THE DISCOVERY OF SPIN

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Supplementary Material for PHY 3305 (Modern Physics)

Harris, Ch. 8.1-8.3

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- The spectral mystery
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REVIEW

- We discussed learning from the wave function
  - expectation values
  - operators
- We applied lessons from the infinite well and harmonic oscillator to new problems:
  - barriers
  - tunneling
- We discussed applications of matter in motion
  - scanning tunneling microscope
  - tunnel diode and SQUID
  - nuclear decay

"BALMER LINES"

434 nm

434 nm

486 nm

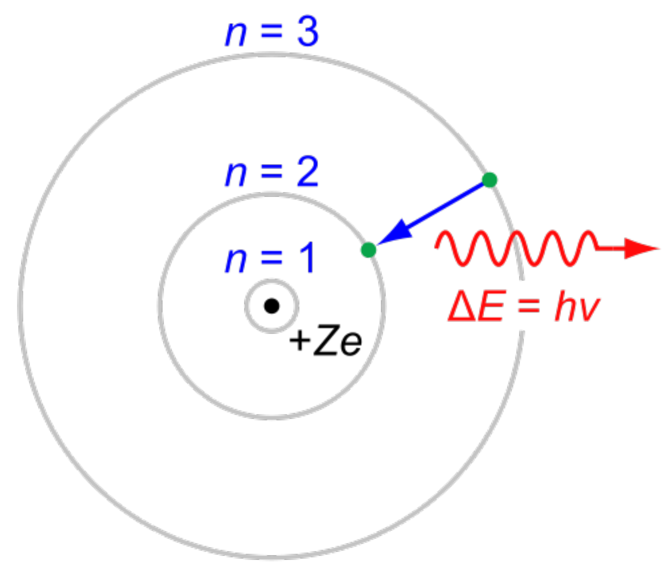
656nm

Balmer's empirical relationship:

$$\frac{1}{\lambda} = 1.097 \times 10^{7}\, m^{-1}\left(1/4 - 1/n^{2}\right)$$

we can write:

$$E = \frac{hc}{\lambda} = (13.6\ \text{eV})\left(1/4 - 1/n^{2}\right)$$

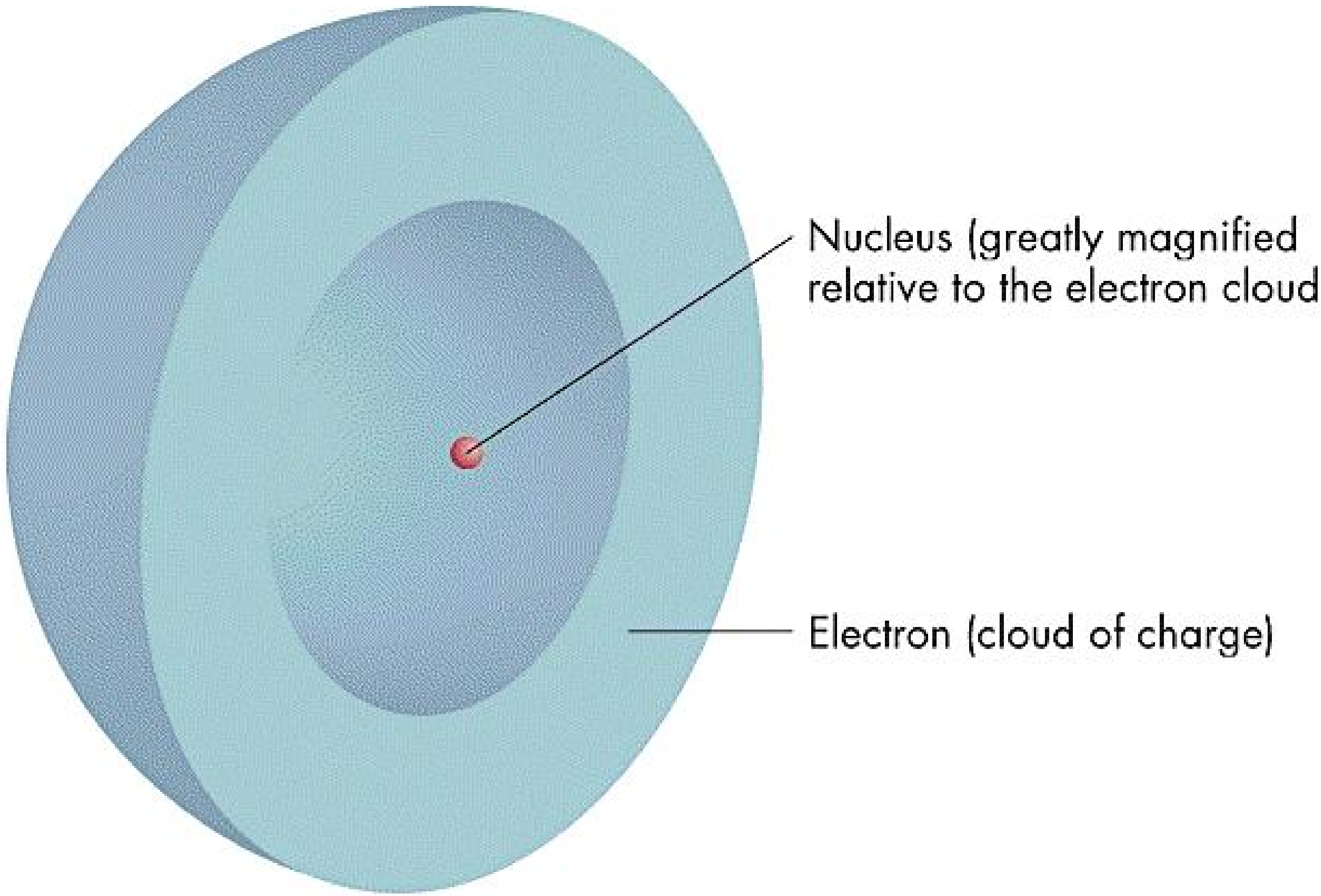


Simplified model of hydrogen (ala Ernest Rutherford)

THE SHROEDINGER WAVE EQUATION

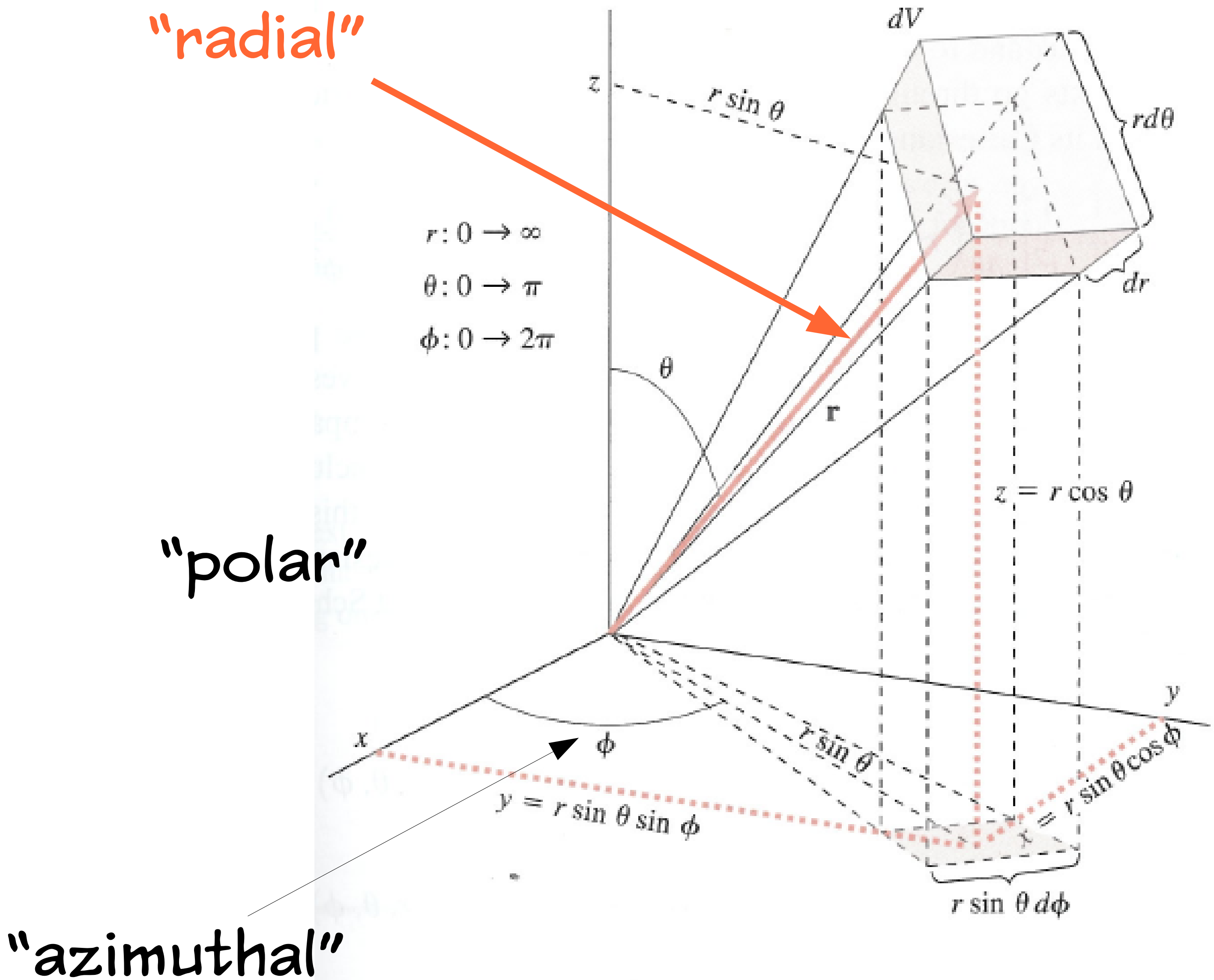
$$\frac{-\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2}+U(x)\Psi(x,t)=i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

