Electrodynamics Exam

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Printed Name ____

DIRECTIONS:

0. If we cannot read it, we cannot grade it.

1. **BOX YOUR FINAL ANSWERS**

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3. Choose **any** 4 (and **ONLY** 4) out of 6 problems. Choosing problems with a cumulative point total that exceeds 100 is OK.
4. Paginate all pages. Label the problem number clearly.
5. Staple your pages together, in order.
6. Good luck!

Q1 25 pts. Near Earth's surface there exists an electric field **E** that points downward with magnitude $E = 100\,\mathrm{V/m}$. Although it isn't true, assume for this problem that this field is due to a uniform charge density ρ throughout Earth's volume.

a) 10 pts Begin by calculating this electric charge density ρ. We need both a symbolic **and** a numerical answer. Numerical *estimates* are fine. **If** you do not know the numerical value of some physical quantity (e.g., Earth's radius R_E), estimate it and label clearly what your estimate is. Box that answer.

b) 5 pts You may be aware that Earth's equatorial diameter is slightly larger than its polar diameter, i.e., Earth bulges about its equator due to its rotation. Model this bulge as a ring of some minor radius r_b. What is the size of r_b as a function of Earth's radius R_E? That is, write $r_b = \mathrm{C}R_E$ and tell us what C is. Estimates are fine!! Box that answer.

c) 10 pts Your model of Earth is now a sphere with a ring around its equator. Each body has a constant charge density ρ you calculated above. Calculate the electric field $\boldsymbol{E_A}$ at a point on the rotational axis of Earth a distance $100R_E$ away from its center. We seek an *analytical* expression in terms of known quantities and be sure to indicate the *direction* of the field. A clear sketch may be useful for the latter.

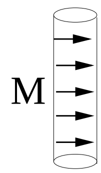
Q2 25 pts. Your job is to design a physics demonstration called Elihu Thomson's jumping ring. There is a working model on the table. The insulated wire is 18 gauge (diameter $= 1.024\,\mathrm{mm}$) copper with resistivity $\rho = 1.68 \times 10^{-8}\,\Omega\,\mathrm{m}$ wound tightly with adjacent coils touching around a hollow cylinder of radius $r = 3\,\mathrm{cm}$ and length $\ell = 30\,\mathrm{cm}$. The cylinder is filled with iron wire which has a relative magnetic permeability μ/μ_0 of approximately 1000 to enhance the magnetic field. Figure out roughly how many turns of wire N should be wrapped around the cylinder so that when it is connected to standard U.S. 120-volt rms 60 Hz alternating current from the wall outlet, the rms current does not exceed 1.5 amp rms (this will not trip the circuit breaker for the room nor blow the fuse in series with the apparatus).

a) 10 pts What is the inductance L of an iron-core solenoid with these dimensions and N turns of wire?

b) 5 pts What is the DC resistance R of the solenoid?

c) 10 pts How many turns N of wire should be used?

Q3 30 pts. Sintered neodymium rare-earth permanent magnets can be cast in almost any shape with magnetization set in any direction. One version is a solid cylinder with a radius of 2 mm and a length of 100 mm with uniform magnetization $\vec{M}$ of 2.4×10^6 A/m frozen perpendicular to the symmetry axis of the cylinder. See the figure.



In the following, give the answers both in symbols and numerically to one significant figure with units.

a) 8 pts Calculate the bound volume current density $\vec{J}_b$ (magnitude and direction).

b) 8 pts Calculate the bound surface current density $\vec{K}_b$ (magnitude and direction).

c) 12 pts Calculate the magnetic field $\vec{B}$ (magnitude and direction) on the symmetry axis. Hint: make the approximation that the magnet is infinitely long and imagine replacing the magnetic material with fine current-carrying wires running parallel to the symmetry axis; then use the standard result for the magnetic field around an infinite straight wire.

d) 2 pts How does this magnetic field strength compare to the Earth's magnetic field strength? Does this seem reasonable?

