Your name:

Electricity and magnetism **Qualifying Ph. D. exam**

June 25, 2010

Instructions. Please solve all three problems. You are allowed to use one textbook of your choice, one math reference, and a calculator. Figures for each problem are shown on a separate page. **In each solution, state clearly the system of units you are using.**

Problem 1. In Bohr's model of a neutral hydrogen atom, a negative charge distribution created by the electron's quantum motion surrounds a pointlike positively charged proton at $r = 0$. The electric potential is given by

$$\varphi(r) = \frac{q}{4\pi\epsilon_0}\frac{e^{-\beta r}}{r}\left(1 + \frac{\beta r}{2}\right),$$

where $\beta = 2/a_0$, and a_0 is Bohr's radius (a known constant). Find the charge density $\rho(\vec{r})$ as a function of distance r from the center of the atom, including at the center $r = 0$. Check that your expression for $\rho(\vec{r})$ reproduces the total charge of the atom, when $\rho(\vec{r})$ is integrated over all space. Find net electric flux through a spherical surface of radius R centered at the atom's nucleus.

Problem 2. A wire loop in the shape of a square with side length ℓ carries current I. The unit vector to the plane of the loop is $\vec{n} = \{n_x, n_y, n_z\}$, and the center of the loop is at the point $M = \{0, y_0, 0\}$. The loop is placed in an external magnetic field with vector potential

$$\vec{A}_K = \frac{\mu_0 K}{4\pi}\ln\left[x^2 + y^2\right]\hat{z}.$$

1. Find the magnetic field $\vec{B}_K$ corresponding to $\vec{A}_K$ and sketch its magnetic field lines. How can such magnetic field be created?
2. Can a different vector potential $\vec{A}_K$ generate the same field $\vec{B}_K$? Explain.
3. Find the torque on the loop in the field $\vec{B}_K$ in the dipole approximation.
4. Find the energy of interaction of the loop with $\vec{B}_K$ in the dipole approximation.
5. Find the magnetic field $\vec{B}_I$ created by the loop at a point $N = \{0, x_0, 0\}$, where $x_0 \gg \ell, y_0$.

Problem 3. The sun is an average star with a mass of 2.0×10^{30} kg. It radiates 3.8×10^{26} W of power. Consider a comet orbitting the sun and composed of spherical, black conducting particles each with radius a and a density of 4000 kg/m^3.

1. Calculate the average radiation force exerted by the sun on the particles in the comet.

2. At what value of a does this force equal the force of gravity from the sun? Does this solution for a depend on the radial distance to the sun, r, and why?

Eventually, the sun will exhaust its supply of hydrogen fuel and ultimately transition to a 'white dwarf' stage in which thermonuclear reactions do not occur. It will shrink to a radius of perhaps 10,000 km. A magnetic dipole field can be produced and may vary in strength. Assume this field has a dipole moment of 10^{44} Am^2 and varies from zero to full magnitude, keeping a constant orientation, with a period of 10 minutes.

3. What fraction of the total power in the magnetic dipole field is radiated within ± 45 degrees of the plane perpendicular to the dipole axis?