THE HYDROGEN ATOM



Cross section of a hydrogen atom

SPHERICAL POLAR COORDINATES



THE 3-D SWE

$$\frac{-\hbar^2}{2m}\vec{\nabla}^2\Psi(r,\theta,\phi,t)+U(r)\Psi(r,\theta,\phi,t)=i\hbar\frac{\partial\Psi(r,\theta,\phi,t)}{\partial t}$$

$$\vec{\nabla}=\left(\frac{\partial}{\partial r},\frac{\partial}{\partial\theta},\frac{\partial}{\partial\phi}\right)$$

The Hydrogen Potential

$$U(r) = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

NEW QUANTIZATIONS

- One-dimensional problems have one quantum number (e.g. "n")
- 3-D problems need three quantum numbers:
  - (n, l, m_l)
  - Think of them as "radial", "polar", and "azimuthal"
  - Total energy (n), total angular momentum (l), and angular momentum along the z-direction (m_l) are all quantized in the atom

NEW QUANTIZATION: AZIMUTHAL ANGLE, φ

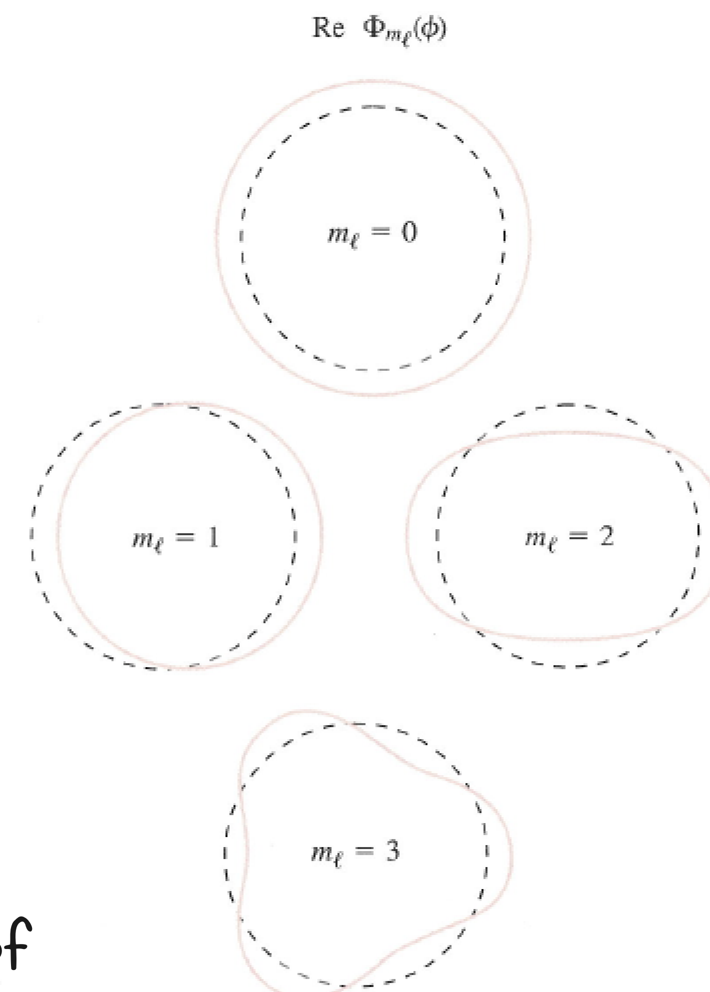
$$\frac{\partial^2}{\partial \phi^2}\Phi(\phi) = -D\,\Phi(\phi)$$

