THINK BIG: STATISTICAL MECHANICS II

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(3/18/2010)
Supplementary Material for
PHY 3305 (Modern Physics)
Harris, Ch. 9.3,9.5-9.7

TABLE OF CONTENTS

- Reminders
- Review of last class

REMINDER

- In-class presentation topics:
  - Deadline: FRIDAY!
  - Consult with me and inform me of your ideas

REVIEW

- Probability - definitions
  - Consider independent (uncorrelated/non-interacting) objects
  - Counting
    - how many unique ways to arrange N objects in N spaces?
      - ANSWER: $N! = N(N-1)(N-2)...1$
    - how many unique ways to arrange N objects in N spaces when some of the spaces are "grouped" and their internal order doesn't matter?

PARK YOUR CAR, WIN FREE COUPONS!

PARK YOUR CAR, WIN FREE COUPONS!

THE QUESTION

- How many ways are there to arrange N cars such that the same N_i cars get free sub coupons without affecting the other $(N-N_i)$ cars?

AN EXAMPLE: 4 SPOTS

- Arrange 4 cars in 4 spots, where 3 of the spots get free food coupons

(free food)

1	2	3	4
1	3	2	4
3	1	2	4
2	1	3	4
2	3	1	4
3	2	1	4

+

18 more, cycling cars 1,2,3 through the fourth parking spot

AN EXAMPLE: 4 SPOTS

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1	2	3	4
1	3	2	4
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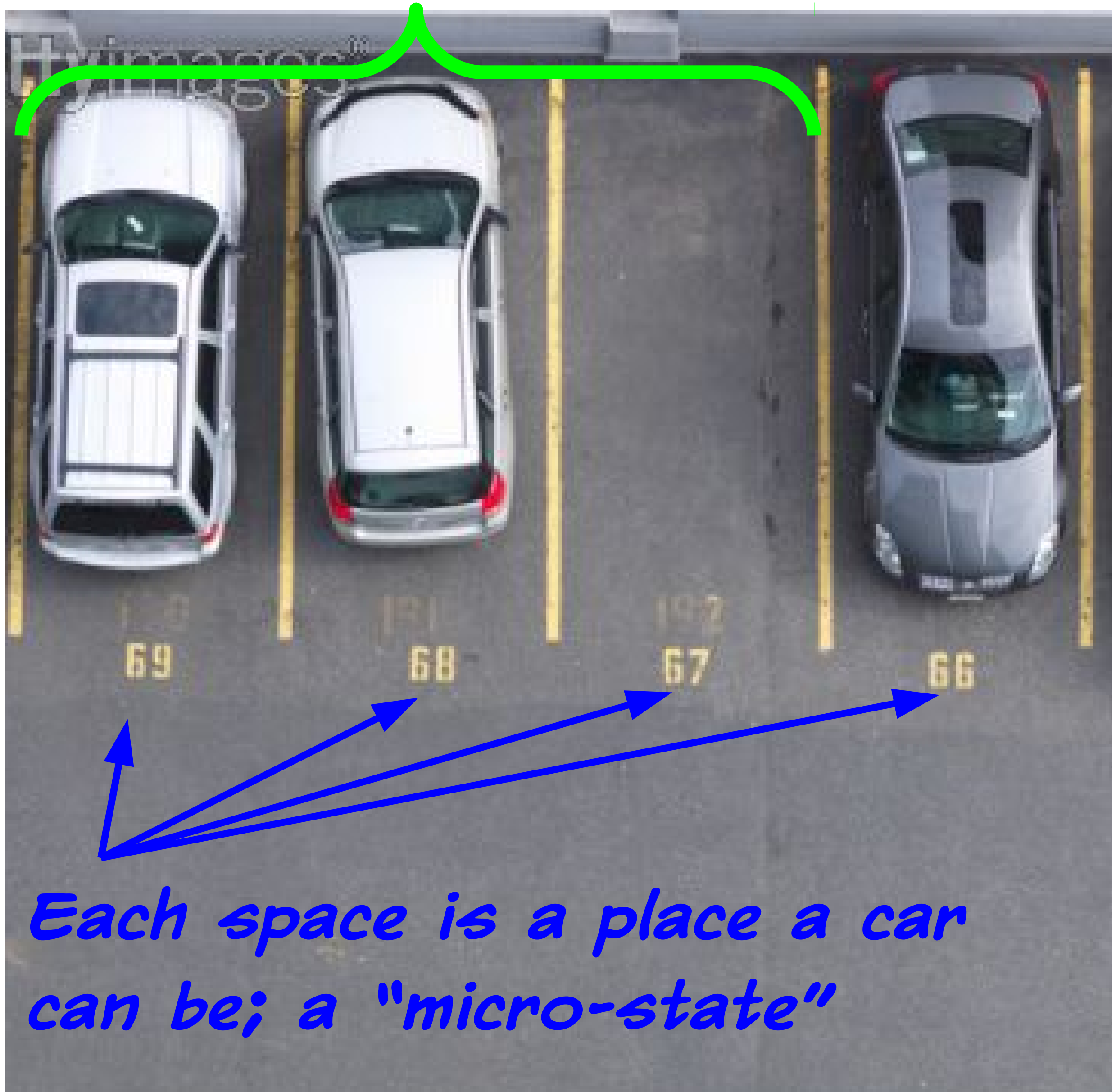
These are not "unique" because in each case the same three people win food without affecting the other (one) person.

