Classical Preliminary Exam

Profs. T.E. Coan and F.I. Olness
Spring 2007

Printed Name ____

BOX YOUR ANSWERS

BOX YOUR ANSWERS

We never make mistakes

Two legs are better than four

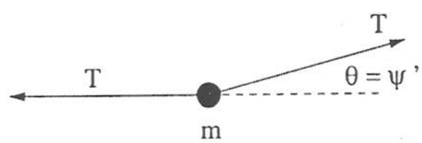
B, B, B, BOX YOUR ANSWERS

Q1. Required. A small uniform sphere of mass m and radius ρ starts at rest from the top of a fixed hemisphere of radius R. The small sphere *rolls without slipping* along the hemisphere's surface. Find the force F_h that the hemisphere exerts on the sphere and determine the height H at which the sphere leaves the hemisphere. The hemisphere's "equator" is parallel to the ground, similar to an orange cut in half and placed face down.

Q2. Required. Consider the transmission and reflection of waves from a mass m at $x = 0$ on a string of linear mass density ρ and tension T. The string stretches from $x = -\infty$ to $x = +\infty$. so that we can assume that the wave $\psi(x,t)$b has the form

$$\begin{aligned}\psi(x,t) &= Ae^{ikx}e^{-i\omega t} + RAe^{-ikx}e^{-i\omega t} && x < 0 \\ &= \tau A e^{ikx}e^{-i\omega t} && x \geq 0,\end{aligned}$$

where ω is the angular frequency of the wave and $k = 2\pi/\lambda$, with λ the wavelength of the wave. Using the abbreviation $\epsilon = (m\omega^2)/(kT)$, determine τ and R. Assume that the mass moves only vertically and that the tension in the string is equal on each side of the mass. This problem requires no special knowledge of waves and is less hard than it might appear. The figure below might help.



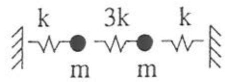
NQ1. Required. *Estimate* the mass of Earth's atmosphere M_a. Make the simple assumptions that the atmosphere's density is independent of altitude and that the acceleration due to gravity is also independent of height above Earth's surface for the heights relevant for this problem. If you do not have a calculator, then approximate (intelligently).

NQ2. Do 1 of 2 optional. *Estimate* the number of atoms N_a in your body.

NQ3. Do 1 of 2 optional. You are an astronaut at rest with respect to Earth at some distance from Earth. You see red neon lights from Las Vegas. *Estimate* how fast you must travel radially away from Earth so that your eye can no longer detect red light? Express your answer β^* as a fraction of the speed of light.

Q3. Required. Pions can be produced in proton-proton collisions via the strong interaction. A beam of protons (each of mass m_p) strikes a stationary target of protons in the reaction $p + p \to p + p + \pi^+\pi^-\pi^0$. Assuming $m_{\pi^\pm} = m_{\pi^0} = m_\pi$, determine the minimum total energy E_p the incident proton must have for the reaction to occur. Express your answer in terms of $\{m_p, m_\pi\}$.

Q4. Required. Two masses m are connected by springs of spring constants $\{k, 3k, k\}$ as shown in the beautiful drawing below. The equilibrium spring length is a. Consider longitudinal motion only and determine the normal modes (also called eigenmodes) of oscillation. Also determine the oscillation frequency for each normal mode.



Farro
Firan
Gibbs
Kasmi

Classical Exam

Prof. T.E. Coan (x8-2497)
Prof. J. Ye
Aug. ~~Jan~~ 2006

PRINTED Name ________________________________

BRAIN ONLY EXAM:
NO REFERENCES ALLOWED.
CALCULATORS OK.

† DO FOUR PROBLEMS.

† BOX your final answers. We will NOT hunt for your answer amid a sea of algebra.

† Scratch out irrelevant calculations so that we can follow your reasoning.

THIS EXAM IS CLOSED BOOK

(1) 25 pts A uniform rod of length a is freely pivoted at one end. It is initially held horizontally and then released from rest.

a) 10 pts Find the angular velocity at the instant when the rod is vertical. When the rod is vertical it breaks at its midpoint. Assume that no impulsive forces are generated when the rod breaks.

b)10 pts Find the largest angle from the vertical reached by the upper part of the rod in its subsequent motion.

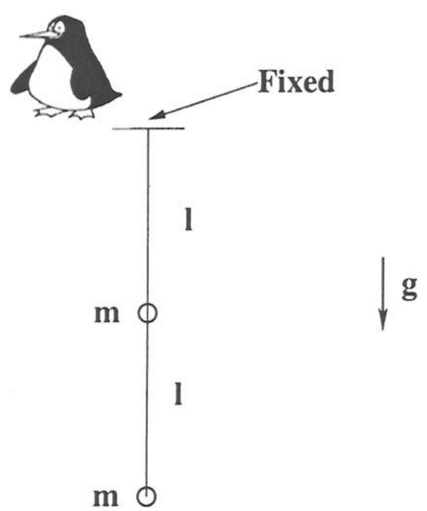
c) 5 pts Describe the motion of the lower part of the rod. Be clear if you use no math.

