Modern Physics (PHY 3305) Lecture Notes

Particles and Interactions: Antiparticles and Conservation Laws

SteveSekula, 27 April 2010 (created 27 April 2010)

no tags

Review

- We discussed the "brief history of human understanding of the universe" - at least, at the level of its building blocks
- we discussed the nature of forces - forces are transmitted by particles exchanged between two other particles. The uncertainty principle allows for such exchanged particles to have arbitrary energy/mass as long as they live a very short time. This is part of what defines the range of a force - the exchange of pions between nucleons was predicted to explain the short, strong nature of nucleon binding.
- we took a look at some of the technology of modern particle physics

Let's conclude the new material in the course with a look at some specific interesting topics:

- antimatter/antiparticles
- conservation laws
- cosmology

Antiparticles

Before we can discuss in detail the fundamental building blocks of the universe, and the fundamental forces that weave them together, we need to discuss antiparticles. What are they?

Antiparticles are simply partners of particles, like the electron, whose properties appear to be all the same with the exception of the reversed electron charge. The positron, for example, has the same mass and spin as the electron but carries positive electric charge.

When particles and antiparticles meet, pair annihilation can occur and leads to the production of pairs of photons. Mass energy is converted to photon energy. You might think that the antineutron has no properties that distinguish it from its antiparticle, but antineutrons have been produced and do, in fact, annihilate with neutrons. This is explained by the fact that the neutron and proton are not fundamental, and the underlying building blocks - the quarks - do have distinct opposing properties from their antiparticles.

What is the theoretical basis for antiparticles? In fact, they were theorized before they were discovered. In order to predict antiparticles, you have to do something that physicists tried to do in the 1920s: marry special relativity and quantum mechanics.

The quantum mechanics we've been discussing, embodied in the SWE, is *NON-RELATIVISTIC*. This, you recall, is encoded in the free-particle formula:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

Which we can short-hand as:

$$K\hat{E}\Psi(x,t) = \hat{E}\Psi(x,t)$$

by recognizing that the derviatives are OPERATORS that act on the wave function and RETURN THESE PROPERTIES OF KINETIC AND TOTAL ENERGY. The SWE is fundamentally an expression of energy conservation, albeit *classical energy conservation*.

Discussion: Energy Conservation and the Relativistic SWE

What is the correct form for full-blown energy conservation? Consider relativity.

Answer:

$$p^2c^2 + m^2c^4 = E^2$$

How might we get at the Relativistic SWE (RSWE)?

Try the operator form:

$$\hat{p}^2c^2\Psi(x,t) + m^2c^4\Psi(x,t) = \hat{E}^2\Psi(x,t)$$

which we can expand into:

$$-c^2\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + m^2c^4\Psi(x,t) = -i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

This is the ***Klein-Gordon*** **Equation**. It has been shown to correctly predict the behavior of spin-less (spin-0) particles at all speeds. It was Paul Dirac who then recognized that spin had to be added to this equation in order to describe useful things like electrons.

In non-rel. QM, we treat spin as a separate property that we add to the wave function but which is not dealt with in the SWE. However, in the Dirac Equation, spin is directly included in the wave function from the beginning. This allows rel. QM to treat electrons at all speeds. The study of the Dirac Equation is in the domain of a future physics course, but we can discuss its implications.

The main difficulty with basing a relativistic matter wave equation on the *Klein-Gordon* equation is that energy appears *squared*, which means that negative energies are allowed by the equation. Either positive or negative energy solutions equally well satisfy this equation.

Another problem with the solutions to the *K-G* equation is that the probability density is not conserved over a volume - the particle can appear or disappear! It takes work, but you CAN form a probability density that doesn't change in time, but then a new problem arises: you can't interpret it as a probability density since it can be either a positive or a negative number.

