Electrodynamics Exam

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Printed Name ____

DIRECTIONS:

0. If we cannot read it, we cannot grade it.

1. **BOX YOUR FINAL ANSWERS**

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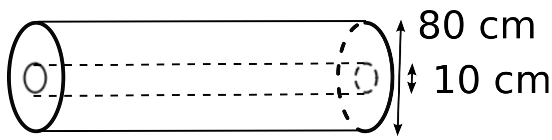
3. Do **all** 6 problems.
4. Paginate all pages. Label the problem number clearly.
5. Staple your pages together, in order.
6. Good luck!

Q1 10 pts. A toroid of rectangular cross-section is wrapped uniformly with fine wire and connected to a DC power supply. The inner radius of the toroid is a, its outer radius b and the number of turns per unit length is n. When connected to the power supply, a steady current I_0 eventually runs through the wire of total electrical resistance R.

a) 5 pts Calculate the energy U_B stored in the toroid associated with the magnetic field produced by the electrical current. Express your answer in terms of quantities defined in the problem and any well-known physical constants. Box that answer.

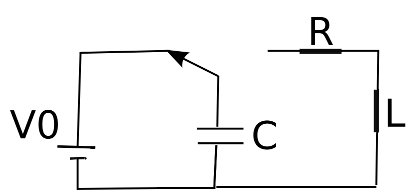
b) 5 pts The voltage of the power supply is increased instantaneously at $t = 0$ by a magnitude $\Delta V = 3I_0R$. Determine the time t' at which the magnetic energy in the toroid *doubles*. Your answer should contain ONLY quantities defined in the problem plus well-known physical constants. No mystery symbols. Box the answer.

Q2 20 pts. The neutrinos that illuminate NOνA's "near detector" at Fermilab are produced by the decays of pions that are themselves the result of protons colliding with a graphite target. After the collisions, the pions are focussed by passing though a pair of aluminum "horns" that are pulsed with an electrical current. The horns have a coaxial geometry such that a toroidal magnetic field is produced in the space between their inner and outer conductors. Current flows along one conductor and then returns along the other. For simplicity, assume that a horn can be approximated as a pair of coaxial conductors with a geometry described by the figure below. A capacitor bank of capacitance C is charged to a voltage V_0 and then discharged through the horns after a switch is thrown to produce the current that runs through the horns.



a) 5 pts Begin by calculating the approximate inductance L of a horn by using the dimensions in the figure. The current of the charged pions and protons is minuscule compared to that in the conductors. Note that $\ln 8 \sim 2$. Box it.

b) 5 pts The simplified circuit describing the horn, its power supply and electrical connections is shown in the figure below. The inductance L represents a horn. After the capacitors $C$are charged, they are discharged through a horn by throwing a switch. Determine the the current as a function of time t that flows through a horn. Your answer should be written in terms of R, L, C, the effective resistance, inductance and capacitance of the effective horn-capacitor bank circuit, respectively. See the figure. Abbreviations are fine. You can assume that the capacitors are to a voltage V_0 before discharge. Box that answer.



c) 5 pts Assume the maximum current through the horns $I = 300\,\text{kA}$. What is the magnitude of the associated magnetic field B at a radial distance 15 cm from the axis of the horn?

d) 5 pts By what angle would a 5 GeV/c pion (rest mass = 138 MeV/c^2) be deflected if it traversed 2 meters of one horn's maximum magnetic field at a radial distance very nearly 15 cm?

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Q3 10 pts. Consider an infinite planar slab of thickness $2a$ lying flat in the x-y plane so the thickness is along the z-axis. This slab contains no free charge but is made from a "linear" dielectric material with constant polarization $\mathbf{P}$ and corresponding permittivity ϵ. However, the *direction* of $\mathbf{P}$ is not necessarily parallel to to the z-axis. There is no material outside the slab.

a) 5 pts Find the electric field $\mathbf{E}$ and electric displacement field $\mathbf{D}$ *inside* the slab. Distinguish between a vector and its scalar magnitude to avoid confusion.

b) 5 pts Find the electric field $\mathbf{E}$ and electric displacement field $\mathbf{D}$ *outside* the slab. You may want to distinguish directions perpendicular and parallel to the slab surface.

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Q4 5 pts. Maxwell's equations are consistent with superconductivity even though superconductivity is a quantum mechanical process. Recall that a substance loses its superconductivity if the B-field in it exceeds a critical value B_c. Consider a lead wire cooled below its "critical temperature ($T_c = 7.2\,\mathrm{K}$) so that it is superconducting. Lead is a "Type-1" superconductor, which means that the B-field inside it is exactly zero when it is in a superconducting state. Electrical current can only flow on its surface. How large a direct current I can this wire carry if its radius $a = 1.0\,\mathrm{mm}$? Recall that lead is non-magnetic. Please box your answer.

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Q5 15 pts. Consider the propagation of a plane electromagnetic wave with angular frequency ω through a poor conductor of conductivity σ, permeability μ and dielectric constant ϵ. You can assume the medium is "linear" and that the electric field $\mathbf{E}$ can be written as $\mathbf{E} = \mathbf{E}_0 e^{i(kz-\omega t)}$.

a) 5 pts Write down the wave equation for the electric field $\mathbf{E}$ in the medium. You may find the identity $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$ useful. Boxes are beautiful.

b) 5 pts Find the real and imaginary part of the wavenumber k. Express k as $k = \alpha + i\beta$. Assume $k > 0$ and that $\frac{\sigma}{\epsilon\omega} \ll 1$. Can you box?

c) 5 pts Determine the attenuation length δ of the wave inside this medium. Recall that the attenuation length is the the distance the wave travels for its amplitude to reduce to e^{-1} of its original value. Your answer should not contain any mystery symbols. Box that answer.

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Q6 15 pts. A semi-infinite solenoid of radius R and n turns per unit length carries a current I. Find an expression for the *radial* component of the magnetic field $B_r(z_0)$ near the axis at the end of the solenoid where $r \ll R$ and $z_0 = 0$. You can assume the coils on the solenoid are circular. Hint: First find a general expression for the magnetic field for a point on the axis of the solenoid. Then, imagine a short, thin cylinder near the end of the solenoid and coaxial with it. Use a well-known implication from Maxwell's equations to do the rest. You can do it. Please box that answer.

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