Closed-book. The pass is set at 60/100. Each problem has 20 points. Name: _____

1. A bullet is fired straight up with an initial speed v_0. Assume quadratic air resistance with the drag constant c_2, show that when the bullet turns back and hits the ground, its speed is $\frac{v_0 v_t}{\sqrt{v_0^2 + v_t^2}}$, with $v_t = \sqrt{mg/\ c_2}$, the terminal speed

2. A mass m moves along the x-axis subject to an attractive force given by $17\beta^2 mx/2$ and a retarding force given by $3\beta m\dot{x}$, where x is its distance from the origin and β is a constant. A driving force given by $mA\cos\omega t$, where A is a constant, is applied to the particle along the x-axis.

(a) What value of ω results in steady-state oscillations about the origin with maximum amplitude?

(b) What is the maximum amplitude?

3. A smooth rod of length l rotates in a horizontal plane with a constant angular velocity ω about an axis fixed at one end of the rod and is perpendicular to the plane of rotation. A bead of mass m is released at the midpoint of the rod. Calculate (a) the position of the bead on the rod as a function of time; (b) the time and velocity (relative to the rod) when the bead leaves the end of the rod.

4. A satellite is placed into a low-lying orbit by launching it with a two-stage rocket from Cape Canaveral with speed v_0 inclined from the vertical by an elevation angle θ_0. On reaching apogee of the initial orbit, the second stage is ignited, generating a velocity boost Δv_1 that places the payload into a circular orbit

   (a) Calculate the additional speed boost Δv_1 required of the second stage to make the final orbit circular.

   (b) Calculate the altitude h of the final orbit. Ignore air resistance and the rotational motion of the Earth. The mass and radius of the Earth are $M_E = 5.98 \times 10^{24}$ kg and $R_E = 6.4 \times 10^3$ km, respectively. Let $v_0 = 6$ km/s and $\theta_0 = 30°$.

5. A uniform chain lies in a heap on a table. If one end is raised vertically with uniform velocity v, show that the upward force that must be exerted on the end of the chain is equal to the weight of a length $z + (v^2/g)$ of the chain, where z is the length that has been uncoiled at any instant.