A conspicuous feature here is that the probability density is correlated with the sign of the energy. Positive energy solutions correspond to one sign of the density, negative energy to the other. Instead, it was interpreted not as a probability density, but a CHARGE DENSITY, which can change sign. This led to an interpretation of the Dirac or *K-G* equations that then predicted the existence of antiparticles.

The charge density must be constant over all space and time, but its sign need not be positive. A negative energy solution, then, describes a charge density whose mass-properties are equal to those of the positive energy solution, but whose charge density sign is opposite that of the positive energy solution. This is an antiparticle.

One interpretation of this picture is to imagine that the universe is like a potential well with closely-spaced energy levels. An electron can only drop into a lower-energy state if one is vacant - a hole, or an "antiparticle". When the hole and the electron meet, energy is released - just like a light-emitting diode. The hole is a state of negative energy, and when the particle falls into it it ceases to participate in interactions (and the hole disappears) and light is emitted.

Another interpretation, which Harris doesn't discuss, is actually a more modern view of these states and was proposed by Stuckelberg and Feynman (in the 1940s). Free particle solutions have a time component that looks like:

$$\Psi(t) \propto e^{-i\omega t} = Ae^{-i(E/\hbar)t}$$

This state has total energy +E.

Imagine a positive energy particle - an electron - that instead of going forward in time goes backward in time:

$$\Psi(t) \propto e^{-i\omega t} = Ae^{-i(E/\hbar)(-t)}$$

Now you get a negative energy solution. This is the *Feynman-Stuckelberg* interpretation of negative energy states: as positive energy states traveling backward in time. This is not the only moment when the idea of "things going backward in time" appears in fundamental particle physics. In fact, time reversal is a common topic of investigation in particle physics.

Conservation Laws

We are used to energy and momentum conservation, as well as conservation of angular momentum. Since spin is a form of internal angular momentum, that, too, is part of the whole conservation of angular momentum. For instance, it's possible for a spin-1 particle to decay to two spin-0 particles, as long as the two final-state particles have ORBITAL ANGULAR MOMENTUM (L=1). Total angular momentum must be conserved.

When we study nature, we look for "rules" that appear to be followed, and from those rules we try to build a picture of the laws of nature. However, those rules are not always follows. Conservation of mass was believed to be another such rule, but it is routinely violated (consider nuclear decay, where the mass of the final state is less than that of the initial state). Instead, thanks to relativity, we view mass as just one more form of energy, and we consider the whole of energy conservation. To date, no violaton of energy or momentum conservation has been observed.

But, there are other rules that nature seems to follow which may or may not stand up to experimental tests.

- Color: *the charge of the strong force, "color," is always conserved.* If a red quark emits a gluon, that gluon can change the color charge of the quark to green, and the gluon itself carries color-imbalance ($r\bar{g}$) which makes it interact aggressively or fragment into other quarks rapidly. A colorless initial state (rgb) will always lead to a colorless final state. All mesons and baryons are colorless states.
- Lepton number: *the number of leptons in any reaction is conserved, and each generation has its OWN lepton number.*
    - for instance, the electron and the electron neutrino have electron number; the muon and muon neutrino have muon number; the tau and tau neutrino have tau number. Leptons have +1, anti-leptons have -1, and non-leptons have 0. The following reaction conserves tau and electron number:

      $$\tau^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\tau$$

      and is realized in nature. This reaction,

      $$\tau^+ \rightarrow \mu^+ \gamma$$

conserves things like charge and energy/momentum but NOT lepton number, and is said to be a "lepton number violating process." A theory of physics beyond the Standard Model of Particle Physics, called Supersymmetry, actually predicts that this process can occur for up to about 1 in every billion tau lepton decays. We have not achieved the sensitivity needed to test this yet. Until 1998, lepton number was believed to be always conserved; however, the confirmation of neutrino oscillation ($\nu_e \to \nu_\mu$, for instance) by studying neutrinos from the sun and cosmic rays showed that lepton number is not conserved (although to date the effect is tiny).

