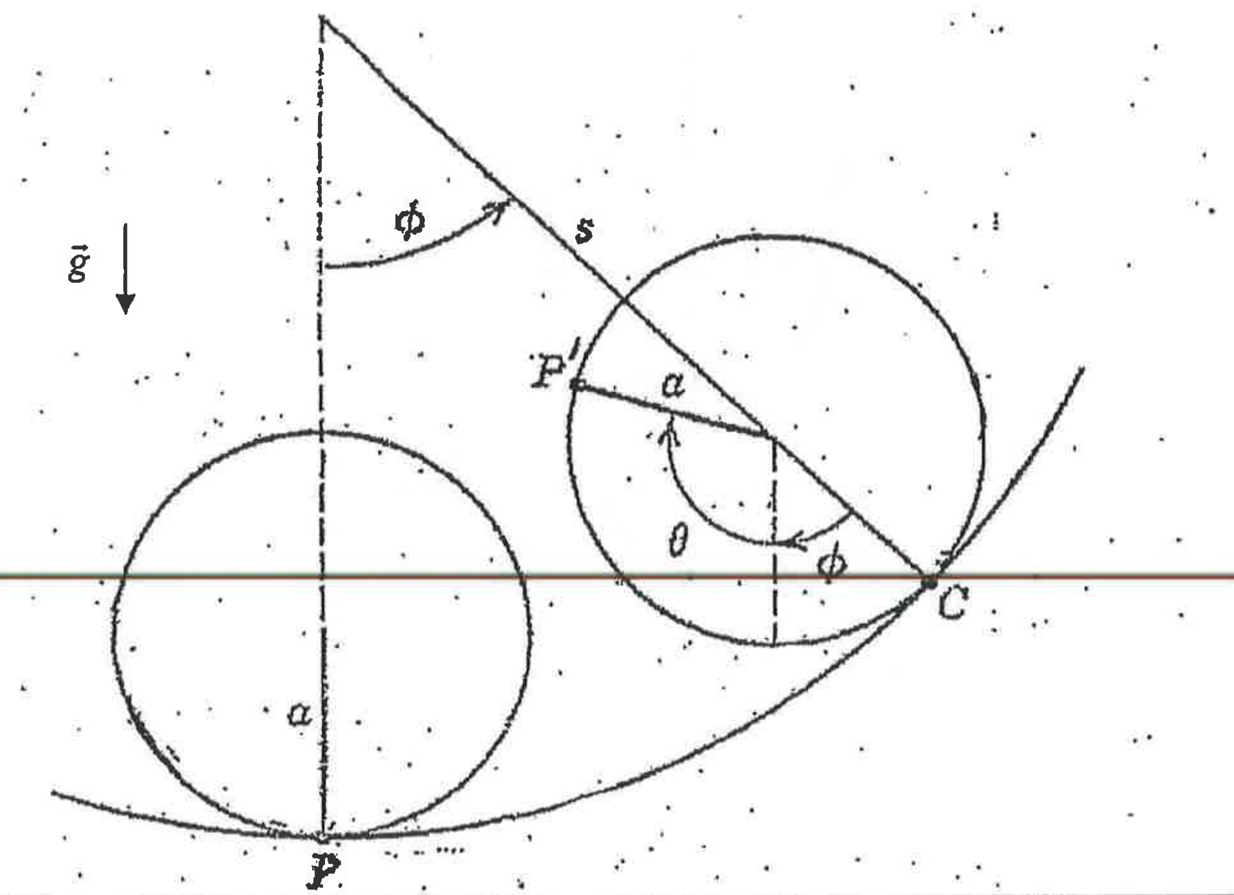
Mechanics Jan 2018

Problem #1 (20 Points out of 100)

A uniform ball bearing of radius a, mass M, and moment of inertia around its center o mass $I = \frac{2}{5}Ma^2$ rolls back and forth without slipping on a cylindrical track of radius s The motion is constrained to the plane of the paper, and a uniform gravitational field of strength g is present, as shown in the figure. The angle ϕ with vertex at the center of th circle of radius s measures the position of the center of mass of the sphere. The sphere rotates through an angle θ when the center of mass moves through an angle ϕ. During this motion, the point P moves to P'.

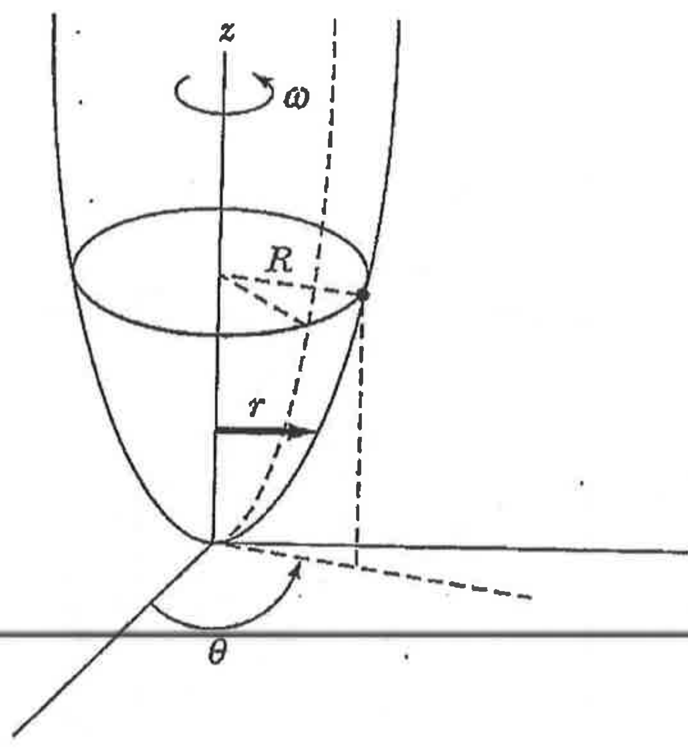
a) Find the equation of motion for the angle ϕ.
b) Find the frequency of the oscillation for small amplitudes
c) Calculate the frequency of small oscillations in the limit $a << s$. Does this frequency equal the one for a pendulum of mass M and length s? Explain.



