanics, to solids. The wave nature of the electron, in the presence of confining potentials exuded by regularly spaced ions, the normal energy levels of an individual atom each split into a band of N levels for N atoms. Electrons are essentially completely free to move around in a band if there are unfilled states in the band, or otherwise must be given enough energy to jump the band gap and get to another band with empty states. Conduction, insulation, semi-conduction - all can be explained by the bands, their capacity, and their spacing. Superconductivity relies on interactions between electrons and ions.
- Most of what we have talked about so far has involved things familiar: electrons, Coulomb potential, photons. These are common topics of study in introductory physics. We are about to enter a world which is less familiar, and is still an active area of study. The nucleus of the atom, neglected in chemistry and biology, plays a central role in the

modern world. To understand the atom, we have to understand all of its parts. We are now going to study the nucleus, its constituents and structure, and see what it can teach us about nature.

- What will we learn? We will learn that nuclear physics is just another application of waves confined in a potential, albeit a new kind of potential. We will learn about nuclear stability and instability. We'll discuss applications of nuclear physics: fission and fusion. These will take us to the cutting edge of modern physics and engineering.

Basic Structure of the Nucleus

- See slides for some historical pieces of atomic and nuclear models ("Plum Pudding Model," disproven by the *Geiger-Marsden* Experiment overseen by Rutherford, who went on to identify the proton in 1919. Chadwick discovers the neutron in 1932).
- Consists of neutrons and protons, collectively called nucleons.
- The number of protons is denoted by Z, the number of neutrons by N, and the total number of nucleons is A = N + Z. Z is the "atomic number," and A is the "mass number"
- chemical behavior depends only on Z.
- Nuclear behavior is changed by changing N, the number of neutrons.
  - Example: most of the hydrogen in nature is ordinary - a single protom in the nucleus. 0.0015% of hydrogen is actually DEUTERIUM, or heavy hydrogen. Deuterium contains a proton and a neutron and is nearly twice the mass of hydrogen. Chemically, they behave the same. The nuclear behavior (e.g. the way in which nucleons rearrange) of deuterium is vastly different from hydrogen.
- Nuclei with the same number of protons but different numbers of neutrons are ISOTOPES of the same element.
  - there are two naturally occurring isotopes of hydrogen.
  - hydrogen and deuterium are both stable - that is, they persist indefinitely
  - hydrogen has a third isotope - tritium - which is highly unstable. This means it is subject to spontaneous nuclear reactions that change it to a different element. This is called radioactive decay.
  - A comment on hydrogen nomenclature: "hydrogen" always means "A=1"; deuterium is A=2, and tritium is A=3.
- The carbon mass is used as a standard unit of measure. 1u is 1/12 of the mass of the carbon atom (including its electrons and nucleons).

No isotopes ABSENT in nature are stable; however, there are unstable isotopes that do occur in nature. Uranium-238 and thorium-232 are both more abundant in nature than silver, and do undergo radioactive decay, but they will be around a long time. Their half-lives (the time it takes a sample of N atoms to decay away to 1/2N of the original number) are billions of years.

Size of the Nucleus

Rutherford-style scattering experiments can be used to probe the size of the nucleus. For instance, you fire alpha particles at a nucleus. While the alpha particle still has too little energy to breach the surface of the nucleus, its scattering is governed by Coulomb repulsion. However, the character of the scattering changes when the alpha is given enough energy to breach the surface. Scattering experiments use this feature, and calculations of the "distance of closest approach" of an alpha to the center of the Coulomb force, to determine the nuclear radius (r).

Experiments have revealed that:

$$r = A^{1/3} \times R_0$$

where $R_0 = 1.2 \times 10^{-15}\ \mathrm{m}$.

This relationship is actually conspicuous. It tells us something deep about the nucleus.