- Baryon number: *the number of baryons (particles containing three quarks) is conserved.* Baryons have +1, anti-baryons have -1, and non-baryons (including mesons - pairs of quarks) have 0. To date, no process has been observed that violates conservation of baryon number. For instance, neutron beta decay,

$$n \to p^+ + e^- + \bar{\nu}_e$$

conserves both baryon and lepton number.

A key goal of many searches in physics are to think of processes that can violate these conservation rules and actively search for them. An observation would be a complete breakthrough in our understanding of the universe. A non-observation puts strong constraints on the possible hypotheses that can explain our universe (especially if such a process is a direct prediction of the hypothesis).

Symmetries

Some of the in-class presentations talked about things that are "asymmetric" - that is, there is clearly a place where it is and where it is not. Time is a good example. It appears to only (dominantly) move in one direction, and in general macroscopic processes are not REVERSIBLE until time. What about microscopic processes?

Symmetries are part of the modern language of physics. They came into power in the 1960s, when symmetries between different quantum numbers were identifed and used to predict the existence of new mesons and baryons. Those predictions were born out, and from these approaches eventually the quarks and gluons were proposed as an explanation of the "particle explosion" of the 1930s-1960s. Some of these symmetries are connected to physical properties with which humans are familiar, and we'll focus on those.

- Parity: *symmetry under a parity transformation means that a process is the same whether the coordinates are defined in a right-handed sense (x,y,z) or a left-handed sense (-x,-y,-z).*
  - Angular momentum: consider an angular momentum in an x,y,z coordinate system. Flip the signs of all the coordinates. Is the rotation direction the same or reversed (remember, in the parity-transformed universe you will consider the z direction to point in the opposite sense of the non-parity-transformed universe). From this consideration, we see that angular momentum is UNCHANGED under a parity transformation (consider the angular momentum equation, $\vec{L} = \vec{r} \times \vec{p}$. Spin is a form of angular momentum, so its direction is UNCHANGED under parity transformation. Linear momentum IS reversed under parity transformation.
  - Consider beta decay of a nucleus. Show that under parity this process is NOT invariant - it doesn't occur at the same rate if the coordinates (z-axis) are inverted, according to experiment.
- Charge conjugation: *symmetry under charge conjugation means that the process is unchanged if you change all particles to anti-particles, and all anti-particles to particles.*
  - Processes involving the decay of weak bosons to final states containing particles, or their anti-particle counterparts, have been studied and shown to NOT be symmetric. However, similar processes that result from photon or gluon processes are unchanged. We say that the weak force is not symmetric under charge conjugation.
  - You can combine symmetries. For instance, if you parity-invert and charge-conjugate, many weak processes are symmetric under that COMBINED transformation. However, in the 1960s we saw violations of CP symmetry that, to this day, are still being scrutinized. CP violation is a necessary, but not sufficient, ingredient in understanding why our universe appears to be dominated by matter and not equally by matter and anti-matter.
- Time: *symmetry under time reversal means that a process happens at the same rate whether time runs forward or backward.*
  - For the most part, processes are invariant under time reversal. There is a theorem that is the basis of quantum field theory that states that all processes are invariant under the COMBINED CPT transformation. To date, no violation of CPT symmetry has been

observed. This implies that processes that violate CP symmetry violate also T symmetry (but conserve CPT). That means that *CP-violating* processes are automatically time-violating. This is also an intriguing thing which continues to be studied.

Cosmology and the Beginning of the Universe.

Discuss current observations of the cosmos, including expansion (supernova), the CMB, motion under gravity, etc. Show that the matter we know about (quarks and leptons) appear to make up less than 5% of the total energy of the universe. Motivate dark matter and dark energy. Motivate colliders are 'time machines" that recreate high-energy, high-density conditions of the early universe, and motivate the LHC and Tevatron as machines that take us back to about 1 picosecond after the big bang.