Topics in Spin - Homework 4

March 31, 2014

Problem SS-10: The New Particle with a Mass of 126 GeV/c^2

The existence of a new particle with a mass of 126 GeV/c^2 was announced on July 4, 2012. At the time, its quantum properties were largely unknown, except that it had been observed to decay via $X \to ZZ$ and $X \to \gamma\gamma$ - both of these are final states involving spin-1 particles.

What if, instead of being spin-0, the new particle was spin-1 or spin-2? Let us take as the "z-axis" the line of flight of the daughter particles in the rest frame of the parent particle (recall that in the rest frame, the daughters will have back-to-back momenta).

1. Using the Clebsch-Gordon Coefficients, write down the representations of a spin-1 or spin-2 parent state in terms of the product space of the daughters into which it can decay. The daughters are always going to have spin-1.

2. The photon is massless, and as a result its spin angular momentum can only have a projection along the z-axis we have chosen (this is a feature that is understood only in relativistic quantum mechanics). Given that information, what possible states of spin-1 or spin-2 remain for the parent?

Problem SS-11: The Dirac Equation and Free-Particle Dirac Spinors

1. Demonstrate that the Gamma Matrices, $\gamma^\mu = \left(\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \right)$, satisfy:

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}.$$

2. Calculate the $\lambda = +\frac{1}{2}$ helicity eigenspinor of an electron of momentum $\vec{p}' = (p\sin\theta, 0, p\cos\theta)$.