Electrodynamics Exam

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Printed Name ________________________________

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DIRECTIONS:

0. If we cannot read it, we cannot grade it.

1. **BOX YOUR FINAL ANSWERS**

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3. Do 2 out of 3 "L" problems. Do 4 out of 5 "S" problems.
4. Paginate all pages. Label the problem number clearly on **EACH** sheet.
5. **DO NOT** staple pages together so we can bulk scan efficiently.
6. Good luck!

L1 16 pts. A circular conducting wire ring of radius R carries current I and sits in the x-y plane with the origin at the center of the ring.

a) What is the volume current density $\mathbf{J}(\mathbf{r})$ in cylindrical polar coordinates? Box that answer.

b) What is the magnetic field $\mathbf{B}(\mathbf{z})$ on the symmetry axis of the ring?

c) Two identical rings are placed at $z = h/2$ and $z = h/2$. What is the magnetic field $B(z)$ on the symmetry axis of the ring?

d) Find the separation distance h that will make the magnetic field at the origin as flat as possible. That is, ensure that both the first and second derivatives of $B(z)$ with respect to z vanish at the origin.

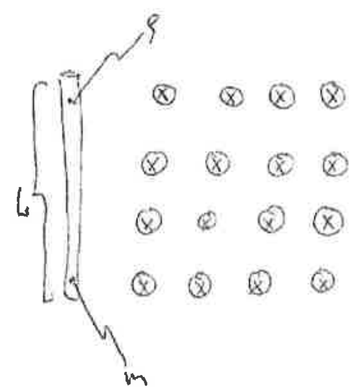
e) If $R = 15\,\text{cm}$ and $I = 1\,\text{amp}$, how many coils N of wire must be wrapped in each coil to give a magnetic field at the origin as strong as the Earths field?

L2 16 pts. Consider a metal rod of mass m, resistivity ρ, length L and cross-sectional area A. The rod is very much longer than its radius and is placed entirely in a region of constant magnetic field pointing into the plane of the paper and of magnitude B. See the figure. If a constant force of magnitude F is applied perpendicularly to the length of the rod at $t = 0$, show that the expression for the speed of the rod as a function of time $v(t)$ is given by

$$v(t) = \frac{F\tau}{m}(1 - e^{-t/\tau})$$

,

where $\tau = \frac{\rho m}{ALB^2}$. Show your work in a clear manner to receive full credit.



L3 16 pts. Consider a flat coil of radius a made form N turns of thin copper wire. The thickness of the coil is much less than its radius. An oscillatory current $I = I_0 \sin \omega_0 t$ flows through each turn of the coil, where the symbols have their conventional meaning. You observe the coil at a distance r away from the coil, where $r \gg b$. You can also assume that the wavelength of the emitted radiation is much greater than the coil radius.

a) 4 pts What is the magnitude m of the magnetic moment in terms of the quantities described above? Box that answer.

b) 4 pts Draw a clear sketch to indicate the *direction* of the coil's magnetic moment.

c) 4 pts If the peak magnitude of the oscillatory current I_0 suddenly doubles, what is the ratio R of the power now emitted by the coil in the form of magnetic dipole radiation to the power emitted *before* I_0 changed. Box the answer.

d) 4 pts If the angular frequency ω_0 of the oscillatory current I_0 suddenly doubles, what is the ratio R of the power now emitted by magnetic dipole radiation to the power emitted before ω_0 changed? Box that answer.

S1 4 pts. Estimate the total charge Q of all the *electrons* in your body.

S2 4 pts. When the LHC is running at design power, the protons will each have 7 TeV of total energy. What will be the difference Δv between the proton speed and c in m/s?

S3 4 pts total. Two conducting spheres have radii R_1 and R_2, and are far apart compared to either radius.

a) 2 pts. What is the capacitance C in MKS units? Box that answer.

b) 2 pts. If both spheres are Earth-sized, what is the capacitance C numerically? Box the answer.

S4 4 pts. A relativistic electron is constrained by a magnetic field in some astrophysical environment to travel in a circle.

a) 2 pts If for some reason the energy of the electron doubles but still travels in the same orbit, what is the ratio R of the power now radiated by the electron to the power it radiated *before* its energy doubled? (Suppose there is some process that replenishes the electron's energy.) Box your answer.

b) 2 pts If the radius of the electron's circular orbit somehow halves but its energy remains constant, what is the ratio R of the power radiated by the electron after the orbit change to the power emitted *before* the orbit change? Box your answer.

S5 4 pts. Consider a solenoid of length L and radius R that has an iron core that extends the entire length of the solenoid's single-layer wire wrapping. Our solenoid is very long ($L >> R$) and has an azimuthal wrapping of N wires per unit length. The iron, with uniform and constant magnetic permeability μ, and uniform and constant electrical conductivity σ, completely fills the interior of the solenoid. Initially, the solenoid has no current running though it but at $t = 0$ current of magnitude I suddenly runs through the solenoid's wires. The current remains at this magnitude indefinitely without changing direction.

What is the magnitude B of the steady-state B-field in the solenoid's interior, well away from its ends? Box your answer.