DISCUSSION:

- Consider the nuclear volume. What is the relationship between nuclear volume and mass number?
    - GUIDE: $V = 4/3\pi r^3$. Inserting the relationship between radius and mass number,

      $$V = A \times \left(4/3\pi R_0^3\right).$$

        - Consider the nuclear density: A/V.
            - Density is constant in nuclei, regardless of the nucleus. The nuclear density of Uranium is the same as that of Copper - volume depends linearly on number, and thus density is a constant.
        - What is the implication of constant density as the size of the nucleus increases?

  - The nucleus behaves like an INCOMPRESSIBLE FLUID - they behave as if they are packed as closely together as they can be. This, in turn, suggests that the nucleus has a strongly repulsive hard core.

The volume allowed each nucleon is then just equal to the total nuclear volume divided by A, or about 1 fm.

Summary of Features so Far

- There is a well-defined experimental relationship between the mass number and the nuclear radius: $r = A^{1/3} \times R_0$.
- Nuclear density is constant as a function of the size of the nucleus
- The nucleus behaves in analogy to an incompressible fluid, where all nucleons are as closely packed as they can be.

But how are nucleons bound to one another? Isn't it weird that same-electric-charge particles like protons can be SO closely packed?

Nuclear Binding

DISCUSSION:

- What is the force that binds the nucleus?
  - Why can't it be the Coulomb Force or Gravity?

$$G_N = 6.67 \times 10^{-11}\ \mathrm{N(m/kg)^2}$$

so over a femtometer, and between 2 nucleons, the binding force is

$$F_{\mathrm{GRAVITY}} = -1.9 \times 10^{-34}\ \mathrm{N}.$$

That's attractive. Compare that to the Coulomb Repulsion between two protons: $F_{\mathrm{COULOMB}} = +230\ \mathrm{N}$. No way gravity is responsible.

On the mid-term, the bonus question was about the Coulomb Force at the sub-nuclear scale (attractive force inside the proton, holding its constituents, quarks, together), and the answer to that problem (comparing the uncertainty in total energy to the binding energy between the quarks) showed that Coulomb is just TOO WEAK to explain anything about the compact binding of the nucleus.

We need a new force. It is called the STRONG NUCLEAR FORCE, and it was originally evoked to explain why the nucleus exists at all. However, over time it evolved into a more general description of how nuclear and sub-nuclear binding occurs between certain classes of particles ("quarks"). We'll revisit the STRONG NUCLEAR FORCE as a more general force in nature in Chapter 12.

As of now, the STRONG FORCE, along with Electromagnetism and Gravity, is one of four forces that appear to exist uniquely in nature in our present universe. Again, we'll revisit this in chapter 12.

Features of the Strong Force:

- Extremely strong over very short distances (within nuclear radii) - one the order of 2 fm (maximal separation between two nucleons).
- Strength does not diminish simply as distance, or distance-squared.
- Strength falls off sharply after that
- Strongest when spins of nucleons point in the same direction (are aligned)

Force	**Strength (relative to Strong Nuclear)**	**Range**
Strong Nuclear	1	~ 1 fm
Electromagnetic	10^{-2}	long-range, proportional to $1/r^2$
Weak Nuclear	10^{-6}	$\sim 10^{-3}$ fm
Gravitation	10^{-39}	long-range, proportional to $1/r^2$

At larger distances than a few femtometers, Coulomb Repulsion becomes a more significant de-stabilizing influence on the nucleus.

DISCUSSION:

- How do you reveal the character of an allegedly new force?
- GUIDE: how did Rutherford reveal the size of the nucleus? Consider scattering of one particle off of another - does the force between them influence how they scatter?

PARTICIPATION:
What might the potential describing the strong nuclear force look like? Draw at the board.

There is no simple formula for this force. We can describe it largely by what we know about it from experiment.

Character of the Strong Nuclear Force

Let us construct a theoretical model of the strong nuclear force from observations of the nucleus. A comprehensive theory of the structure of the nucleus has eluded discovery, so we are resigned to "model building" - assembling pieces of experimental observation into a framework that can make predictions, but which itself is not a fundamental mathematical law.