(2) 25 pts This problem requires no special knowledge of particle physics. The proposed NOνA experiment at Fermilab seeks to determine, among several things, the mass ordering of the neutrino mass eigenstates. The experimental technique relies on generating a beam of energetic muon neutrinos ν_μ. This is accomplished by first colliding a beam of protons into a carbon target, and then letting the subsequently produced charged pions decay in flight. The charged pions have a typical energy of 10 GeV and decay isotropically in their *rest* frame into a muon and a muon neutrino ($\pi^+ \to \mu^+ + \nu_\mu$). Useful information includes $m_\pi = 140\,\mathrm{MeV}$ and $m_\mu = 106\,\mathrm{MeV}$. Assume $m_\nu < 1\,\mathrm{eV}$.

10 pts a) What is the neutrino energy E_ν and the muon energy E_μ in the pion *rest* frame? Express your compact answers as functions of m_π, m_ν and m_μ.

10 pts b) Show that for neutrinos produced in the *lab* frame and parallel to the flight direction of the pions, the neutrino energy $E_\nu \simeq 0.43 E_\pi$, where E_π is the energy of the pion in the lab frame.

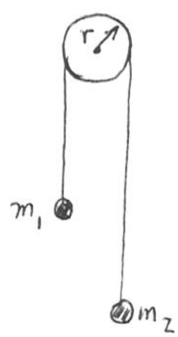
(3) 25 pts The AMANDA and ICECUBE Antarctic neutrino experiments rely for signal detection on photomultiplier tubes (PMTs) strung along vertical cables frozen in the ice. Holes are first drilled in the ice, a PMT string is lowered and the ice is then allowed to refreeze, fixing the positions of the PMTs. For simplicity, assume each such string has only 2 PMTs, each of identical mass m, and let the distance between the 2 PMTs and the distance between the top most PMT and the surface be l. (See the figure.)



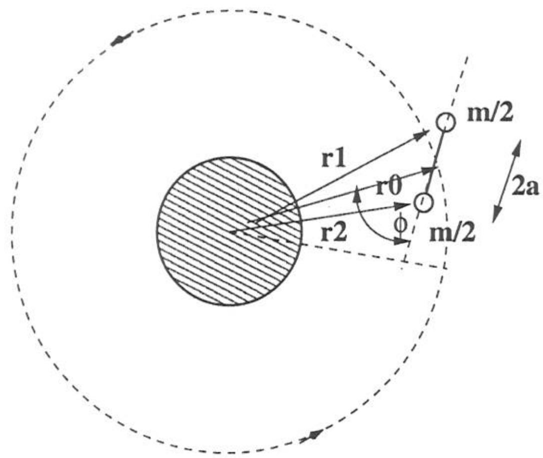
a) 15 pts If one end of the PMT string is always fixed at the surface, what are the frequencies of the normal modes of oscillation of the PMTs before the ice refreezes? Assume the hole diameter is large enough so that it does not interfere w/ a swinging PMT and ignore the effects of the ice water in the bore hole.

b) 5 pts Sketch each normal modes and match it with its respective eigenfrequency.

(4) (25 points) Atwood's machine consists of two weights of mass m_1 and m_2 connected by a massless, non-stretchable string of length l. The string passes over a pulley of radius r and moment of inertia I. The pulley is fixed in space but can rotate without any friction. The string does not slide on the pulley. Choose a proper coordinate and find the equation of motion for this system from the Lagrange equations, assuming the system is stationary at the beginning.



5) 25 **pts** We are interested in the stability of the orientation or "attitude" of a dumbbell-shaped satellite in its circular orbit about earth. The satellite is composed of two small spherical masses of identical mass $m/2$ joined by a thin massless cylinder of length $2a$. See the figure. Let the polar coordinates r and θ describe the position of the center-of-mass of the satellite and let the angle ϕ describe the attitude or angular orientation of the satellite axis relative to the radius vector $\vec{r}_0$.



a) 5 pts) To start, consider the motion of a point mass m moving in 1 dimension and in a conservative force field whose potential energy function is $V(q)$, where q denotes the particle's displacement from the origin. Also assume that $q = 0$ is a point of equilibrium. After Taylor series expanding $V(q)$ around $q = 0$ and keeping only the lowest non-vanishing term, write the lagrangian L for this system. Ignore arbitrary constants in the series expansion. (By arbitrary, I mean those terms that can be set to zero without loss of generality.)

b) 5pts What is the equation of motion for this simple system? (Be as explicit as you can. The symbol L should not appear in this equation.)

c) 5pts For motion near $q = 0$, what is the condition on $(d^2V/dq^2)|_q = 0$ for oscillatory motion about $q = 0$ and what is the angular frequency of oscillation ω in terms of m and $(d^2V/dq^2)|_{q=0}$?

d) 5pts Back to the problem at hand. Write the potential energy of the satellite in terms of r_1, r_2, m, M_E and G. The symbols have an obvious meaning.

