THE DISCOVERY
OF SPIN

Prof. Stephen Sekula
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Supplementary Material for
PHY 3305 (Modern Physics)
Harris, Ch. 8.1-8.3
TABLE OF CONTENTS

- Presentation
  - guidelines
  - ideas and how to have them
- Review of last class
- The spectral mystery
- The hydrogen atom in 15 min.
- New Quantizations
- The Stern-Gerlach Experiment
- The Exclusion Principle
We discussed learning from the wave function
  - expectation values
  - operators
We applied lessons from the infinite well and harmonic oscillator to new problems:
  - barriers
  - tunneling
We discussed applications of matter in motion
  - scanning tunneling microscope
  - tunnel diode and SQUID
  - nuclear decay
"Balmer Lines"

Balmer's empirical relationship:

\[
\frac{1}{\lambda} = 1.097 \times 10^7 \text{ m}^{-1} \left( \frac{1}{4} - \frac{1}{n^2} \right)
\]

we can write:

\[
E = \frac{hc}{\lambda} = (13.6 \text{ eV}) \left( \frac{1}{4} - \frac{1}{n^2} \right)
\]
THE SHROEDINGER WAVE EQUATION

\[
-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + U(x) \Psi(x,t) = i \hbar \frac{\partial \Psi(x,t)}{\partial t}
\]
THE HYDROGEN ATOM

Cross section of a hydrogen atom

Nucleus (greatly magnified relative to the electron cloud)

Electron (cloud of charge)
Spherical polar coordinates

"radial"

"polar"

"azimuthal"
THE 3-D SWE

\[-\hbar^2 \frac{\nabla^2}{2m} \Psi(r, \theta, \phi, t) + U(r) \Psi(r, \theta, \phi, t) = i\hbar \frac{\partial \Psi(r, \theta, \phi, t)}{\partial t}\]

\[\vec{\nabla} = \nabla = \begin{pmatrix} \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \end{pmatrix} \]
THE HYDROGEN POTENTIAL

\[ U(r) = -\frac{1}{4\pi \varepsilon_0} \frac{e^2}{r} \]
NEW QUANTIZATIONS

- One-dimensional problems have one quantum number (e.g. “n”)
- 3-D problems need three quantum numbers:
  - \((n, l, m_l)\)
  - Think of them as “radial”, “polar”, and “azimuthal”
  - Total energy \((n)\), total angular momentum \((l)\), and angular momentum along the z-direction \((m_l)\) are all quantized in the atom
NEW QUANTIZATION: AZIMUTHAL ANGLE, $\phi$

\[
\frac{\partial^2}{\partial \phi^2} \Phi(\phi) = -D \Phi(\phi)
\]

\[
\Phi(\phi) = e^{i m_l \phi}
\]

$m_l = 0, \pm 1, \pm 2, \ldots$

$m_l \hbar = L_z$

Quantization of angular momentum along the $z$-axis
NEW QUANTIZATION: POLAR ANGLE, $\theta$

- Considerations of the polar-angle-only SWE leads to:

  - Only certain TOTAL angular momenta are allowed
    - quantization of total angular momentum
    - also: $L_z \leq L$, so $m_l = 0, \pm1, \pm2, \ldots, \pm l$

\[ |L| = \sqrt{l(l+1)}\hbar \]
VISUALIZING $L,L_z$ QUANTIZATION

For $\ell = 1$:

- $|L| = \sqrt{2} \hbar$
- $L_z = \{ +1 \hbar, 0, -1 \hbar \}$

For $\ell = 2$:

- $|L| = \sqrt{6} \hbar$
- $L_z = \{ +2 \hbar, +1 \hbar, 0, -1 \hbar, -2 \hbar \}$

Possible states:

- $\Delta z \neq 0$
- $\Delta p_z \neq 0$

Impossible state:

- $z = 0$
- $p_z = 0$
**WAVE FUNCTIONS (SOLUTIONS)**

\[ \psi_{n\ell m}(r, \vartheta, \varphi) = \sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n - \ell - 1)!}{2n(n + \ell)!}} e^{-\rho/2} \rho^\ell L_{n-\ell-1}^{2\ell+1}(\rho) \cdot Y_\ell^m(\vartheta, \varphi) \]
HYDROGEN ORBITALS
(2-D)

\[ |\psi(r, \theta, \phi)|^2 = R^2(r) \Theta^2(\theta) \]

\[(n, \ell, m_\ell) = (1, 0, 0)\]

1s

\[(n, \ell, m_\ell) = (2, 0, 0)\]

2s

\[(n, \ell, m_\ell) = (2, 1, 0)\]

2p

\[(n, \ell, m_\ell) = (2, 1, \pm 1)\]

\[(n, \ell, m_\ell) = (3, 0, 0)\]

3s

\[(n, \ell, m_\ell) = (3, 1, 0)\]

3p

\[(n, \ell, m_\ell) = (3, 1, \pm 1)\]

\[(n, \ell, m_\ell) = (3, 2, 0)\]

3d

\[(n, \ell, m_\ell) = (3, 2, \pm 1)\]

\[(n, \ell, m_\ell) = (3, 2, \pm 2)\]
ENERGY LEVELS

\[ E_n = \frac{m_e e^4}{32 \pi^2 \varepsilon_0^2 \hbar^2} \frac{1}{n^2} = 13.6 \text{eV} \frac{1}{n^2} \]

\((n = 1, 2, 3, \ldots)\)
PROVE IT

- Quantization of angular momentum was a new concept
- Prove it! Prove that it's quantized!
  - The Stern-Gerlach Experiment
MAGNETISM AND ANGULAR MOMENTUM

- Consider a loop of current
  - a single electron going in a circle
  - what is the “magnetic moment” (a susceptibility to magnetic force)?
  - consider a dipole in a magnetic field
    - consider what happens to the ground state of hydrogen in a field
Stern-Gerlach Experiment

What was actually observed

Inhomogeneous magnetic field

Hydrogen atoms

Source
ROADMAP

- **Statistical Mechanics**
  - or, “what happens when a bunch of particles do stuff”
- **Solid-state physics**
  - quantum mechanics and the structure of atomic matter
- **Nuclear physics**
  - quantum mechanics and the structure of the atomic nucleus
- **Particle physics**
  - quantum mechanics, relativity, and the fundamental structure of the universe
NEXT TIME

• Statistical Mechanics
  • Probabilities and Thermodynamics
  • The Boltzmann Distribution

• Reading: Harris Ch. 9.1-9.3