AN EXAMPLE: 4 SPOTS

- Arrange 4 cars in 4 spots, where 3 of the spots get free food coupons
- There are 4! ways of arranging the cars . . .
- . . . but only 4 of them (4!/3!) have unique outcomes
    - There are only 4 ways of arranging all the cars such that the order of cars in the free-food spots DOESN'T AFFECT the order of the cars in the other spots and thus the outcome (= "free food").

A collection of spaces that achieves the same outcome (e.g. the same coupon winners) is a "macro-state"



NUMBER OF WAYS (W)

- The number of ways N objects can be organized in N spaces:

$$\boxed{W = N!}$$

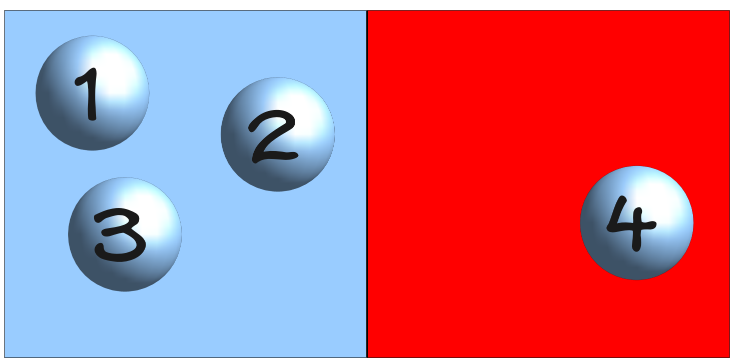
- The number of ways N objects can be arranged such that Ni of them don't affect the order of the (N-Ni):

$$\boxed{W = N!/N_i!}$$

- And if there are M macro-states:

$$\boxed{W = N!/\prod_{i=1}^{M} N_i!}$$

TWO-STATE BOX



Four gas molecules: how many ways to get ...

all on the left side?

$$W_0^4 = \binom{N}{N_R} = \frac{N!}{N_R!(N-N_R)!} = \frac{4!}{0!4!} = 1$$

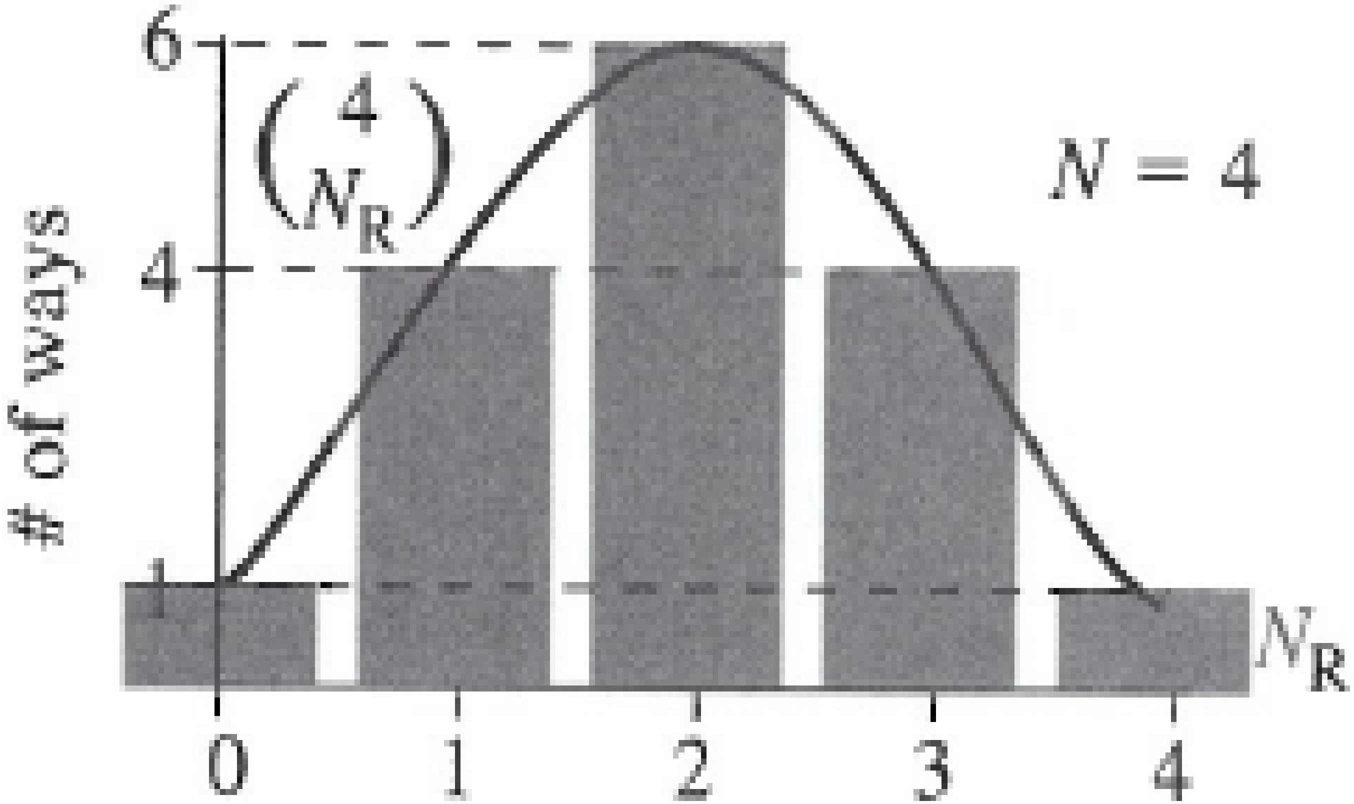
three on the left side?