e) 5pts Examining the figure and from the law of cosines, we can write

$$r_{1,2} = (r_0^2 \pm 2r_0 a \cos\phi)^{1/2} = (r_0^2 + a^2)^{1/2}(1 \pm \epsilon \cos\phi)^{1/2},$$

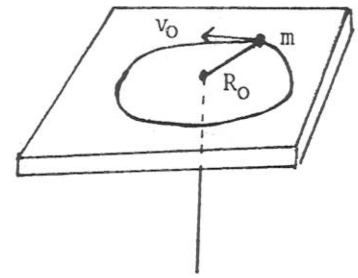
where $\epsilon = 2r_0 a/(r_0^2 + a^2)$. Since $a \ll r_0$, ϵ is small and we can then use the binomial series expansion $(1+x)^{-1/2} = 1 - x/2 + 3x^2/8 + \cdots$, where $x = \pm\epsilon\cos\phi$. With these approximations, simplify your answer for V in part d) and rewrite it as

$$V(\phi) = -\frac{GM_E m}{r_0}\{\cdots\},$$

where *you* need to fill in the $\{\cdots\}$. Just keep the lowest order terms.

f) 5 pts Determine the value of ϕ for *stable* oscillations. Sketch the attitude of the satellite in its stable configuration.

4. A mass m moves on a smooth horizontal plane with velocity v_o at a radius R_o. The mass is attached to a string which passes through a smooth hole in the plane, as shown. ("Smooth" means frictionless.)



(a) What is the tension in the string?

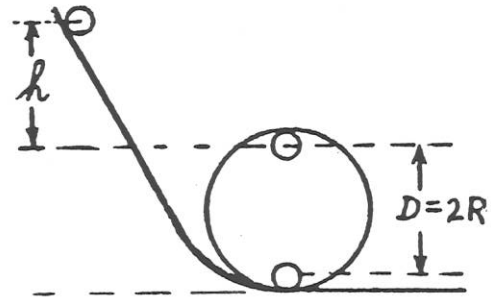
(b) What is the angular momentum of m?

(c) What is the kinetic energy of m?

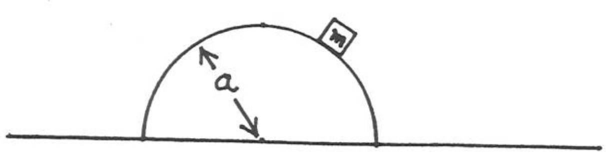
(d) The tension in the string is increased gradually and finally m moves in a circle of radius $R_o/2$. What is the final value of the kinetic energy?

(e) Why is it important that the string be pulled gradually?

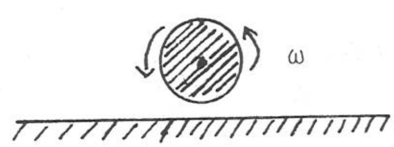
9. Find the minimum height h (above the top position in the loop) that will permit a spherical ball of radius r (which rolls without slipping) to maintain constant contact with the rail of the loop. (The moment of inertia of a sphere about the center is $2/5\ m\ r^2$.)



3. **A mass m slides without friction on a hemispherical hill of radius a. If it starts at rest at the top of the hill (unstable equilibrium), at what height h does it leave (separate from) the hill?**



12. A coin spinning about its axis of symmetry with angular frequency ω is set down on a horizontal surface. After it stops slipping, with what velocity does it roll away?



Classical Exam

Prof. T.E. Coan (x8-2497)
May 2005

PRINTED Name ______________________________

BRAIN ONLY EXAM:
NO REFERENCES ALLOWED
NO CALCULATORS ALLOWED.
OK, YOU CAN DRINK BEER.

† DO ALL PROBLEMS.

† BOX your final answers. I will NOT hunt for your answer amid a sea of algebra.

† Scratch out irrelevant calculations so that I can follow your reasoning.

THIS EXAM IS CLOSED BOOK

(1) 10 pts Here are some easy questions to start with. Answer **only five** of them. All answers should have at least 1 significant figure precision and use proper "conventional" units such as meters, kilograms, seconds, centimeters, grams, etc. I do **NOT** want to see energy units for mass.

1) What is the density ρ_{Fe} of iron?

2) What is the mass $M_\oplus$ of the earth?

3) What is the value of Avogadro's number N_0?

4) What is the mass m_p of a proton?

5) What is the mass m_e of an electron?

6) What is the wavelength λ of visible light?

7) What is the mass $M_\odot$ of the sun?

8)What is the mean radius R_{ES} of earth's orbit about the sun?

9) What is the value of atmospheric pressure P at sea level? (Do NOT write "1 atmosphere.")

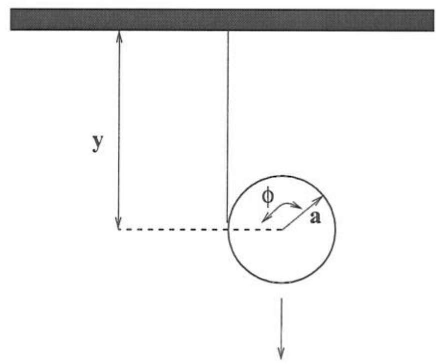
10) What is the value of the air density ρ in this room?

