Topics in Spin - Homework 3

March 21, 2014

Problem SS-7: The Eigenvectors of the Pauli Spin Matrices, and Spin Measurement

The Pauli Spin Matrices are the operators that "measure" spin projections along different space axes (x, y, and z). You already have the normalized eigenvectors of $S_z = \frac{1}{2}\hbar\sigma_z$.

1. Determine the normalized eigenvectors of S_x and S_y.

2. You prepare an electron in a pure eigenstate of spin projection along the positive x-direction (e.g. by using a filter that only allows electrons with this spin projection to pass). You then pass the electron through a filter that measures its component of spin projected along the y-direction. What is the *probability* of finding the electron, prepared as stated, to have a projection of spin along the positive y-direction? Use the definition of probability, $P = |\langle w|v\rangle|^2$, where $|v\rangle$ is the original prepared state and $\langle w|$ represents the state of being projected along a specific direction (that may not be the original prepared direction).

3. Continuing with the premise in part 2, what are the *expectation values* of the S_x and S_y operators for the state, as originally prepared above?

4. Explain what is going on in part 2 and part 3 of this problem, at a physical level (that is, what physics principle(s) is/are at play that would explain what is going on here?) *HINT: is this result what one would expect in a classical physics setting?*

Problem SS-8: The Two-Particle S^2 Operator

Show that

$$S^2 = S_1^2 + S_2^2 + 2\left(S_{1z}S_{2z} + \frac{1}{2}\left(S_{1+}S_{2-} + S_{1-}S_{2+}\right)\right)$$

and that the middle block of the S^2 matrix elements (those for the original basis vectors $|+-\rangle$ and $|-+\rangle$) are all $1\hbar^2$ (and thus that these vectors are not eigenvectors of this matrix).

Problem SS-9: A Baryonic System of Particles

This problem will require the use of something like Mathematica. Consider a baryonic system of three spin-$\frac{1}{2}$ particles (the proton would be one such arrangement - three quarks in a bound-state, each carrying $\frac{1}{2}\hbar$ unit of internal spin angular momentum projected along the z-direction).

1. Determine the normalized eigenstates of the three-particle system (that is, the simultaneous eigenstates of S^2 and S_z that guarantee they are diagonal, and thus commutable). *HINT: Solve the eigenvalue equation, which involves the determinant of an nxn matrix, using a computational tool. Attach your worksheet from the tool to your solutions. Graph paper will be helpful in writing out the matrix elements of S^2 before diagonalization.*

2. Discuss what kind of spectrum of states, based only on spin angular momentum, you expect as a result of determining these eigenstates. Which of the states are likely to be physically represented by the observed proton, the lowest-mass stable baryon in nature? Defend your answer.