$$W_1^4 = \frac{4!}{1!3!} = 4$$

half on the left side?

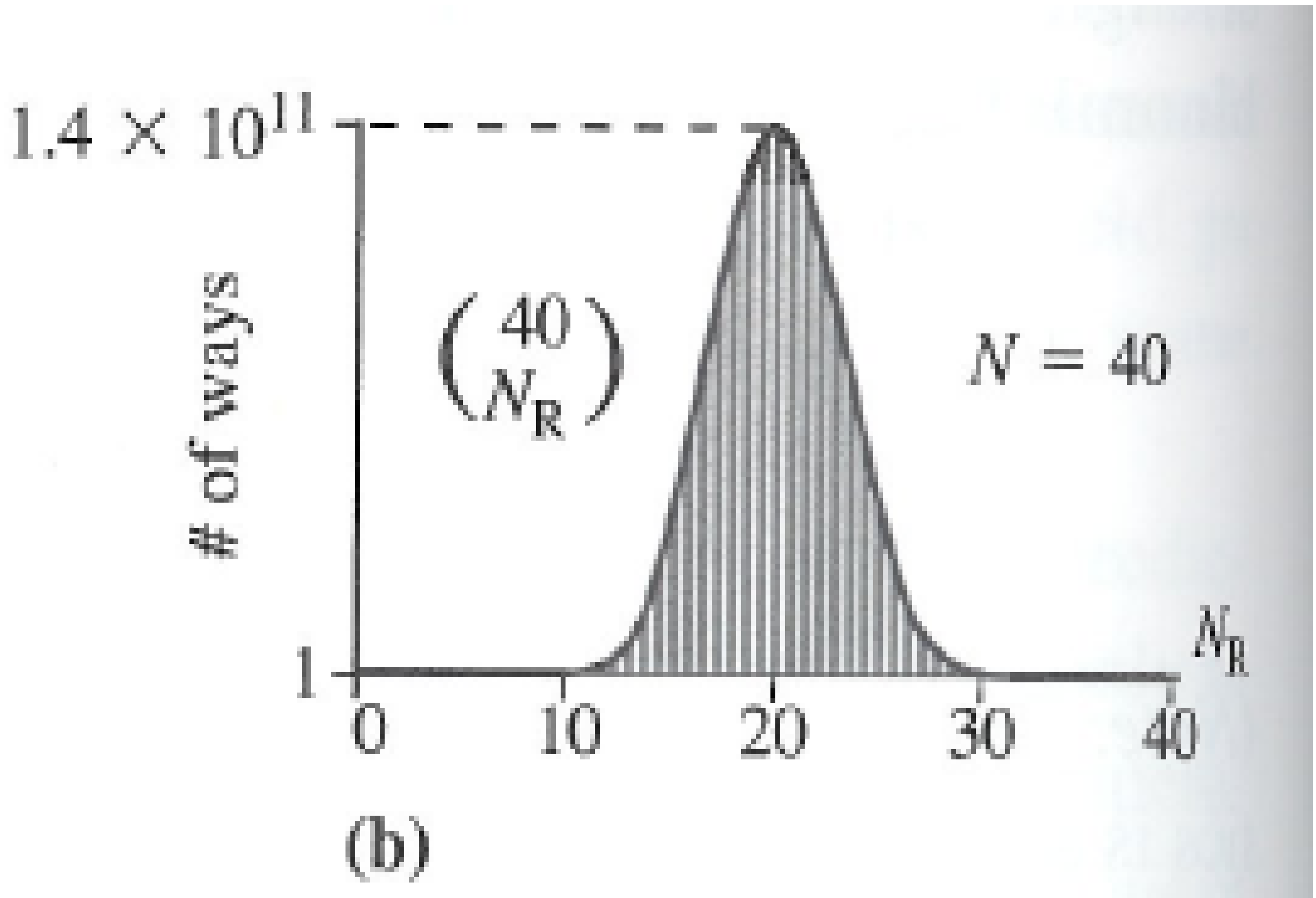