(2) 20 pts This problem requires no special knowledge of particle physics. A charged pion (π^+) decays into a muon μ^+ and a muon neutrino ν_μ. Let the mass of these particles be denoted in an obvious notation by m_π, m_μ and m_ν.

10 pts a) What is the neutrino three-momentum p_ν in the pion *rest* frame? Express your answer in terms of the three particle masses and a well-known physical constant.

10 pts b) What must be the pion total energy E_π as measured in the *lab* frame if the neutrino is produced at rest in the *lab* frame? Now you see why producing neutrinos at rest is very difficult. Perhaps this could be your thesis topic.

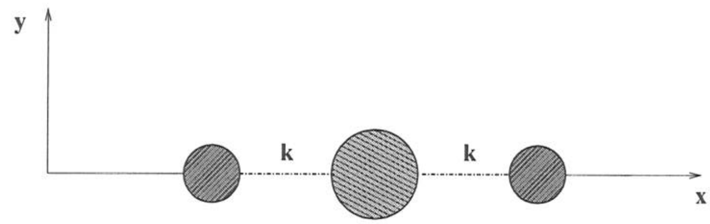
(3) 20 pts Consider the behavior of a yo-yo. (This does not refer to the famous cellist Yo-Yo Ma.) A yo-yo is a toy in the form of a disk with a string wrapped around it. Suppose our yo-yo falls under the influence of gravity only and that the trajectory of the disk's center-of-mass is completely vertical. Our yo-yo has the end of its string that is not attached to the disk, attached to a fixed support. The figure summarizes the situation. Let the yo-yo have a mass m, radius a and start falling from rest. Ignore the mass of the string. To keep me sane while I try to grade these, use the variable names as shown in the figure.



10 pts a) Find the equations of motion for the falling disk. You do NOT need to integrate them: you can just give them to me as uncoupled differential equations.

10 pts b) Find the forces of constraint and tell me their *physical* significance.

(4) 20 pts There is strong evidence that earth's atmosphere is heating up. One of the molecules involved in this process is CO_2. Let's see if we can understand something about this molecule's vibrational modes. CO_2 is a symmetric molecule with all 3 atoms in a line, as shown in the figure. Let the mass of the carbon and oxygen atoms be m_C and m_O, respectively. Let k be the effective spring constant between the carbon and oxygen atoms.



10 pts a) What are the three eigenfrequencies and eigenmodes ("normal modes") for the vibrational motion of CO_2?

8 pts b) For the mode(s) with non-zero eigenfrequency, what is the algebraic relation between the non-zero amplitudes of motion of the relevant atoms? Sketch the normal mode associated with each eigenfrequency, including any possible zero-frequency eigenmode.

2 pts c) What is the *numerical* value of the ratio(s) of the non-zero eigenfrequencies?

For Pavel, Sophia

Classical Exam

Prof. T.E. Coan (x8-2497)
Prof. Y Gao
Spring 2003

Name ______________________________

BOX your final answers

Do ALL problems

BOX your final answers

Do ALL problems

Do What with your answers?

Classical 1. Most of the surface area of earth is liquid water and the core of the earth is presumably liquid iron. Consider earth as an isothermal, homogeneous liquid sphere with the same mean mass density and radius as the real earth. Ignore rotational and surface tension effects.

i) Calculate the radius R_p at which the hydrostatic pressure inside this fully liquid earth is a *maximum*.

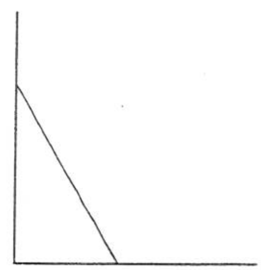
ii) What is this maximum pressure P_{max}? I need both a formula and a number. If you do not have a calculator, estimate the calculation. No units, no credit.

Classical 2. Consider two identical pendula joined as in the figure. Treat the rods as inflexible and massless. The rods each have a length l and the bobs each have mass m. Consider motion only in the plane of the pendula.

i) Find Lagrange's equations of motion for the system. Do not assume small angles.

ii) For the case of small angles ($\sin\theta \sim \theta$), what are the eigenfrequencies?

Classical 3. A ladder is leaning against a wall at some angle θ as shown in the figure. The ladder now starts to fall down in such a way that its ends are always in contact with the two surfaces. For simplicity, assume that there is no friction between the ladder ends and the wall and floor surfaces.



i) Calculate the equation(s) of motion for the system.

ii) Determine the forces that the wall and floor exert on the ladder.