$$\Phi(\phi) = e^{i m_l \phi}$$

$m_l = 0, \pm 1, \pm 2, \ldots$

$$m_l \hbar = L_z$$

Quantization of angular momentum along the z-axis



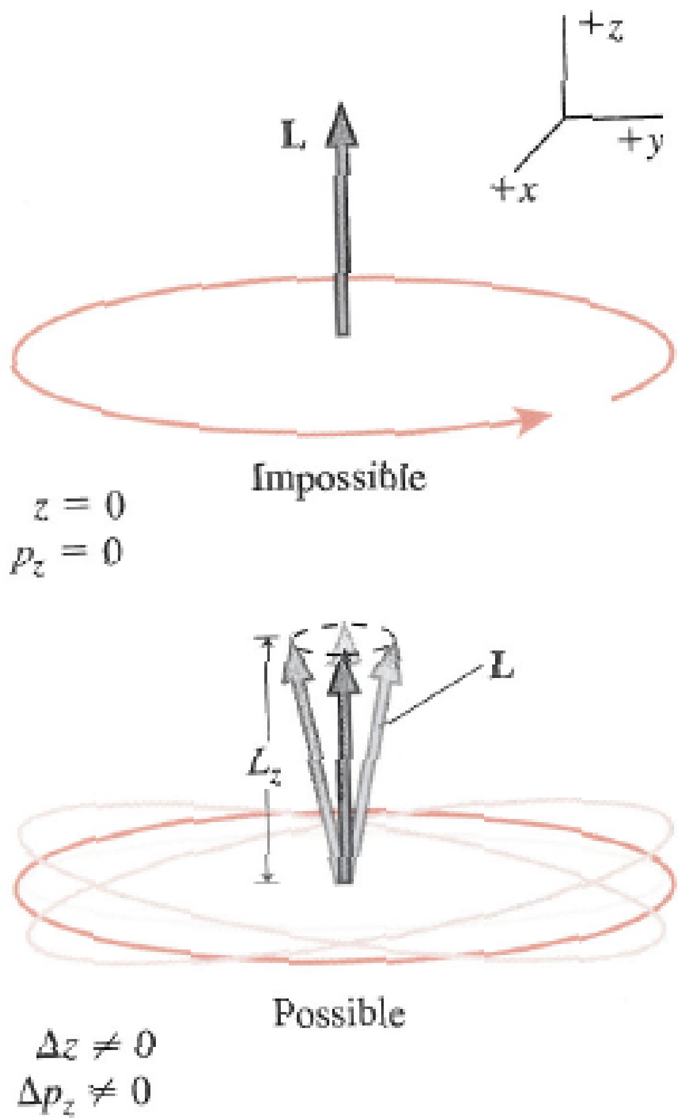
New Quantization: Polar Angle, θ

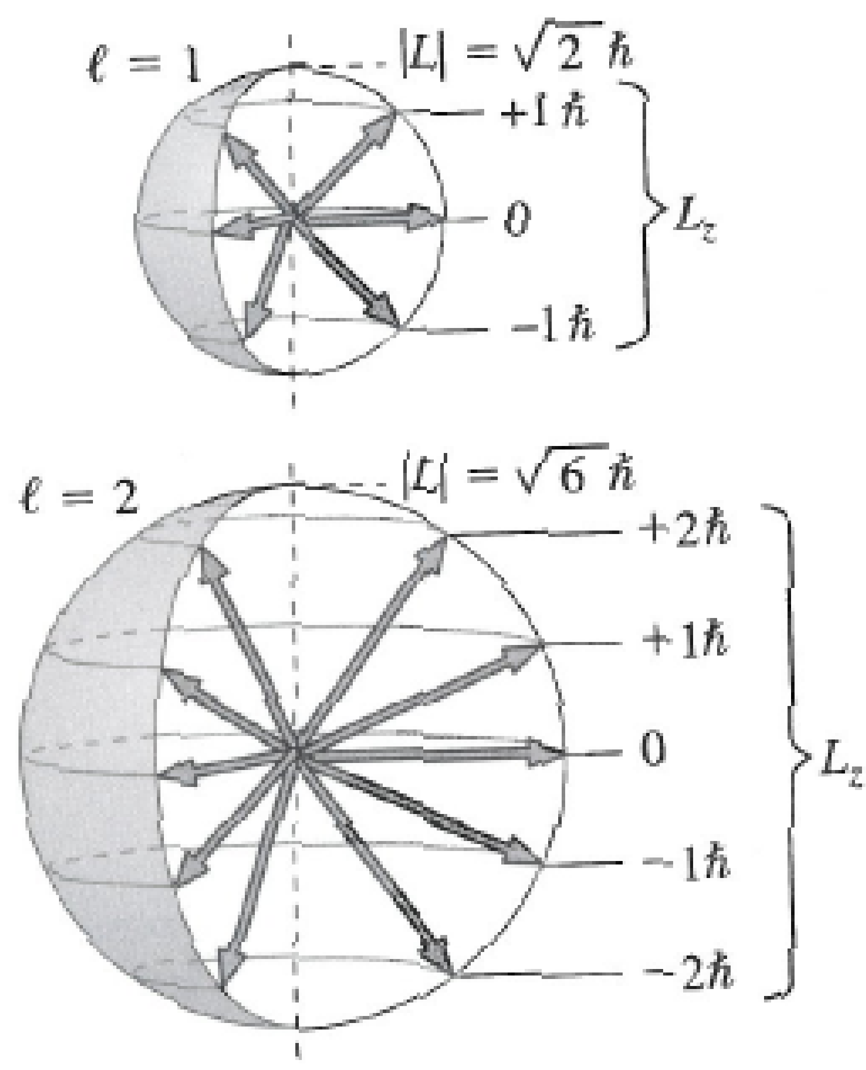
- Considerations of the polar-angle-only SWE leads to:

$$|L| = \sqrt{l(l+1)}\hbar$$

- Only certain TOTAL angular momenta are allowed
  - quantization of total angular momentum
  - also: $L_z \leq L$, so $m_l = 0, \pm 1, \pm 2, \ldots, \pm l$

VISUALIZING L, L_z QUANTIZATION



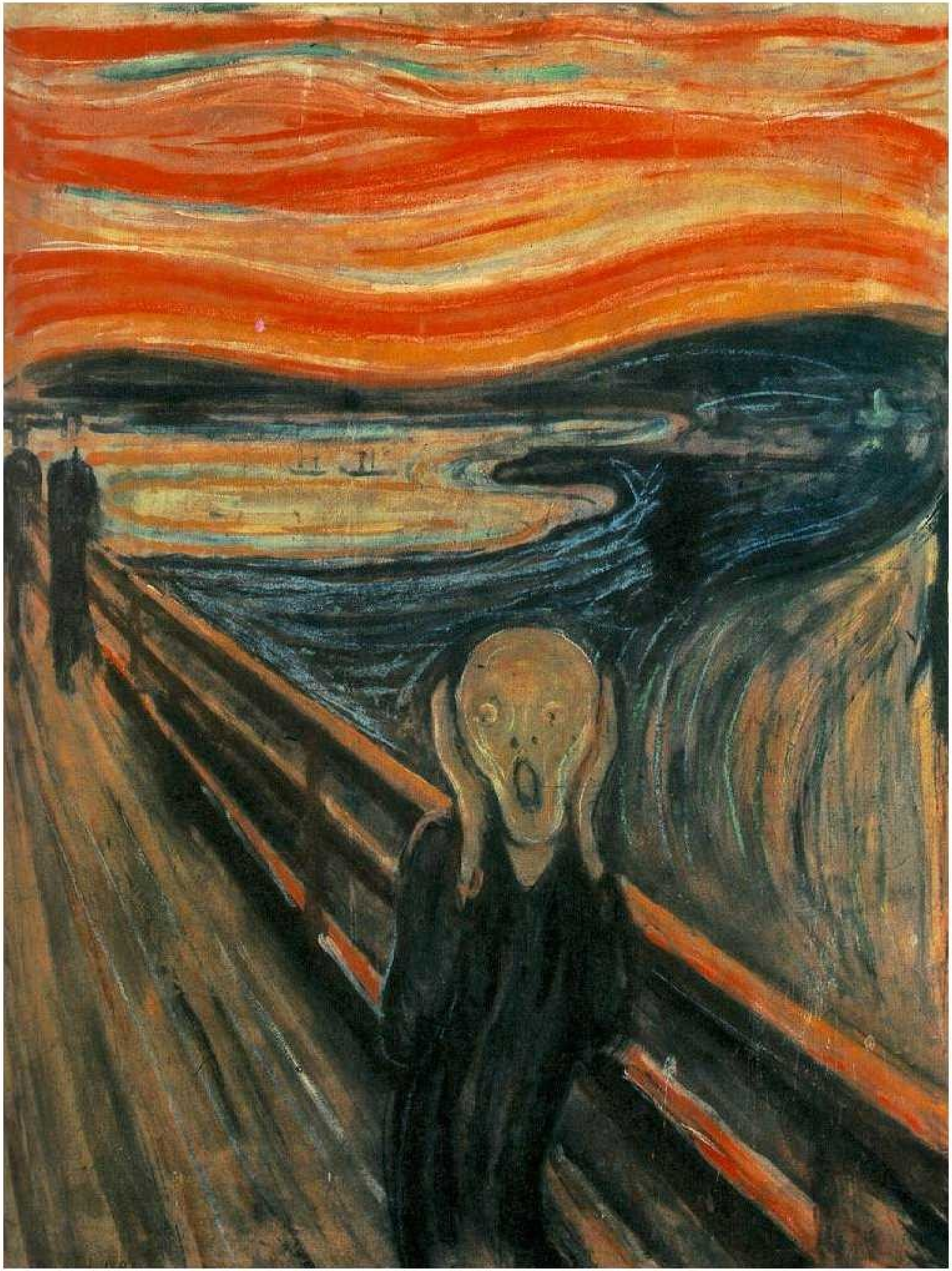


WAVE FUNCTIONS (SOLUTIONS)

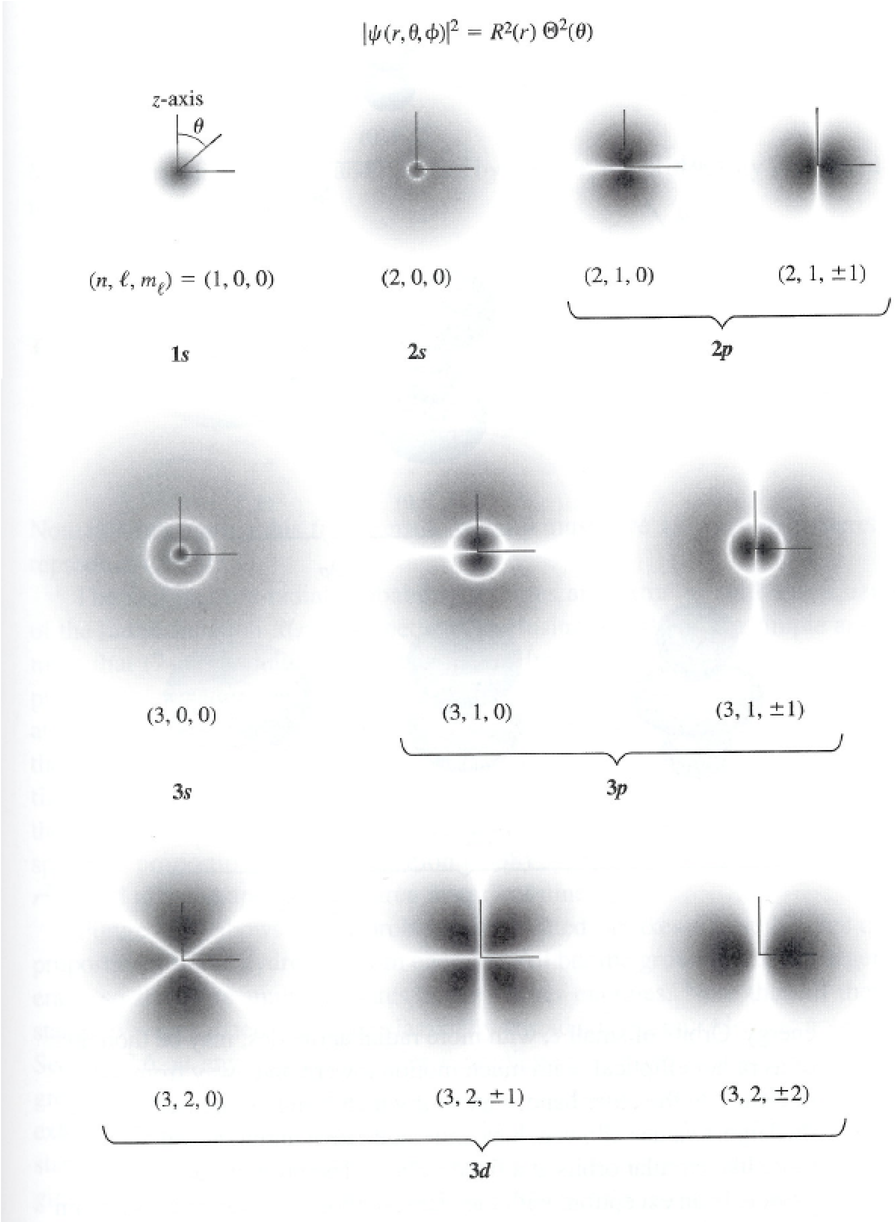
$$\psi_{n\ell m}(r,\vartheta,\varphi) = \sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-\ell-1)!}{2n(n+\ell)!}} e^{-\rho/2} \rho^\ell L_{n-\ell-1}^{2\ell+1}(\rho) \cdot Y_\ell^m(\vartheta,\varphi)$$

General Laguerre Polynomials (all radial)

Spherical Harmonics (all angular)



HYDROGEN ORBITALS (2-D)



Energy Levels

$$E_n = \frac{m_e e^4}{32\pi^2 \varepsilon_0^2 \hbar^2} \frac{1}{n^2} = 13.6\,\text{eV}\,\frac{1}{n^2}$$

(n = 1, 2, 3, ...)