$$W_2^4 = \frac{4!}{2!2!} = 6$$

RELIABLE PREDICTIONS

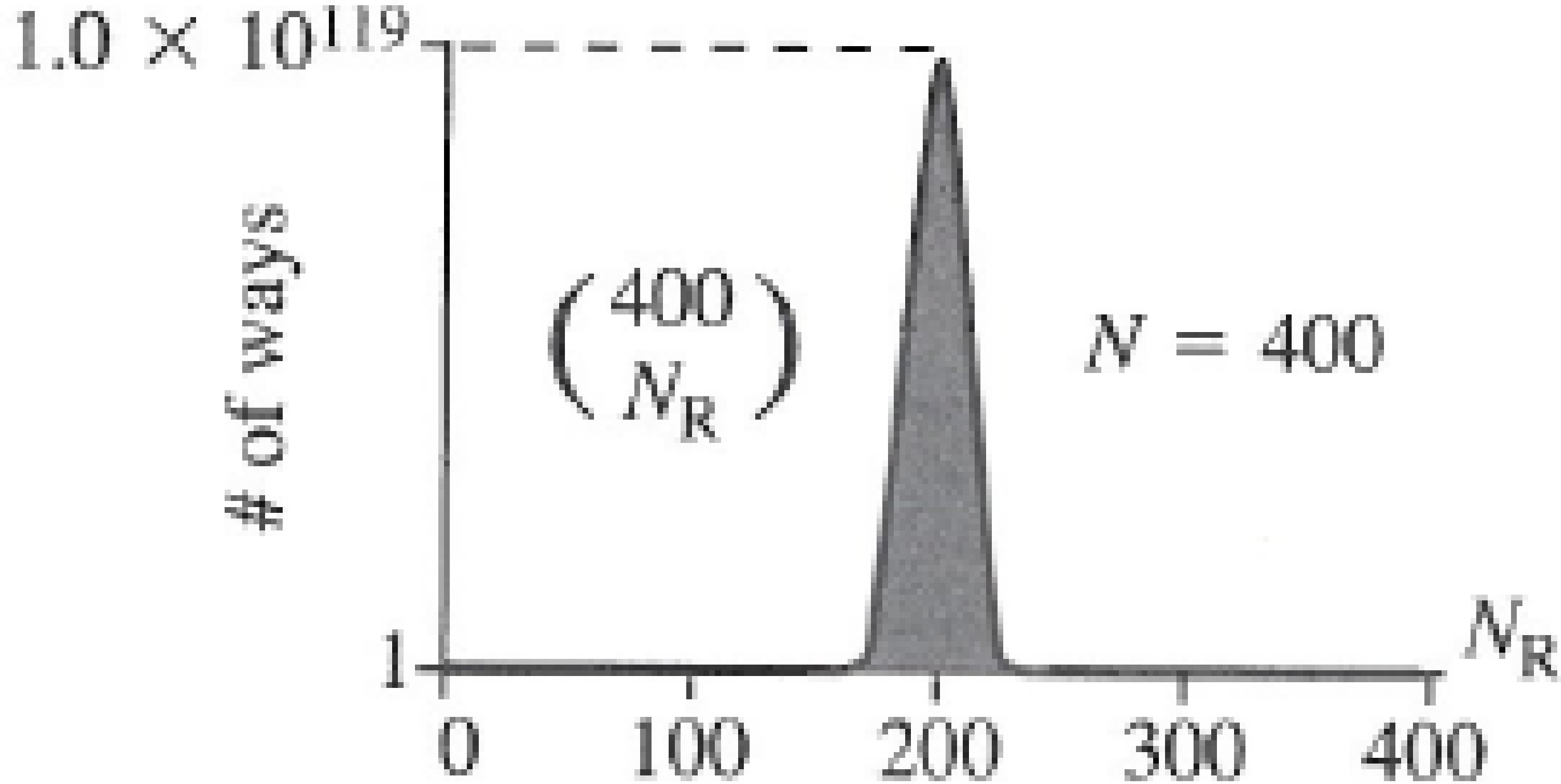


(a)

