Electrodynamics Exam

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Printed Name ______________________________

DIRECTIONS:

0. If we cannot read it, we cannot grade it.
1. **BOX YOUR FINAL ANSWERS**
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3. Do all 6 problems.
4. Paginate all pages. Label the problem number clearly.
5. Staple your pages together, in order.
6. Good luck.

Q1 10 pts. Although air is generally considered an insulator, it does have some conductivity which will cause charge to leak off of conductors placed in air, such as the dome of a van de Graaff accelerator. The sea-level conductivity of air $\sigma \simeq 3 \times 10^{-14}\,(\Omega \cdot \mathrm{m})^{-1}$ is due largely to ions formed by Earth's radioactivity and by the ionizing radiation of cosmic rays. Estimate the characteristic time τ for charge to leak off a conductor, of *arbitrary* shape. **Hint:** You will need to appeal to general principles. Certainly one of these is the conservation of electric charge, $\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$, where J is the current density and ρ is the charge density. Another is Ohm's law, in its appropriate form. Consider a region of an arbitrarily shaped conductor and ask yourself what is the current density J flowing away from this region. Now consider what is happening for the entire surface of the conductor. Express your answer τ in seconds (or minutes) and more than one form of the answer is correct.

Q2 10 pts. A parallel plate capacitor with plate area A and plate separation d has a potential difference V applied to its plates by connecting them to a battery. A dielectric of thickness t ($t < d$), dielectric constant ϵ, and of the same area as the plates is inserted between them such that there is an air gap between the dielectric and both of the plates. What is the capacitance C of this structure? Ignore edge effects.

Q3 10 pts. An isolated bubble of radius 1 cm is at a potential of 100 volts. If the bubble collapses to a spherical drop of radius 1 mm, what is the change ΔW in its electrostatic energy? You can assume the bubble and the drop are uniformly charged and that the charge is on the surface.

Q4 10 pts. A horizontal wire of length L and mass m is sliding without friction down a rectangular, *vertical* wire frame of internal resistance R (see Fig. 1). A constant, uniform magnetic field B is applied normal to the plane of the frame, covering the area over which the wire moves. The wire was released from rest at time $t = 0$. Find the velocity $v(t)$ of the wire as a function of time. Note that an integral of the form $\int \frac{dx}{a+bx}$, where a and b are constants, can be simplified using a simple change of variables.

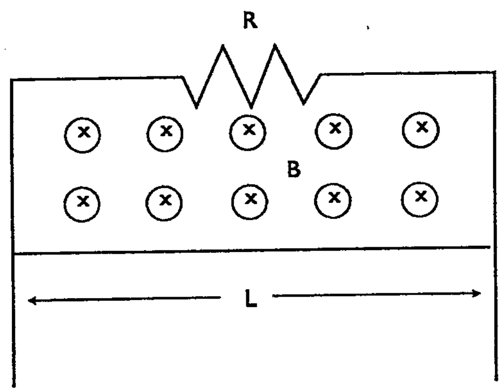

Figure 1: A horizontal wire of length L and mass m slides without friction down a rectangular, vertical wire frame of internal resistance R.

Q5 15 pts. The Zeeman effect observed in the spectra of sunspots reveals the existence of magnetic fields as large as 0.4 Tesla. Assume that the magnetic field is due to a disk of electrons of radius $r = 10^7$ meters that rotates at an angular velocity $\omega = 3 \times 10^{-2}$ rads/sec. You can further assume the thickness of the disk is small compared to its radius.

(a) 10 pts. Calculate the surface density σ of *electrons* required to achieve a B-field of 0.4 Tesla.

(b) 5 pts. Calculate the total current I in the disk.

Q6 15 pts. A non-relativistic positron of charge e and speed v_1 ($v_1 \ll c$) impinges head-on on a fixed nucleus of positive charge Ze. The positron, which comes from far away, is decelerated until it comes to rest and is then accelerated in the opposite direction until it reaches some terminal velocity v_2. Taking radiation loss into account (but assuming it is small) find v_2 as a function of v_1 and Z. **Hint:** First find the speed squared v^2 of the positron as a function of v_1 and r_0, the minimum separation between the positron and the nucleus. Use the fact that radiation losses are small. **Another hint:** How much energy does the positron lose to radiation? For your final answer, you may find the following definite integral useful,

$$\int_a^\infty \frac{dr}{r^3\sqrt{r^2 - ar}} = \left.\frac{2\sqrt{r^2 - ar}(3a^2 + 4ar + 8r^2)}{15a^3r^3}\right|_a^\infty = \frac{16}{15a^3}.$$