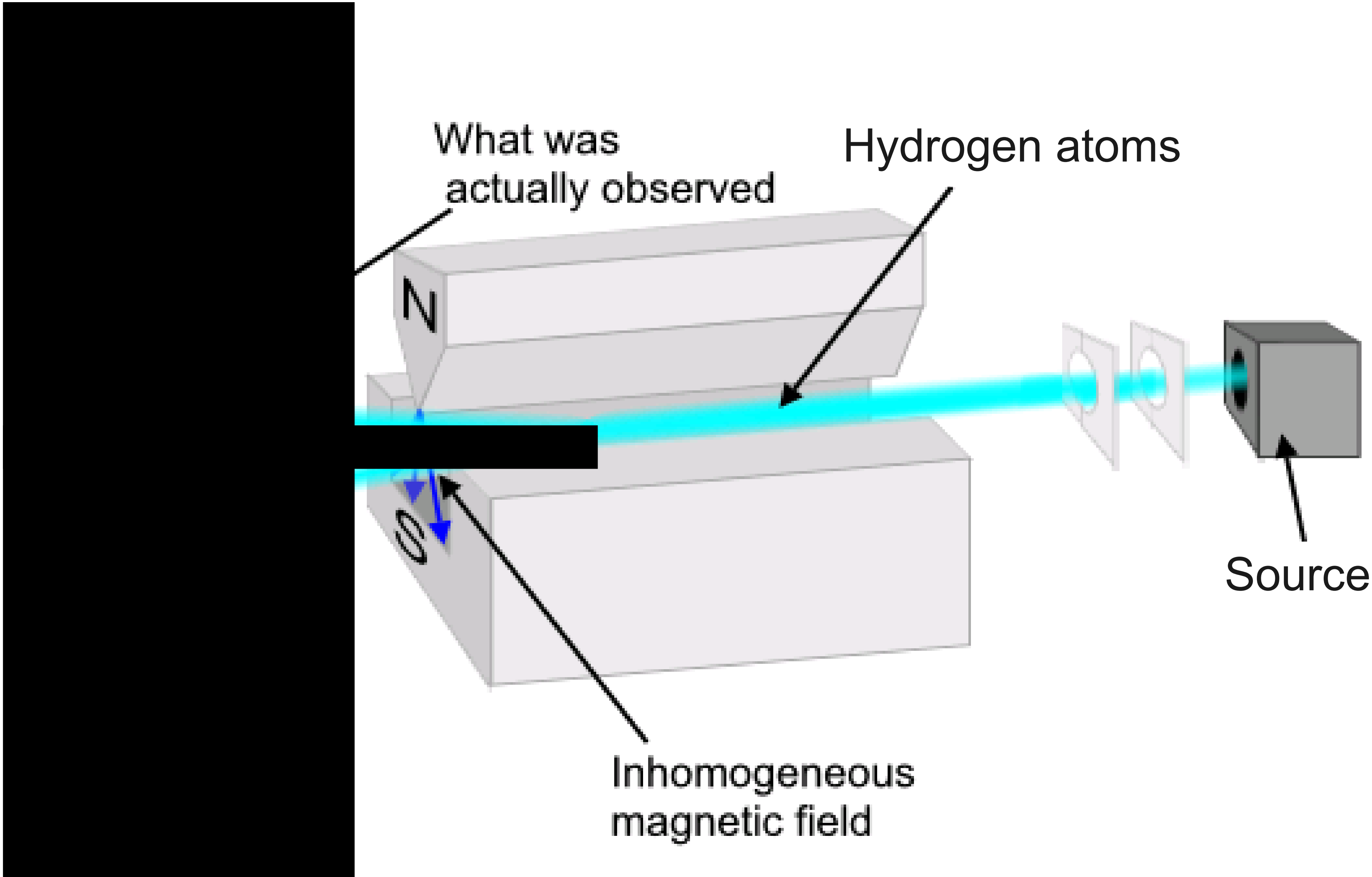
PROVE IT

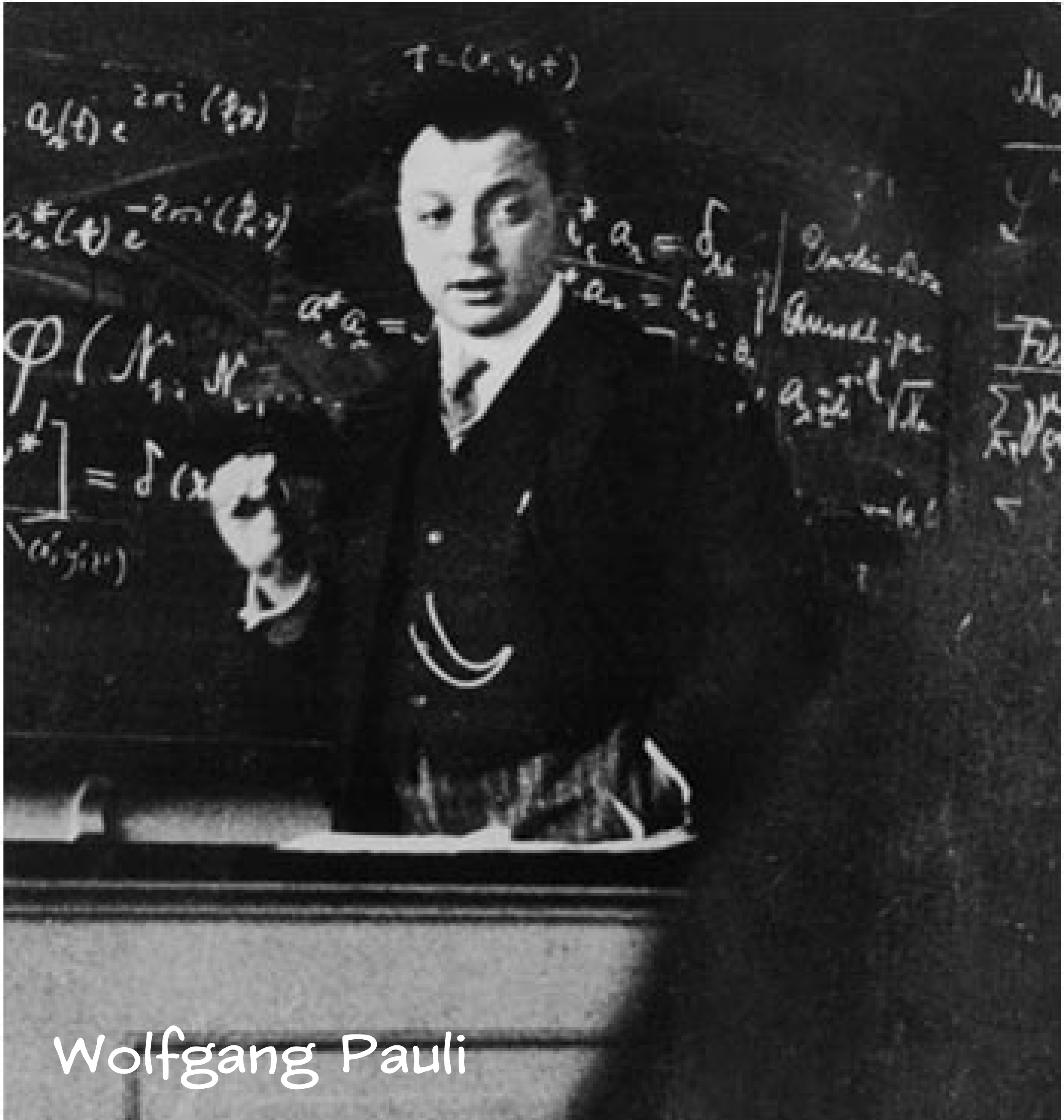
- Quantization of angular momentum was a new concept
- Prove it! Prove that it's quantized!
    - The Stern-Gerlach Experiment

Magnetism and Angular Momentum

- Consider a loop of current
  - a single electron going in a circle
  - what is the "magnetic moment" (a susceptibility to magnetic force)?
  - consider a dipole in a magnetic field
    - consider what happens to the ground state of hydrogen in a field

STERN-GERLACH EXPERIMENT





Wolfgang Pauli

ROADMAP

- Statistical Mechanics
  - or, "what happens when a bunch of particles do stuff"
- Solid-state physics
  - quantum mechanics and the structure of atomic matter
- Nuclear physics
  - quantum mechanics and the structure of the atomic nucleus
- Particle physics
  - quantum mechanics, relativity, and the fundamental structure of the universe

NEXT TIME

- Statistical Mechanics
  - Probabilities and Thermodynamics
  - The Boltzmann Distribution
- Reading: Harris Ch. 9.1-9.3