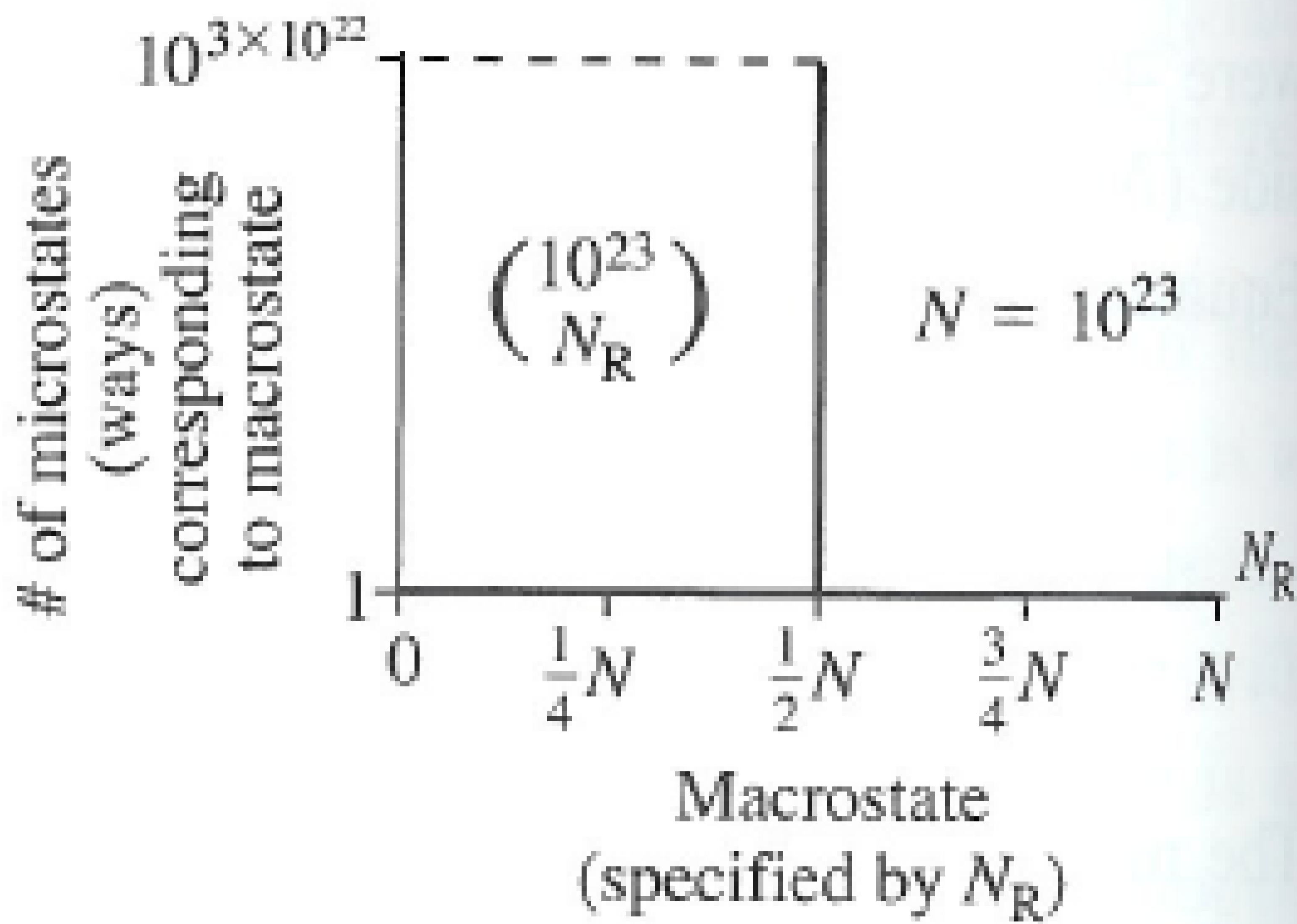
RELIABLE PREDICTIONS



RELIABLE PREDICTIONS



RELIABLE PREDICTIONS



TERMINOLOGY

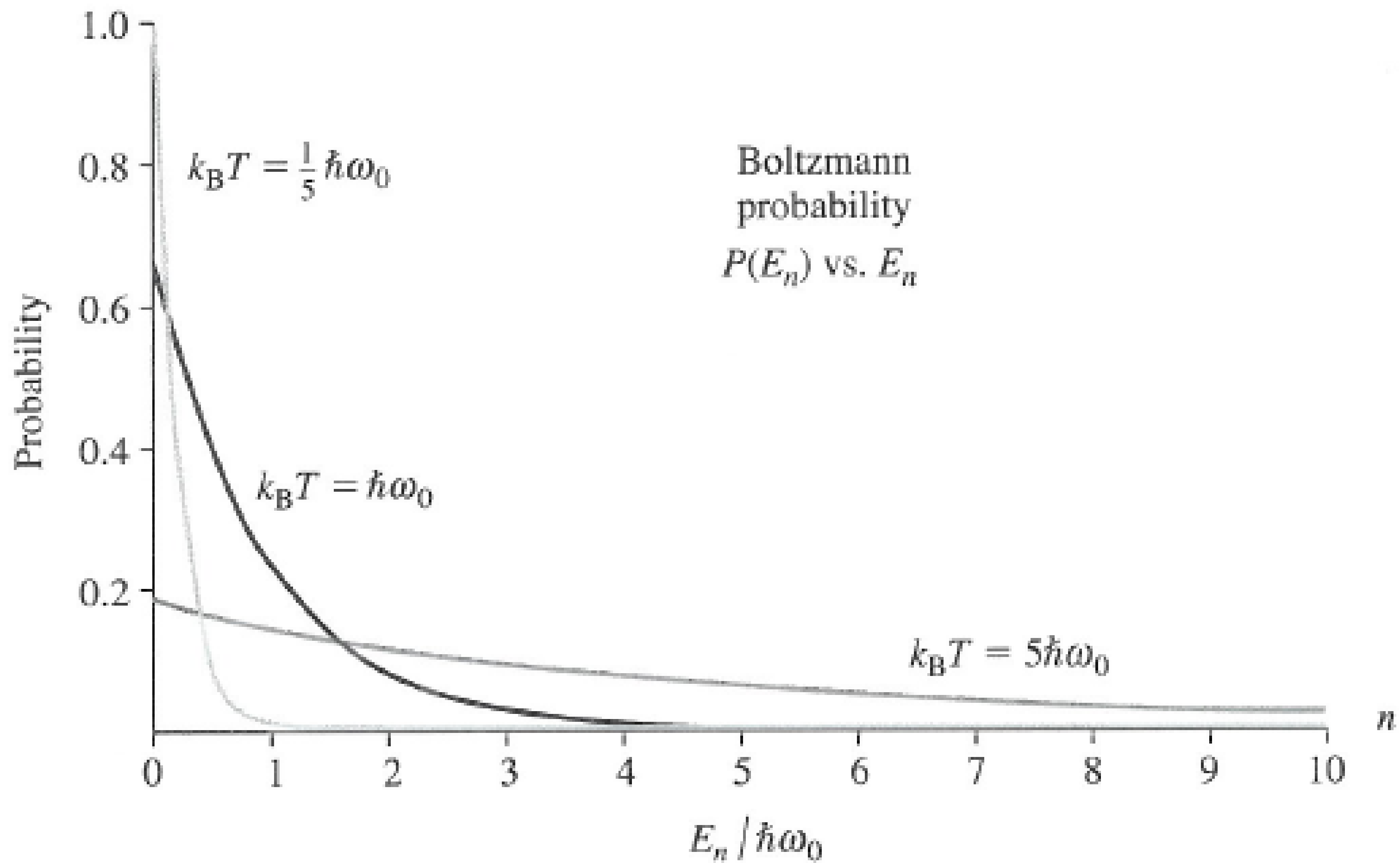
- micro-state: each individual way of obtaining a distribution
  - e.g. each way of getting ½ of the molecules on one side of the room
  - the state of the system, given COMPLETE microscopic knowledge of the states of each individual particle.
- macro-state: the property of the system that doesn't depend on the exact microscopic states
  - e.g. temperature, pressure, density, number, energy, volume, etc.
  - in our box example, "being on the right side" is a macro-state
- equilibrium state: the most probable macro-state
