Modern Physics (PHY 3305) <small>Lecture Notes</small>

Nuclear Physics: Fission and Fusion (11.7)

SteveSekula, 19 April 2010 (created 12 April 2010)

Review

no tags

- We completed our discussion of nuclear modeling with a discussion of the liquid drop and shell models
- We began discussing radioactivity

Radioactive Decay - More Detail

Last time, radioactivity was discussed in a cursory way. Today, we'll go into more depth and then discuss fission and fusion. The language of radioactive decay is to compute the quantity Q:

$$Q = (m_i - m_f)c^2$$

which represents the kinetic energy released by the nucleus when it undergoes a nuclear reaction.

- $Q > 0$: the reaction results in additional energy, which can go into the motion of final-state particles (daughter nuclei, the radiation particle)
- $Q < 0$: the reaction requires the input of energy, and is unfavorable under equilibrium conditions

Alpha Decay

As we discussed last time, this is the emission of a Helium nucleus (2p2n). It has $Z = N = 2$ and reduces each of the N and Z by an even number. The original nucleus is the parent, the resulting nucleus is the daughter, and the radiation is an alpha. This kind of decay is possible (and likely, when possible) because of the mass of the helium nucleus, for its size, is unusually far from the sum of the masses of its constituents. This makes such a unit very tightly bound, and likely to be ejected as a unit.

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Example: the decay of Uranium

Uranium is one of the most abundant radioactive isotopes in nature (U-238). It attains a
lower energy state by alpha decay. What is the daughter nucleus, and how much kinetic
energy is released?

Uranium-238 has Z=92 and A=238. Thus, N=238-92=146. When it emits an alpha, Z=92 -> Z=90
and N=146 -> N=144. Z=90 is Thorium, with mass number 234. Thus Uranium-238 becomes
Thorium-234.

What about the kinetic energy released? Calculating Q from the masses of Uranium,
Thorium, and the alpha:

Q = (238.050784u - (234.043593u + 4.002603u)) * 931.5 MeV/u = 4.27 MeV

These are the atomic masses of Uranium and Thorium, but since the same number of
electrons are present in both their masses are automatically subtracted.
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Beta Decay

As briefly mentioned last time, beta decay is the emission of an electron from the nucleus. The electron is not already present in the nucleus, so something must produce it. These are denoted "β^- particles."

The simplest beta decay is that of a FREE NEUTRON. While neutrons in a nucleus are relatively stable over long periods, a free neutron is slightly heavier than a proton and thus decays (within about 20 minutes). The decay is:

$$n \rightarrow p^+ + \beta^- + \bar{\nu}$$

To account for the change in nucleon number and charge, the beta is often written as: $^{0}_{-1}\beta^-$.

For instance, the decay of Boron-12 to Carbon 12 is denoted:

$$^{1}2_5 B \rightarrow^1 2_6 C +^{0}_{-1}\beta^- + \bar{n}u$$

For a particle with no electric charge, the anti-particle is denoted by drawing a "bar" over the top. The neutrino emitted in this decay is known to be an anti-neutrino. To calculate Q, we need to take into account that there are 5 electrons in Boron-12 and 6 in Carbon-12. We can write the Q

as:

$$Q = (M_i - (M_f - m_e) + m_e)c^2$$

Since we use ATOMIC MASSES in the calculation, and we have to subtract an additional electron mass from the final atomic mass to get the nuclear mass of the isotopes, we wind up canceling the beta mass and obtain:

$$Q_{\beta^-} = (M_i - M_f)c^2$$

This reaction has a Q of 13.4 *MeV*.

Complimentary to β^- decay is β^+ decay. This is the emission of an anti-matter electron - a positron! For instance:

$$^{12}_7N \rightarrow ^{12}_6C + ^0_{+1}\beta^+ + \nu$$
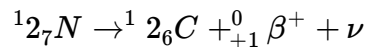
How does the Q calculation get affected here? In this case, Carbon-12 has 1 FEWER electron than Nitrogen-12. Instead of canceling, the atomic electron mass and the β^+ mass ADD:

$$Q_{\beta^+} = (M_i - M_f - 2m_e)c^2$$

One last form of electron-related nuclear reaction is the capture of an atomic electron by the nucleus, resulting in a loss of a neutron and the addition of a proton. This is possible because the electron is not at a well-defined radius from the nucleus - it's a wave, with some probability of being very close to or in the nucleus. Electron capture leads to reactions like:

$$^{11}_6C + ^0_{-1}\beta^- \rightarrow ^{11}_5B + \nu$$
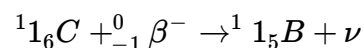
The Q-value depends only on the difference of the atomic masses:

$$Q_{capture} = (M_i - M_f)c^2$$

Decay Series

The goal of an unstable nucleus is to decay in such a way that it gets onto the curve of stability. This can lead to a sequence of decays ("branches") until finally the nucleus arrives at a stable place.

Gamma Decay

The emission of a photon from a nucleus occurs when it transitions from an excited state to a lower energy state. The energy of the photon is characteristic of the kind of nucleus (since the number of nucleons affects the spacing of energy levels), and serves as a nuclear fingerprint to help identify a nucleus.