- A model: well-informed but simplified guesswork.

As we discuss pieces of the model, we won't state explicitly which combinations of nucleons will be stable - only that a given factor will tend to lend or rob a nucleus of stability.

- In general, a nucleus is more stable when its constituent are assembled in a state of lower energy - energy is all-important, as usual.

A lower-energy state requires MORE energy to remove constituents from the state.

Consider the simplest nuclei - pairs of nucleons: (A=2)

- p-p
- p-n
- n-n

DISCUSSION:

- Which is the most stable?
- GUIDE: I like to think about what the universe would "look" like if various forces were switched on and off at will. Imagine a bunch of light switches, one for each force. Consider switching off all of them but the STRONG NUCLEAR force. Is there any difference between the

three pairs? What about if we switch off all but the Electromagnetic force? Are they the same. Now, switch on both the strong and EM forces. Which pairs are more/less stable?

DISCUSSION:
The reality is that only n-p pairs form stable bound states. Why?

- consider the following observed fact: the inter-nucleon attraction is strongest when the spins of each nucleon point in the same direction (are aligned).
- what does this mean for p-p and n-n combinations? Sketch the problem out by making up some quantum numbers for this problem, treating the nucleons as if they are in a deep well.

Experimentally, the n-p combination (a "deuteron") is BARELY bound. If their spins are anti-aligned, they do not form a bound state. They both have to be in the same state, or the bound state doesn't exist at all. n-p occupy a lone bound state that results from their mutual attraction.

More complicated cases: arbitrary nucleon number (A>2)

- Strong inter-nucleon attraction

You expect that a pair of nucleons will have a single bond, one to the other. Adding more nucleons to the picture, they will arrange (pack) themselves in such a way that they try to share a bond with as many of one another as possible. Consider having four nucleons, then six (one plane of four, one above the plane, one below the plane), then more. Each nucleon has bonds with each of the closest neighbors, but due to the fall-off of the nuclear force it doesn't go much further than that.

We think about shared bonds as follows. This for a 2-nucleon situation, each has half a bond. For four nucleons, you have six bonds, or 1.5 bonds per nucleon. It's harder to extract a nucleon from a 4-nucleon group than a 2-nucleon group. However, this trend does not continue indefinitely because at some point, adding more nucleons doesn't increase the bonding to any single nucleon due to their distance from that nucleon. As a result, each nucleon has the same maximum number of per nucleon bonds (nucleons on the "surface" of the packed structure have fewer, but as A increases they become a decreasing fraction of the total number of nucleons).

The force "saturates". Draw this on the board - binding energy per nucleon vs. A.

Instead of talking about binds per nucleon, we talk about binding energy per nucleon - that's the energy required to remove a single nucleon from the nucleus. Binding energy is the energy required to pull ALL nucleons apart, and binding energy per nucleon the energy required to remove one representative nucleon.

- Coulomb Repulsion

All pairs of protons in the nucleus repel. This adds positive potential energy to the negative strong force binding potential energy, and can be depicted as a "bifurcated well" (draw on the board).

Consider a A=4, Z=2 case. The lowest energy level available to the neutron pair is lower than that available to the proton pair.

For a small number of nucleons, all nucleons "touch" and their strong attraction overcomes the weaker electrostatic repulsion between protons. However, as the number of nucleons increases past some point, the protons are further from one another and cannot bind; there is then only a net repulsion in the nucleus, which destabilizes it and LOWERS the binding energy per nucleon.

Draw this on the board.

This suggests there should be a mass number A that is more stable than all others.

- The Exclusion Principle

We have a situation with two groups of identical, close-packed particles, creating situations of indistinguishability.

Considering only strong attraction and coulomb repulsion, it would seem that an all-neutron nucleus is the most stable. But the exclusion principle requires that no two identical fermions occupy the same state. So in an all-neutron nucleus, only a maximum of two neutrons can occupy any available energy level in the well.