  - by "most-probable," I mean the one with the most corresponding micro-states that achieve it.

ENERGY AND STATES

- In a system of particles, varying the energy of just one particle causes sharp changes in the way energy is then distributed amongst all the other particles.
- The greatest freedom to distribute energy amongst particles occurs when that one particle under consideration has the least energy possible.
- Therefore, the more probable state for a given particle, the state in which the number of ways of distributing energy among all particles is greatest, is one of lower energy.

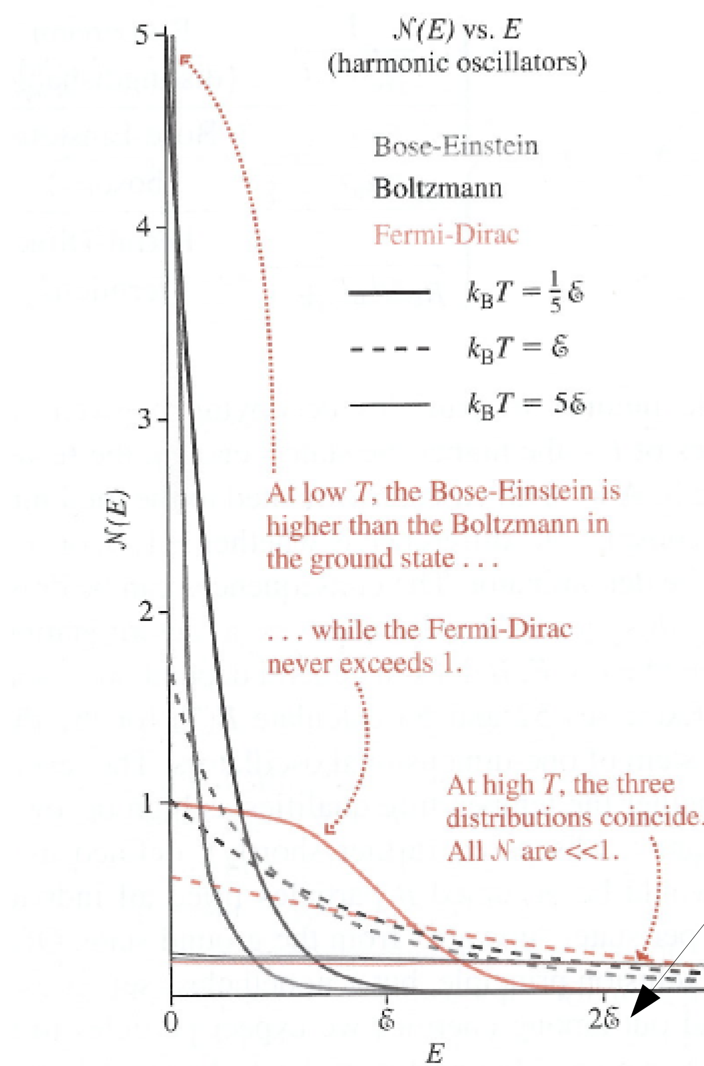
BOLTZMANN: P VS. E FOR DIFFERENT T

Figure 9.7 Variation of Boltzmann probability from $k_BT < \hbar\omega_0$ to $k_BT > \hbar\omega_0$.