Problem #2 (20 Points out of 100)

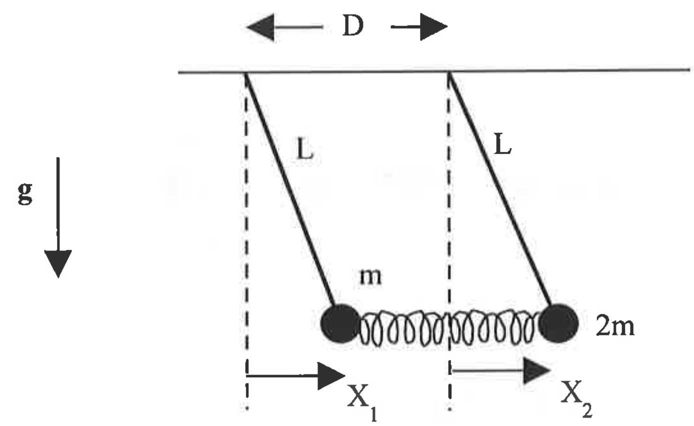
A bead slides frictionless along a wire bent in the shape of a parabola $z = cr^2$. The wire is rotating about its vertical symmetry axis with angular velocity ω.
Choose r, θ, and z as the generalized coordinates for the problem.

a) Find the kinetic energy of the bead.
b) Find the potential energy of the bead choosing $U = 0$ at $z = 0$.
c) Write the equations of constraint for the system. How many degrees of freedom does the system have?
d) Find Lagrange's equations of motion for the bead.
e) Find the value of c that causes the bead to rotate in a circle of fixed radius.



Problem #3 (20 Points out of 100)

Two pendulums are coupled by a massless spring with spring constant k. Both pendulums have massless rods of length L. They are separated by distance D. The masses are m and 2m. Consider small oscillations.



a. Solve for the normal modes of the pendulums.
b. Determine the normal coordinates that undergo simple harmonic motion.

Problem #4 (20 Points out of 100)

An elementary relativistic particle of rest mass m_i moving with relativistic velocity v_i in the lab frame collides with another identical particle which is at rest in the lab frame, and they become a new single particle of rest mass M as in FIG. 3. Shortly afterwards, it decays into two identical particles of rest mass m_f which fly apart with the same speed v_f with an angle θ from the original direction as shown in FIG. 4. If you prefer, you can use the unit system where the speed of light is unity, $c = 1$.

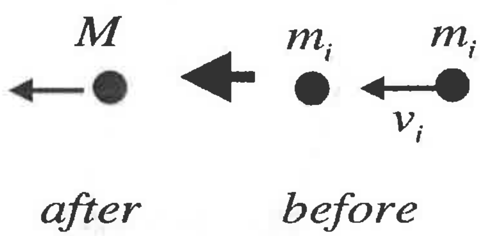


FIG. 3. Figure for (a) and (b).

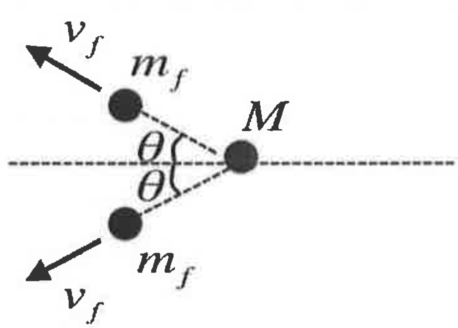


FIG. 4. Figure for (c).

(a) Find the rest mass M of the intermediate particle before its decay, in terms of m_i and v_i.

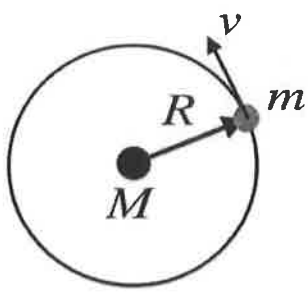
(b) Find the velocity of the intermediate particle in the lab frame before its decay, in terms of m_i and v_i.

(c) Find the final velocity v_f and the angle θ in the lab frame, in terms of m_i, v_i, and m_f.

(d) Show or argue that in the center of mass frame, the final two particles in this case fly apart in a direction perpendicular to the original incident direction.

Problem #5 (20 Points out of 100)

. A small meteoroid of mass m is in the circular orbit of radius R around a very heavy stellar object of mass M. Let the Newton's gravitational constant be G.



(a) Find the speed v of the meteoroid, its angular momentum L, and its total mechanical energy E.

(b) Suppose the stellar object suddenly explodes in a spherical symmetric way without gaining any momentum, and loses part of its mass to a region far away from the system, and becomes a neutron star of mass $M_f < M$. Assume that the explosion happens very fast compared to the meteoroid motion, and the meteoroid motion is not disrupted during the process, and the meteoroid moves in the gravitational fed of the newly born neutron star. Find the minimum value of M_f that the meteoroid orbit is still bound to the neutron star.

(c) Supposing that $M_f = \frac{3}{4}M$, f nd the farthest distance from the neutron star R_m and the velocity at that point v_m.