Spontaneous Fission

In this case, a heavy nucleus splits into two nearly equal (intermediate) mass daughter nuclei. This results in extra neutrons which are not needed by either daughter and are so loosely bound that they are immediately freed. Spontaneous Fission results in the emission of neutrons, as well as the production of lighter nuclei.

The driving force behind fission is the desire to relieve the Coulomb repulsion. While splitting into to smaller nuclei RAISES the overall surface area of the system, it decreases the number of protons acting on one another in each nuclei. These two competing forces are in play, and help determine which fissions are most likely. Coulomb repulsion is relaxed and compensates for the increased strength of the surface term, ala the Liquid Drop Model.

The neutrons are emitted because intermediate mass nuclei are more stable when $N \approx Z$. Thus they shed the additional neutrons, releasing energy in the process.

Let's move on to a full-blown discussion of Nuclear Reactions.

Nuclear Reactions

We have all the pieces. We have discussed different forms of nuclear decay. We can now more generally discuss "Nuclear Reactions," which are *any*

occurrance in which nucleons are changed or exchanged between nuclei, much as electrons are exchanged in a chemical reaction.

- Radioactivity is a FORM of nuclear reaction, albeit a spontaneous one.
- Other reactions can be INDUCED by striking a nucleus with another particle

For instance,

$$^{12}_{5}B +^{1}_{0}n \rightarrow^{7}_{3}Li +^{4}_{2}He$$
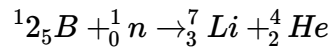
While boron-12 is stable, the addition of a neutron destabilizes the nucleus. It then tries to attain a lower energy state and breaks into two pieces. This reaction has Q=2.79MeV - it releases energy and thus results in an overall low-internal-energy situation. This is the nuclear equivalent of an EXOTHERMIC REACTION.

Are there ENDOTHERMIC REACTIONS? Yes. If the initial state nuclei already have kinetic energy, and their interaction can result in an overall lower-kinetic-energy state, they will do so. The above reaction can be reversed, and used as a source of mono-energetic neutrons.

The EXOTHERMIC reactions are of most interest, because they result in the release of energy as the nuclei attain an overall lower-mass state.

There are two ways to achieve this. Mass numbers near A=60 have the highest BE/nucleon, and thus represent states of the highest stability. Inducing heavier or lighter nuclei to move toward A=60 is this one way to RELEASE energy. There are two opposing ways, then, of releasing energy from nuclear reactions:
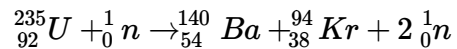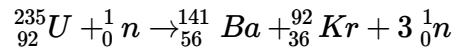
- Fission: the splitting of heavier nuclei to intermediate nuclei, leading to lower-energy nuclei and thus an exothermic reaction
- Fusion: the combination of lighter nuclei to form intermediate nuclei, again leading to a release of energy

Fission Reactions

The number of naturally occurring fission reactions is few. Oklo in Gabon is

a rare example of a natural, self-sustaining nuclear fission reaction (see slides).

Inducing fission is much more attractive. In this case, we stimulate the release of energy at a time of our choosing. Certain nuclei are particularly susceptible to this, e.g. Uranium-235:

$$\,^{235}_{92}U +\,^1_0 n \rightarrow\,^{141}_{56} Ba +\,^{92}_{36} Kr + 3\,^1_0 n$$

$$\,^{235}_{92}U +\,^1_0 n \rightarrow\,^{140}_{54} Ba +\,^{94}_{38} Kr + 2\,^1_0 n$$

$$\,^{235}_{92}U +\,^1_0 n \rightarrow\,^{132}_{50} Ba +\,^{101}_{42} Kr + 3\,^1_0 n$$

Not only does the addition of a neutron stimulate the fission of the Uranium nucleus; it results in more neutrons. Plutonium-239 is another notable example of such an easily stimulated decay. The subsequent isotopes are also unstable and decay further by a variety of means (beta decay, primarily). This makes it hard to exactly compute Q, but an estimate based on mass number changes suggests a per-nucleus release of about 1 *MeV*. Thus, 200 Uranium nuclei undergoing this process at the same time release 200 *MeV* of energy.

The process of MULTIPLE neutrons in the final state is fateful. If you get enough Uranium together in one place and induce a fission reaction, a CHAIN REACTION can occur in which neutrons freed from the nucleus induce further fission.

DISCUSSION:

- Two applications of this idea are a bomb and a reactor. What, do you think, is the difference between a nuclear bomb and a nuclear reactor?
    - density of the nuclear fuel - a bomb requires critical assembly of the fuel material
    - regulation of the neutrons - material can be introduced to absorb neutrons and slow the rate of reaction, preventing it from running away. Also, losses due to absorption by other nuclei (daughters, contaminants) poisons the reaction.

If a single neutron induces a reaction that produces n neutrons, and each subsequent neutron induces a reaction that also then produces n neutrons, the energy released by the jth generation of the reaction is:

$$E = E_0 n^j$$

It would EXPONENTIALLY INCREASE WITH TIME.