N(E) VS. E

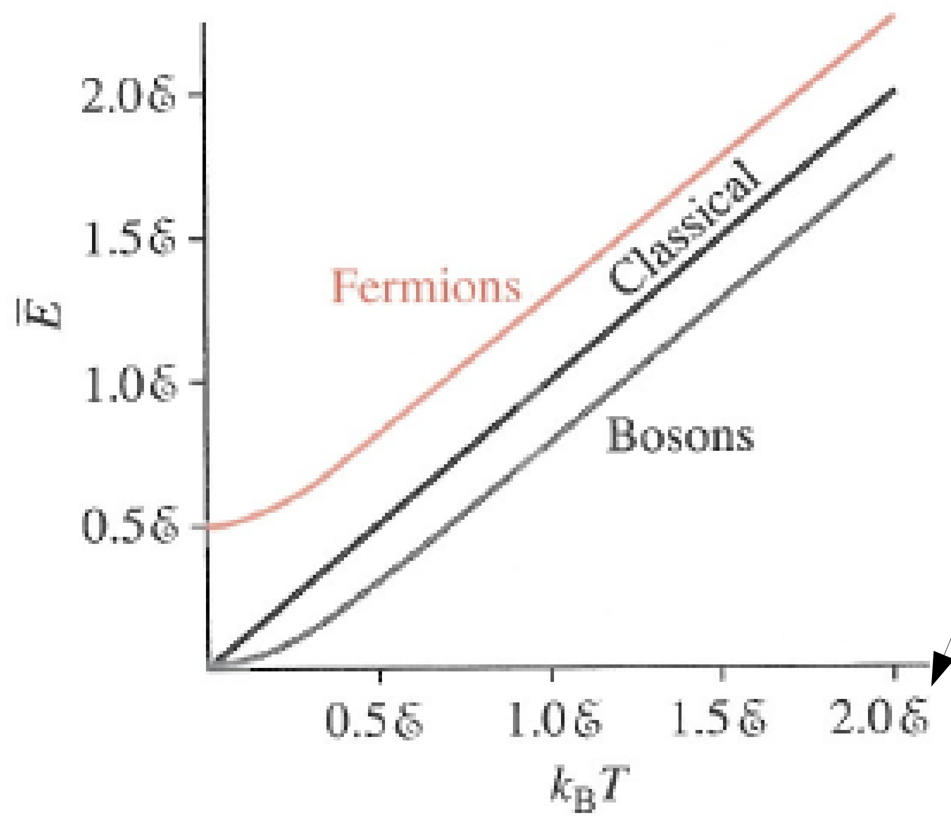
Figure 9.10 The three distributions for oscillators at low, intermediate, and high temperature.



By definition, the energy below which individual particles fill all available states

AVERAGE ENERGY VS. T

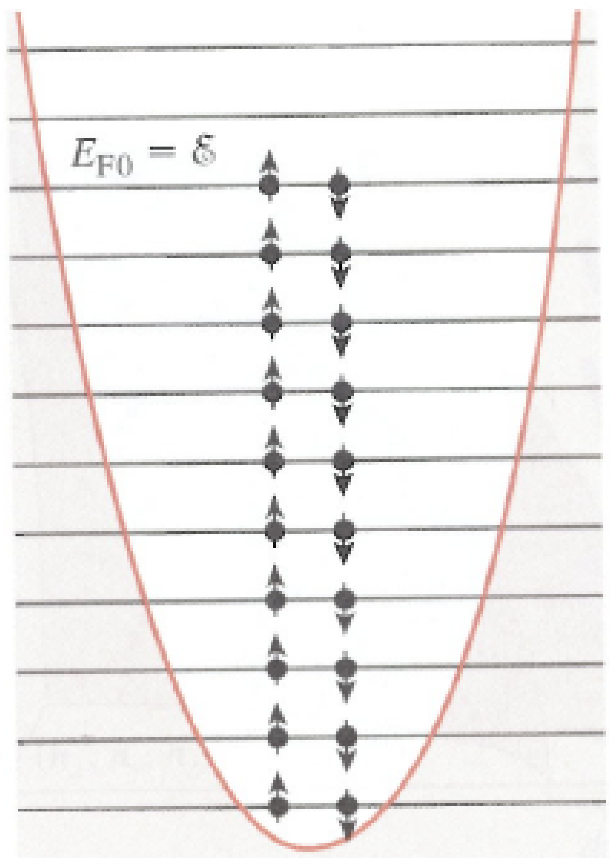
Figure 9.11 Average oscillator energy versus temperature for the three types of particles.



