Topics in Spin - Homework 2

March 18, 2014

Problem SS-4: The Raising and Lowering Operators, $L_+$and L_-

Write the basis vectors in angular momentum space in terms of ℓ, m as $|\ell, m\rangle$. In this basis, determine the coefficients $C_\pm(\ell, m)$ in the following equations for the raising and lowering operators:

$$\begin{aligned} L_+ |\ell, m\rangle &= C_+(\ell, m) |\ell, m+1\rangle \\ L_- |\ell, m\rangle &= C_-(\ell, m) |\ell, m-1\rangle \end{aligned}$$

Problem SS-5: The Matrix Elements of the Orbital Angular Momentum Operators

Use what you have learned about the action of the operators L^2, L_z, L_+, and L_- on the basis vectors to construct the matrix elements of L^2, L_x,L_y, and L_z. Do this for $\ell = 0, 1$. Reports results in the form of a 4×4 matrix. *HINT: label your basis vectors $|1\rangle = |0,0\rangle$, $|2\rangle = |1,1\rangle$, $|3\rangle = |1,0\rangle$, etc. if it helps you to think about how to start this problem.*

Problem SS-6: Thinking Ahead: What if the electron were really spinning?

Let us pretend for a moment that "spin" corresponds to a real, mechanical property of the electron. The classical radius of the electron is given by equating the electrostatic potential energy of a sphere of charge, e, and radius, r_0, with the rest energy of the electron, $U = e^2/r_0 = m_e c^2$. One can then solve for the radius and, inserting the current values for the electron charge and mass, obtain, $r_0 = 2.818 \times 10^{-15}$m.

Let us assume that, in fact, the electron is spinning and has total spin angular momentum $S^2 = \frac{3\hbar^2}{4}$ and projection $S_z = \frac{1}{2}\hbar$. At what maximum linear speed is the "surface" of the electron charge sphere rotating? Please comment on your results.