Classical 4. Refer to the figure below. A star, located at O is at rest with respect to you, located at P. The star explodes, producing a spherically expanding shell of gas moving with speed v. You observe the light from this expanding shell of gas. Assume that there is vacuum between you and the star and let the distance $\overline{OP} = R$. You can assume $R \gg r$, where r is the radius of the shell. Show that the apparent ('observed') speed of then expanding shell can exceed the speed of light in vacuum. Specifically,

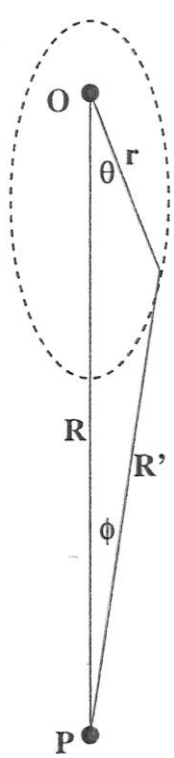
i) Show that the *observed* speed $u_\perp$ of the shell at right angles to the line of sight is given by

$$u_\perp = \frac{v \sin\theta}{1 - (v/c)\cos\theta}.$$

Hints: Star and you are at rest relative to each other. Define 3 times,

t'_e = emission time of light;
t'_0 = explosion time of star (start time of expanding shell);
t'_{obs} = time when shell light reaches P.

Construct relations between times and the geometry of the problem.



10 pts Here are some easy questions to start with. Answer **only five** of them. All answers should have at least 1 significant figure precision and use proper "conventional" units such as meters, kilograms, seconds, centimeters, grams, etc. I do **NOT** want to see energy units for mass.

1) What is the radius R_E of the earth?

2) What is the mass M_E of the earth?

3) What is the value of Avogadro's number N_0?

4) What is the mass of a proton?

5) What is the mass of an electron?

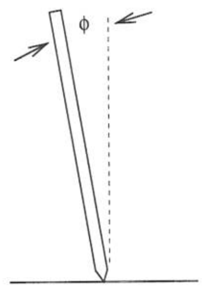
6) What is the value of Newton's gravitational constant G?

7) What is the mass M_S of the sun?

8)What is the mean radius R_{ES} of earth's orbit about the sun?

9) What is the value of the gravitational acceleration g in vacuum of an object dropped near the surface of the earth?

10 pts A homogeneous pencil of total length L and total mass M is placed nearly vertically on a table and allowed to fall over. See the figure. Just *after* the pencil is released, what is the force F_T that the table exerts on the pencil?



Classical Exam

Prof. T.E. Coan (x8-2497)
Fall 2002

Name ________________________________

CLOSED BOOK: NO REFERENCES OR CALCULATORS ALLOWED.

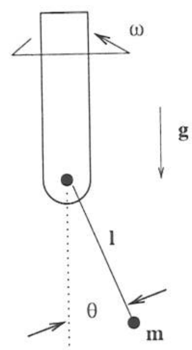
† DO ALL PROBLEMS.

† BOX your final answers. I will NOT hunt for your answer amid a sea of algebra.

† Scratch out irrelevant calculations so that I can follow your reasoning.

THIS EXAM IS CLOSED BOOK

20 pts The bearing of a rigid pendulum is forced to rotate uniformly about a vertical axis with angular speed ω. The pendulum has a rod length l and a bob mass m. Let θ be the angle between the pendulum and the vertical. See the figure. Neglect the inertia of the bearing as well as the rod connecting it to the mass. Neglect friction but do include the effects of gravity.



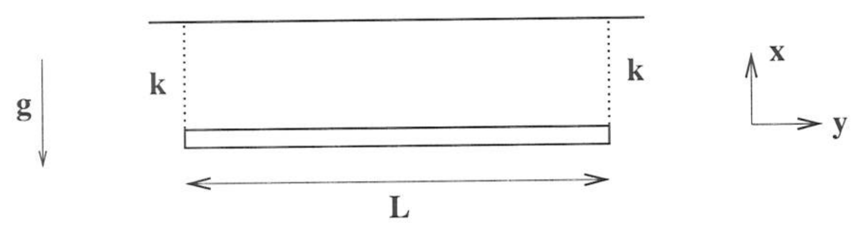
a) Determine the differential equation for θ.

b) At what angular speed ω_c does the stationary point $\theta = 0$ become unstable?

c) For $\omega > \omega_c$, what is the stable equilibrium value of θ?

d) What is the frequency Ω of small oscillation about this point?

20 pts total A rigid homogeneous bar of total length L and mass M is suspended in equilibrium in a horizontal position by identical massless springs of spring constant k at each of its two ends. See the figure. Assume that the bar's center-of-mass can move *only* parallel to the x-direction and that the bar is further constrained to move *only* in the x-y plane.



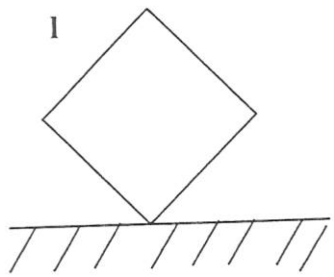
15 pts a) What are the eigenfrequencies of oscillation?