By definition, the energy below which individual particles fill all available states

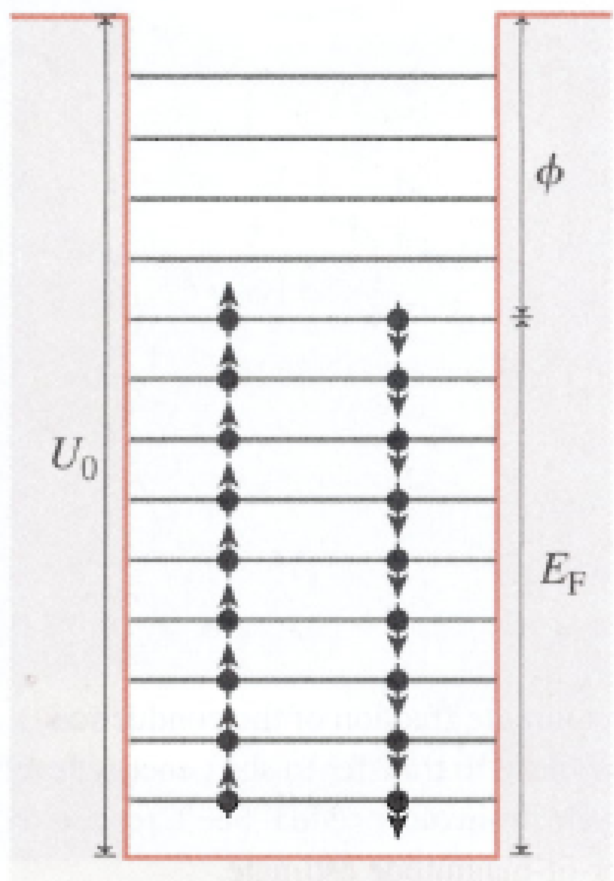
FERMI ENERGY

Figure 9.12 Fermion oscillators in the lowest possible energy state.



METALS - WORK FUNCTION

Figure 9.17 Electron energies in a "cold" metal.



Planck's Blackbody Spectrum

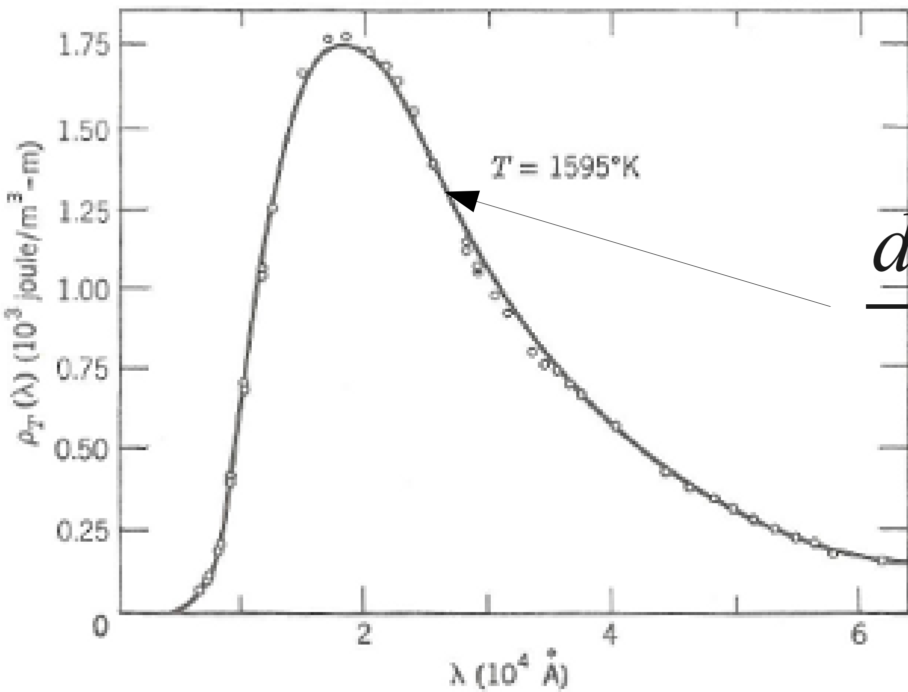


FIGURE I–11

Planck's energy density prediction (solid line) compared to the experimental results (circles) for the energy density of a blackbody. The data were reported by Coblentz in 1916 and apply to a temperature of 1595°K. The author remarked in his paper that after drawing the spectral energy curves resulting from his measurements, "owing to eye fatigue it was impossible for months thereafter to give attention to the reduction of the data." The data, when finally reduced, led to a value for Planck's constant of 6.57 × 10^{-34} joule-sec.

$$\frac{dU_{photons}}{df} = \frac{hf^3}{c^3} 8\pi V \frac{1}{e^{hf/k_B T} - 1}$$

ROADMAP

- Solid-state physics
  - quantum mechanics and the structure of atomic matter (crystals, metals, (semi)conduction)
- Nuclear physics
  - quantum mechanics and the structure of the atomic nucleus
- Particle physics
  - quantum mechanics, relativity, and the fundamental structure of the universe

NEXT TIME

- Statistical Mechanics II
  - The Boltzmann Distribution
  - Classical Averages
  - Quantum Distributions
- Reading: Harris Ch. 9.3-9.5