Let's ignore Coulomb repulsion for a moment. The state of lowest energy would contain equal numbers of protons and neutrons. Draw a well on the board an illustrate this fact.

- Consider the following situation. For a fixed A=12, draw a potential well with Z=N=6. Now, keep A fixed and convert one of the protons to a neutron (or vice versa). A=12, but Z=5 and N=7. That extra neutron cannot occupy the same top-level the system had before - it's already filled with neutrons. It must go one level higher, adding more energy to the total energy of the nucleus.

- For fixed A, it would seem that Z=N is the lowest energy.

Now, add back Coulomb repulsion. The lowest proton energy level starts higher than the lowest neutron energy level. Carrying out the same exercise, fixing A, we see that now the lowest energy situation is one where N>Z. This case is most concerning for large nuclei, where there is an un-canceled net Coulomb Repulsion. (Draw a bifurcated well, with an extra lowest energy state for neutrons that is NOT in common between the two sides of the well.

Summary: for small A, where the Coulomb repulsion is easily overcome, the most stable nuclei have $N \approx Z$. For larger A, where there is a net Coulomb repulsion, you need more neutrons to "glue" the nucleus together, and thus the most stable states are where $N > Z$.

Stability: The Experimental Truth

We have a model, and it makes some predictions. Let's put them to the test.

First of all, we need to figure out how to compute binding energy per nucleon.

DISCUSSION:
Do we have to go and measure the attraction between individual nucleons? How do we figure out the binding energy of the nucleus, and thus the

binding energy per nucleon?

GUIDE: what is a measure of the total internal energy of a system?

ANSWER: measure the mass of nuclei. Compare the mass to the sum of the masses of the constituents. If those differ, then there is binding between the components and that adds mass to the total. The size of that additional mass tells us about the strength of the binding.

Consider the deuteron: see slides.

- Imagine pulling the deuteron apart. Binding energy means we have to ADD energy to the system to break apart the deuteron, which necessarily INCREASES its mass. Is this true?

- Yes. The sum of the parts is 2.22 MeV/c^{2} heavier than the deuteron. We would have to add that much energy to break it apart.

Consider the hydrogen atom. How much energy does it take to remove the electron? 13.6 eV - that's the binding energy. One expects the mass of the Hydrogen atom to be correspondingly SMALLER than the sum of the masses of the proton and the electron, but the binding energy is SO SMALL compared to that of the strong force that it's a negligible contribution to the total mass.

It's useful to know that $m_H = m_p + m_e = 1.007825\text{ u}$, or that $1u \times c^2 = 931.5\text{MeV}$.

Binding Energy

A useful formula for calculating the binding energy is:

$$BE = \left(Zm_H + Nm_n - M_{{}^{A}_{Z}X} \right) c^2$$

where the first term is the total mass of electrons and protons in an atom with Z protons (hydrogen's mass is derived almost entirely from the sum of its constituents), the second term is the total mass of the neutrons, and the third term subtracts the atomic mass of the element, including all its protons, neutrons, electrons, and binding energy.

Comparing the Model to Experiment

See slides. Experimental measurements of the masses of stable elements shows that they do follow the expected trends. For small A, stability occurs when N = Z, and for large A when N>Z. The binding energy per nucleon rises to a maximum around elemental iron, an isotope of iron, and an isotope of nickel. It falls thereafter.

This is useful because if you want to liberate energy, you want to get the best bang for your buck. If your buck is binding energy, then the total binding energy of J nucleons arranged into nuclei of A nucleons each is (J/A)*BE, where BE is the total binding energy of each nucleus. We see that, re-grouping terms, for a fixed J you want to maximize J*(BE/A), which means maximizing BE/A, the binding energy per nucleon.

That's why we stress BE per nucleon SO MUCH. For energy applications, this is the quantity that matters, because releasing that energy means getting lots of energy for work.

Next Time

- Specific models of the nucleus - liquid drop and shell models
- Radioactive decay
- Applications of Nuclear Theory
- Harris: Ch. 11.3-11.6