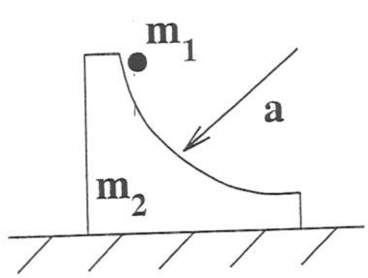
5 pts b) Sketch clearly the normal modes of oscillation and identify the normal mode with its correct eigenfrequency.

20 pts A proton with $\gamma = 1/\sqrt{(1 - v^2/c^2)}$ collides elastically with a proton at rest. If the two protons rebound with equal energies, what is the angle between them?

for Eliana Viahello
by Tom Caan

A homogeneous cube, of edge length l, is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then given a small displacement and allowed to fall. Find the angular velocity when one face strikes the plane if sliding can occur without friction.





Refer to the figure above. A particle of mass m_1 slides down a smooth circular surface of radius of curvature a of a wedge of mass m_2 that is free to move horizontally along the smooth horizontal surface on which it rests.

a. Find the equations of motion for each mass.

b. Find the force of constraint exerted by the wedge on the particle.

8/21/01

Classical Exam

Prof. T.E. Coan (x8-2497)
Fall 2001

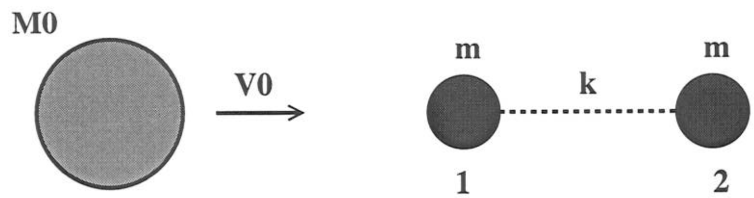
Name ______________________________

BOX YOUR FINAL ANSWERS

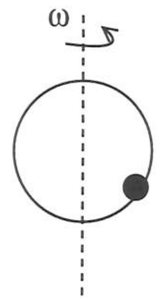
BOX YOUR FINAL ANSWERS

Classical 1. Estimate the mass of earth's atmosphere. Do not write down just a number, but show your reasoning based on familiar physical quantities. If you do not know the numerical value of some quantity, indicate clearly what your assumption is.

What fraction of earth's mass is this?



Classical 2. A ball of mass M_0 moves with velocity V_0 on a horizontal frictionless surface and strikes the first of two stationary target balls of mass m each. The balls are joined together by a massless spring of spring constant k. See the figure. You can consider the collision as head-on, elastic and instantaneous. Describes the motion of target ball 2 as a function of M_0, m, V_0, k and the time after collision t. Assume that there is only 1 collision between M_0 and the target balls.



Classical 3. A particle of mass m is constrained to move without friction on a circular wire of radius a that rotates with constant angular velocity ω about a vertical diameter. See the figure. Recall that Lagrange's equations are:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_j}) - \frac{\partial L}{\partial q_j} = 0, \quad j = 1, 2, \ldots$$

A Find the equilibrium position of the particle.

B Calculate the frequency of small oscillations about the equilibrium position.

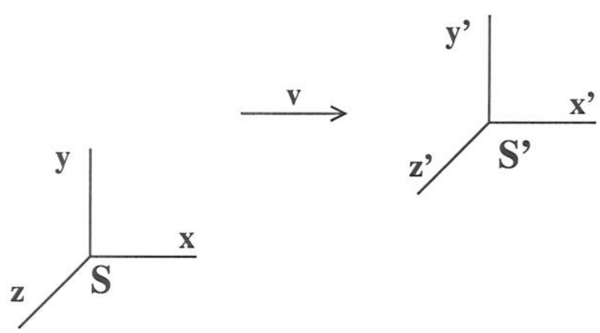
Classical 4. It is estimated that earth's mass accumulation rate due to meteorites is 10^9 grams/yr. Assuming that this mass accumulation rate has been constant since earth's formation and that the mass is accumulated uniformly over earth's surface, calculate the total change in the length of the day due to this effect. Indicate whether the day has increased or decreased. If you do not know the value of some physical quantity, clearly indicate what your assumption is.

Classical 5. An astronaut accelerates radially away from earth at a uniform rate (as measured from earth) of $g = 9.8\,\mathrm{m/sec}^2$.

A Calculate the astronaut's speed at which the red neon lights of Las Vegas become invisible to the human eye. If you do not know the value of a physical parameter, guess it and clearly label your assumption.

B Calculate how long T it takes, as seen from earth, for the astronaut to reach this speed. **Hint:** You can transform continuously to the astronaut's reference frame so that for each transformation the astronaut's proper speed is zero. Useful transformation formulae in an obvious notation are below and refer to the figure. The table contains a useful integral.