Impurities and effects at the surface of the material (losses of neutrons out of the solid) cause the reaction to slow. Therefore, a more relevant way of computing the energy released is to use the k factor:

$$E = E_0 k^j$$

- $k > 1$ leads to a self-sustaining reaction (a bomb)
- $k = 1$ means subsequent generations all release the same amount of energy (a reactor)
- $k < 1$ means energy decreases with each generation, extinguishing the chain reaction

Nuclear reactor moderators exist for several reasons:

- neutrons produced by Uranium fission are "fast" - they typically have KE in the 1 *MeV* range.
- Uranium nuclei more easily absorb SLOW neutrons
- A regulator slows neutrons produced from the fission
- Good choices for a regulator:
    - something that provides targets of roughly equal size to neutrons (neutrons or protons)
    - something that doesn't itself absorb neutrons
- Possibilities:
    - Water: unfortunately, hydrogen likes to capture neutrons. Still, this is a widely used regulator given how cheap it is.
    - Graphite, helium, "heavy water" - better choices, given their low capture rate for neutrons.

A "Breeder Reactor" benefits from the presence of U-238, which doesn't fission as easily as U-235. U-238 can absorb a neutron, yielding U-239. It then quickly decays via beta radiation (twice) to Pu-239, which then serves as fuel for the reactor.

Nuclear Fusion

If light nuclei are forced together, it's possible to get lower mass an increased kinetic energy. For example,

$$^2_1H + ^1_1H \rightarrow ^3_2He + \gamma$$
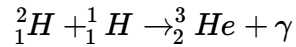
with $Q = 5.48 MeV$. This yields a gamma ray! This is called Fusion.

Fusion is the primary means by which stars produce energy. The most abundant element in stars is Hydrogen, and a series of fusion reactions, known as "the proton-proton cycle". leads inexorably toward a tightly bound Helium-4 nucleus. Each step releases energy. (see slides)
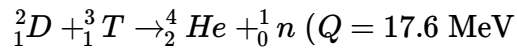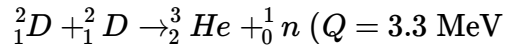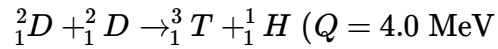
Heavier stars than ours also can have a "carbon cycle". involving the fusion of helium-4 to Beryllium-8, then fusion between Be-8 and He-4 to C-12. This begins the cycle, but given Be-8's tendency to decay quickly can only occur in sufficiently dense stars where the second fusion is more likely to happen quickly. The carbon is a catalyst for reactions down through nitrogen and oxygen to Helium-4 again, leaving excess Carbon-12 from the reaction.

As the temperature goes up, even heavier elements can be formed. We see that stars are FACTORIES for the very elements upon which planets and life are formed. We owe more than just our pleasant daily temperature to the stars.

While fusion is possible, it doesn't happen spontaneously, as does fission. Fusion requires you to first overcome the Coulomb repulsion between two nuclei, so you have to put in energy. That's why high density, high temperature situations are required before fusion can occur. The first unnatural fusion reaction was the hydrogen bomb (see slides for a demonstration of the Russian Tsar Hydrogen Bomb), in which an atomic bomb first creates a high-density, high-temperature environment for the subsequent fusion of Hydrogen fuel. Pound-for-pound, fusion delivers more energy then fission. This is simply because the binding energy per nucleon rises more steeply toward A=60 than it falls off after A=60.

Fusion Reactors

The greater the charge of the nuclei, the harder it is to generate fusion. You want to balance energy output with energy input to get the "biggest bang for your buck." The following reactions are so far the most promising:

$$_1^2D + _1^2D \rightarrow _1^3T + _1^1H \; (Q = 4.0 \text{ MeV}$$

$$_1^2D + _1^2D \rightarrow _2^3He + _0^1n \; (Q = 3.3 \text{ MeV}$$

$$_1^2D + _1^3T \rightarrow _2^4He + _0^1n \; (Q = 17.6 \text{ MeV}$$

The two *D-D* reactions occur with about equal probability. However, this doesn't negate the need for extreme temperature and density, which is what makes applied commercial fusion SO DIFFICULT (and so far impractical). Our sun is able to sustain fusion reactions at a temperature of "only" about 10^7K. In order to achieve that in the lab, you have to make sure the fusion reaction products do not contact material in the reactor, lest they be vaporized and contaminate the reaction. Isolation of the reaction from material is one of the hardest problems in fusion, and a few means to do this have been devised:

Magnetic Confinement

The fuel is ionized and placed in a magnetic field that keeps it confined to a small volume while it moves through the field. Non-uniformities in the field, however, lead to diffusion of the fusion fuel to the outer edges of the field, and eventually to loss. Another magnetic field is used to correct for this effect. There are several running and planned experiments using this technology. ITER is the most notable, given it's status as a global collaboration (see slides). It's goal is to achieve and SUSTAIN 9which is the key) a power amplification of 5 - that is, a 5:1 ratio of energy out to energy in. It may wind up being the first successful fusion reactor, albeit still an experimental one.

Inertial Confinement

Here, you apply force to the fusion material uniformly in all directions, so that it has no chance to escape. Lasers are a favorite tool for this procedure.

Summary