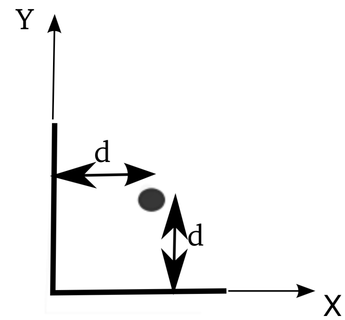
Q4 25 pts. You are well aware that real materials have a wide range of electrical conductivity σ, or equivalently, resistivity. Those with high conductivity we call "conductors" and those with low conductivity "insulators."

a) 10 pts It is well known that sea water is conductive. Its average conductivity σ for both oceans is $\sigma = 3.3\,\mathrm{S/m}$ in SI units, changing somewhat with temperature and salinity. You build an underwater drone, shaped as a shark, to scare away pesky scuba divers from some pirate treasure you have discovered. You communicate with the submerged drone by sending it radio waves with a typical frequency $f = 1000\,\mathrm{Hz}$. Estimate the maximum depth d beneath the ocean surface the drone should "swim" in order for you to communicate with it efficiently. Box your answer.

b) 15 pts You are well aware that the differential form of Ohm's law in a material is $\mathbf{J}_f = \sigma\mathbf{E}$. Here, $\mathbf{J}_f$ is the free current density in the material, σ is the material's electrical conductivity and $\mathbf{E}$ is electric field in the material. You are also aware the charge is conserved and that the continuity relation expresses this, $\nabla \cdot \mathbf{J}_f + \frac{\partial \rho}{\partial t} = 0$, where ρ_f is the density of free charge and t is time.

A certain amount of free charge is deposited in a plate of glass. *Roughly*, how long T does it take the charge to migrate to the surface? The glass' conductivity $\sigma = 10^{-12}\,\mathrm{S/m}$ and its permittivity $\epsilon = 2.25\epsilon_0$, where ϵ_0 is the permittivity of free space. **Hint:** Use the above relations in the first paragraph and Maxwell's equations to derive a simple differential equation for $\rho_f(t)$. This differential equation is so simple you can solve it by inspection! Your answer should be written as a symbolic answer **and** also include a number with proper units. Box both parts of your answer .

Q5 20 pts. A charge Q is placed near the corner of a perfectly conducting folded sheet held at ground potential. The charge's position is equidistant from each surface. Each half-plane of the folded sheet can be considered to be infinite in extent and the half-planes are perpendicular to one another. See the figure. Compute the force $\mathbf{F}$ on the charge due to the image charges formed by this configuration. Be sure to indicate the direction of the force in some sensible and unambiguous way. Of course, you will box your answer.



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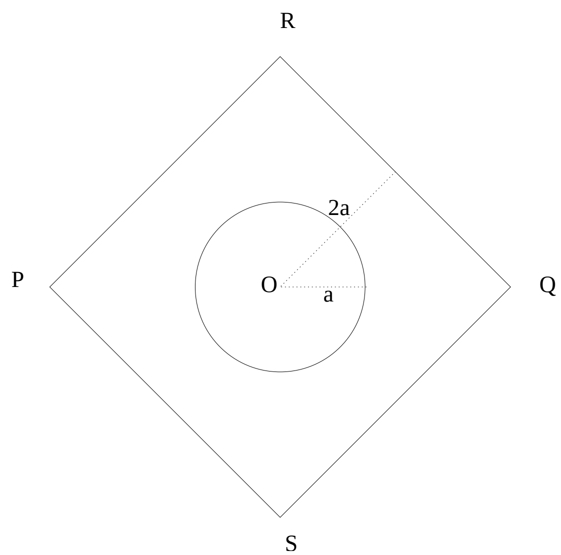
Q6 30 pts. The Aharonov-Bohm effect is a fascinating quantum mechanical phenomenon in which the reality of the vector potential field is demonstrated by changing the phase of a charged particle's wave function as seen in shifting interference patterns. A pair of in-phase electrons travel from P to Q along the two different paths PRQ and PSQ. If the line integral of the vector potential along the closed loop PRQSP is zero, then the electrons will arrive in phase.



The vector potential is given in cylindrical polar coordinates $\{s, \phi, z\}$ as

$$\vec{A}(\vec{r}) = \mu_0 k s \hat{\phi} H(a-s) + \mu_0 k \frac{a^2}{s} \hat{\phi} H(s-a)$$

for constants k and a where H is the Heaviside step function.

The curl in cylindrical polar coordinates is

$$\vec{\nabla} \times \vec{v} = \left[\frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi} \right] \hat{z}$$

a) 10 pts What is the magnetic field $\vec{B}(\vec{r})$ everywhere (magnitude and direction)?

b) 10 pts What is the volume current density $\vec{J}(\vec{r})$ everywhere (magnitude and direction)?

c) 10 pts What is the line integral of the vector potential along the closed loop PRQSP $\oint \vec{A}(\vec{r}) \cdot d\vec{\ell}$? Hint: Use Stokes' Theorem to write this in terms of the magnetic flux Φ_B enclosed by the square PRQSP.