$u_x = \text{x velocity in frame S}$	$u_y = \text{y velocity in frame S}$	$\int \frac{d\beta}{1-\beta^2} = \frac{1}{2}\ln\frac{1+\beta}{1-\beta}$
$u'_x = \text{x velocity in frame S}'$	$u'_y = \text{y velocity in frame S}'$	
$u_x = \frac{u'_x + v}{1+vu'_x/c^2}$	$u_y = \frac{u'_y/\gamma}{1+vu'_y/c^2}$	$t = \gamma(t' + vx'/c^2)$
$a_x = \frac{a'_x}{\gamma^3(1+vu'x/c^2)^3}$	$a_y = \frac{a'_y}{\gamma^2(1+vu'_x/c^2)} - \frac{(vu'_y/c^2)a'_x}{\gamma^2(1+vu'_x/c^2)^3}$	



Du, Magar, Wang

Classical Exam

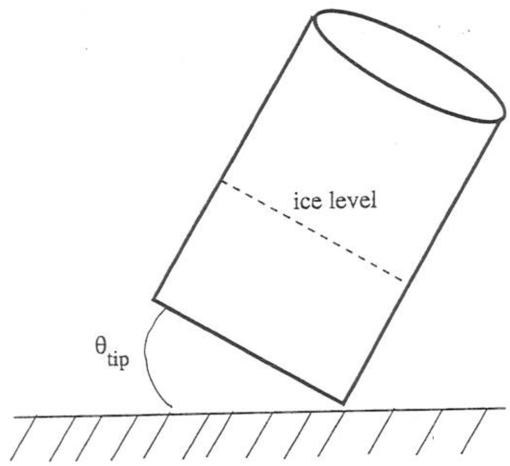
Prof. T.E. Coan
Spring 2000

Name ______________________________

Both problems are required.

BOX YOUR FINAL ANSWERS

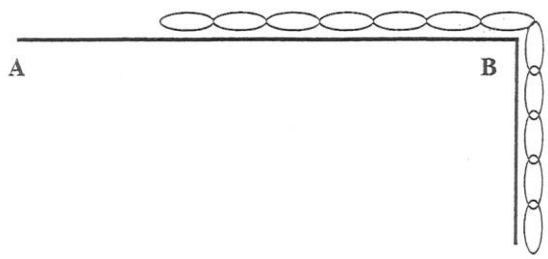
BOX YOUR FINAL ANSWERS



Classical 1. Refer to the figure above. The Texas heat has melted your brain and you have taken to eating ice. A cylindrical glass full of ice has a mass four times that of the empty glass. At what intermediate level of filling is the glass *least* likely to tip over? (The tip angle θ_{tip} is defined in the figure.) I am looking for the height h of the ice level above the bottom of the glass. For simplicity, ignore the mass at the bottom of the glass. Let m be the empty glass mass, ρ be the ice density, r be the glass radius, h_0 be the glass length, and h be the ice level. Assume that the top surface of the ice is always parallel to the bottom of the glass as shown in the figure.

(a) First, convince yourself (and me!) that the maximum tip angle corresponds to the *minimum* center-of-mass (CM) height. Do this by deriving a simple relationship between Z_{CM}, the center of mass of the ice+glass system measured from the glass bottom and along the symmetry axis of the glass, θ_{tip} and the glass radius r. You may draw figures to aid your mathematical explanation.

(b) Determine h. Hint: the actual value of r is totally irrelevant and the mass of the ice in the glass is proportional to h.



Classical 2. A uniform chain of total length a has a portion $0 < b < a$ hanging over the edge of a smooth table AB. (Refer to the figure above.) Show that the time t for the chain to slide off the table if it starts frorm rest is $t = \sqrt{a/g}\,\ln[\frac{(a+\sqrt{a^2-b^2})}{b}]$. You may find the integral $\int \frac{dx}{\sqrt{x^2-c^2}} = \ln[2\sqrt{x^2-c^2}+2x]$, where c =constant, useful.

Daeschler, Trout – Aug 08
by Coan/Fao

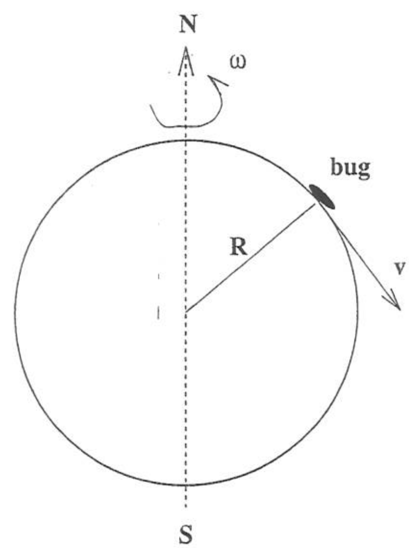
A chain of mass M and length l is suspended vertically above the horizontal surface of a scale so that its lower end just touches the scale. The chain is released and falls onto the scale. What is the scale reading when a length of chain x has fallen? Neglect the size of the chain links.

