Electrodynamics Exam

Prof. T.E. Coan (x8-2497)
Prof J. Cooley
Prof R. Kehoe
Fall 2011

Printed Name ______________________________

DIRECTIONS:

0. If we cannot read it, we cannot grade it.

1. **BOX YOUR FINAL ANSWERS**

2. **BOX YOUR FINAL ANSWERS**

3. Do all 8 problems.
4. Paginate all pages. Label the problem number clearly.
5. Staple your pages together, in order.
6. Good luck.

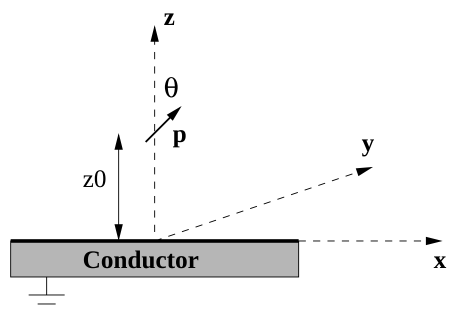
Q1 10 pts. Recall that the magnetic field $\mathbf{B}_{\text{dip}}$ of a dipole with magnetic moment $\mathbf{m}$ can be written in coordinate-free from as:

$$\mathbf{B}_{\text{dip}} = \frac{\mu_0}{4\pi}\frac{1}{r^3}[3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}].$$

For our earth, the magnitude of $\mathbf{m}$ is $m \sim 10^{23}\,\mathrm{A\,m^2}$. Estimate the magnetostatic energy E_B in earth's magnetic field in the region exterior to earth's surface. Be sure to include the proper energy unit and explicitly assume any quantity you do not remember.

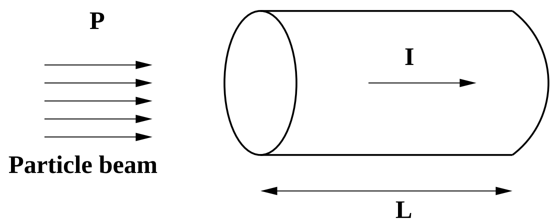
Q2 10 pts. A total charge Q is placed on a uniform, perfectly conducting spherical balloon of initial radius R_0 and total, fixed mass m_B. The balloon is initially constrained so that its radius is fixed. At time $t = 0$ the constraint is released without disturbing the spherical shape of the balloon. Assuming the skin of the balloon does not break and offers no resistance to either stretching or shrinking, what is the final radial speed dr/dt of the balloon skin? Indicate also, possibly with a simple sketch, the radial direction of motion of the balloon skin. Ignore the effects of gravity.

Q3 10 pts. A dipole **p** is placed a perpendicular distance z_0 away from a grounded conducting plane that lies in the xy plane of a right handed coordinate system. See the figure. The dipole **p** makes an angle θ with respect to the normal to the plane and lies in the xz plane. Find the torque τ exerted on the dipole and be sure to indicate its direction. What are the equilibrium values of θ?



Q4 10 pts. Earth's magnetic field can be modeled as a dipole whose axis is tilted approximately 11 degrees with respect to earth's rotational axis. Estimate the power radiated by earth due to this situation. Take the magnitude of dipole moment m_0 of earth to be $m_0 = 10^{23}\,\mathrm{A}\cdot\mathrm{m}^2$. Estimate any quantity you do not know.

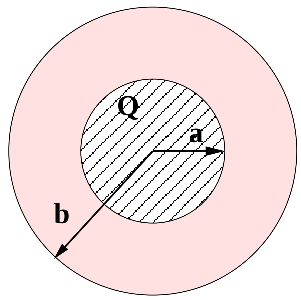
Q5 20 pts. To produce a useful neutrino beam at an accelerator like the main injector at Fermilab, neutrino experiments rely on "magnetic horns" to focus charged particles (e.g., pions and kaons) produced in collisions between energetic protons and a stationary carbon target. The focused charged hadrons then decay to produce a beam-like flux of neutrinos. A magnetic horn is nothing more than a hollow axisymmetric aluminum conductor that has a very large current ($I \simeq 250\,\mathrm{kA}$) run though it when the original protons strike the carbon target. We want to understand something about how the magnetic horn works so let us model it as a solid cylinder of length L that carries a current I parallel to its axis. See the figure below.



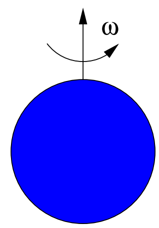
(a) 5 points. Find the magnitude and direction of the magnetic field **B** everywhere *inside* the cylinder. (You can use r to indicate the radial position from the cylinder's symmetry axis and ignore end effects.)

(b) 15 points. A beam of particles (e.g., charged pions), each with momentum P parallel to the axis of the cylinder and each with positive charge q strike the cylinder on its left end. Show that after passing through the cylinder the particle beam is focused to a point and compute the focal length f of the cylinder. (Make a "thin lens" approximation and assume the cylinder is much shorter than the focal length. Neglect the scattering and slowing of the particle beam by the material of the cylinder.) Hint: Ask yourself what is the force on each particle in the beam as it passes through the cylinder? Extra hint: Assume the momentum parallel to the cylinder axis does not change.

Q6 10 pts. A metal sphere of radius a carries a charge Q. It is surrounded, out to radius b, by a linear dielectric material of permittivity ϵ. (Recall that permittivity is the constant connecting $\mathbf{D}$ with $\mathbf{E}$.) See the figure. Find the induced surface bound charge density σ on the dielectric surfaces at $r = a$ and $r = b$.



Q7 10 pts. A total charge Q is distributed uniformly over a spherical shell of radius R and of negligible thickness. The shell rotates with a uniform angular velocity ω. See the figure. Calculate the so-called gyromagnetic ratio of the shell. The gyromagnetic ratio g is defined by the relation $\mathbf{m} = g\mathbf{L}$, for an object with magnetic moment $\mathbf{m}$ and angular momentum $\mathbf{L}$. Hint: It may help if you decompose the sphere into a set of coaxial thin rings.



Q8 10 pts. A charge q of mass m is held at rest a distance s_0 above a perfectly conducting infinite plane held at zero potential. See the figure below. The charge is then released. How long T does the charge take to reach the plane? You may find the following indefinite integral useful:

$$\int -\sqrt{\frac{cs}{s_0 - s}}\, ds = \sqrt{c}\left(\sqrt{s_0 s - s^2} - s_0 \tan^{-1}\sqrt{\frac{s}{s_0 - s}}\right),$$

where c is a constant. Also, be careful about ($\pm$) signs!