A globe rotates freely without friction with an intial angular velocity ω_0. A small bug of mass m starts at the North pole and travels to the South pole along a meridian with constant velocity v. The axis of rotation of the globe is fixed. See the figure below. Let M be the mass of the globe and R be the globe radius. You can assume that the globe is solid with uniform mass density so that its moment of inertia $I_0 = \frac{2}{5}MR^2$. Finally, suppose that the time of the bug's trip from pole-to-pole is T. Show that, during the time the bug is travelling, the globe rotates through the angle

$$\Delta\theta = \frac{\pi\omega_0 R}{v}\sqrt{2M/(2M+5m)}.$$

A useful integral is

$$\int_0^{2\pi} \frac{dx}{a + b\cos x} = \frac{2\pi}{\sqrt{a^2 - b^2}}, \qquad (a^2 > b^2).$$

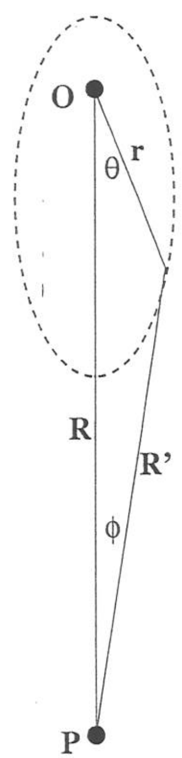


Refer to the figure below. A star, located at O, is at rest with respect to you, located at P. The star explodes, producing a spherically expanding shell of gas moving with speed v. You observe the light from the expanding shell of gas. Assume that there is vacuum between you and the star and let the distance $\overline{OP} = R$. You can assume also that $R \gg r$, where r is the radius of the shell. Show that the *apparent* ('observed') speed of the expanding shell can exceed the speed of light in vacuum c. Specifically,

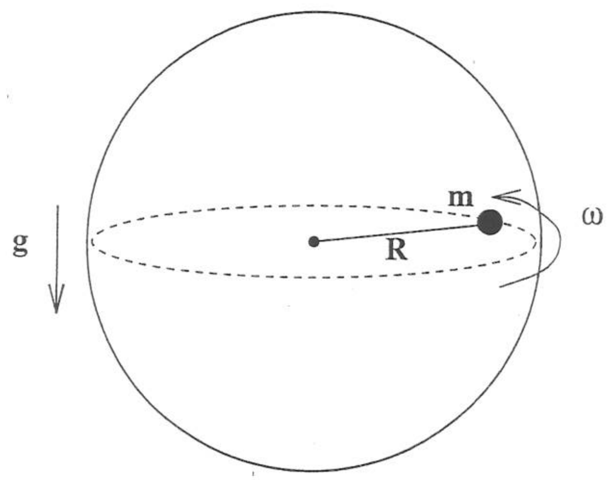
a) **20 pts** Show that the *observed* speed $u_\perp$ of the shell at right angles to the line of sight is given by

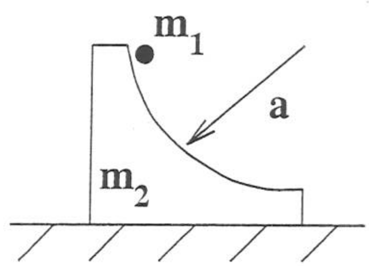
$$u_\perp = \frac{v \sin\theta}{1 - (v/c)\cos\theta}.$$

b) **5 pts** Determine the maximum value $u_{\perp,\max}$ of $u_\perp$ for fixed v, and the angle θ for which this occurs.



A particle constrained to move on the surface of a hollow spherical shell of radius R is projected horizontally from a point at the level of the sphere's center so that its angular velocity about the vertical symmetry axis of the sphere is ω. See the figure below. (You can imagine that the particle is shot from the outside through a small hole in the shell which is then magically sealed.) If $\omega^2 R \gg g$, where g is the acceleration due to gravity, show that the particle's maximum depth below the level of the center is approximately $z \simeq \frac{2g}{\omega^2}\sin^2(\frac{\omega t}{2})$.





Refer to the figure above. A particle of mass m_1 slides down a smooth circular surface of radius of curvature a of a wedge of mass m_2 that is free to move horizontally along the smooth horizontal surface on which it rests.

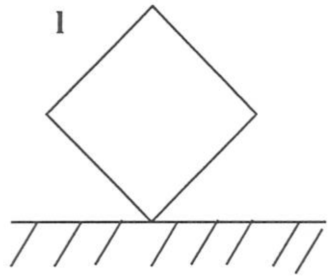
a. Find the equations of motion for each mass.

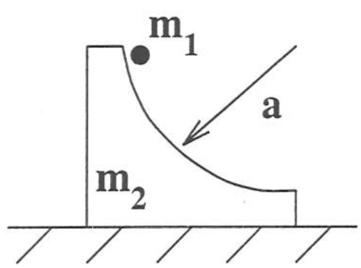
b. Find the force of constraint exerted by the wedge on the particle.

MAY 99

for Eliana Vianello
by Tom Caan

A homogeneous cube, of edge length l, is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then given a small displacement and allowed to fall. Find the angular velocity when one face strikes the plane if sliding can occur without friction.





Refer to the figure above. A particle of mass m_1 slides down a smooth circular surface of radius of curvature a of a wedge of mass m_2 that is free to move horizontally along the smooth horizontal surface on which it rests.

a. Find the equations of motion for each mass.

b. Find the force of constraint exerted by the wedge on the particle.