Classical Exam

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Spring 2009

Printed Name _____

DIRECTIONS:

0. If we can't read it, we can't grade it.
1. # BOX YOUR FINAL ANSWERS
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3. Paginate all pages. Label the problem number clearly.
4. Staple your pages together, in order.
5. Good luck.

Complete **any 3** of the following problems. (The total points for the three problems are 15.) I am interested in order of magnitude estimates only. Show clearly your reasoning and estimates of key numbers.

FP 1. Estimate the number N_B of atoms in your body.

FP 2. Estimate the mass M_A of earth's atmosphere

FP 3. How much slower Δv than the speed of light does a 10 TeV proton travel? Make any approximations you consider reasonable.

FP 4. What fraction of the Sun's mass is converted to energy during its 10 Gigayear lifetime? (Assume the luminosity of the sun is constant.)

2 15 pts. An impoverished graduate student in need of money takes a part time job for NASA. She designs a pendulum clock for use on a gravity-free spaceship. The mechanism is a simple pendulum (a mass m at the end of a massless rod of length l) hung from a pivot, about which it can swing in a plane. To provide artificial gravity, the pivot is forced to rotate at an angular frequency ω in a circle of radius R in the same plane as the rod. See the figure. Show that this scheme succeeds, i.e., that the possible motions $\theta(t)$ of this pendulum are *identical* to the motions $\theta(t)$ of a simple pendulum in a uniform gravitational field of strength $g = \omega^2 R$, not just for small oscillations, but for any amplitude, and for any length l, even for $l > R$.
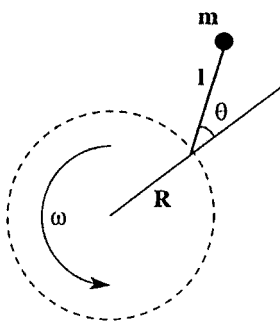
3. 15 pts. A mass m_1 with initial velocity V_0 strikes a mass-string system m_2, initially at rest but able to recoil. The spring is massless with spring constant k, see the figure. There is no friction. What is the maximum compression of the spring?
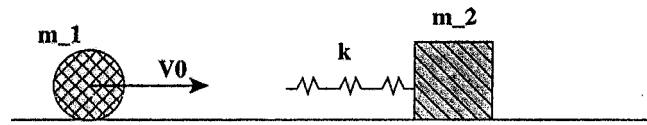
4. **15 pts** An atom in its ground state has mass m. It is initially at rest in an excited state of excitation energy ΔE. It makes a transition to its ground state by emitting one photon.

a) Find the frequency ν of the photon, accounting for the relativistic recoil of the atom.

b) Express your answer also in terms of the mass M of the excited atom.

5. 15 pts. A particle moves in the potential $V(r) = -C/(3r^3)$, $C > 0.$. Assume its angular momentum L is known. Let the particle come in from infinity with speed v_0 and impact parameter b. In terms of C, m and v_0, what is the largest value of b (call it b_{max}) for which the particle is captured by the potential? In other words, what is the "cross section" for capture, πb^2_{max}, for this potential?

6. 15 pts. The Coriolis force arises due to the earth's rotation.

a) Compute the angular velocity Ω for the earth. What is the direction of Ω? Be sure and indicate the units. A sketch may help.

b) You place a mass M on a weight scale and place the entire apparatus in a truck. What *direction* should you drive the truck so that the Coriolis force maximally reduces the apparent mass as measured by the scale? (Assume the truck is driving level on earth's surface.)

c) The mass M is dropped from a *stationary* hot-air balloon which is a height h meters above earth's surface. The balloon is directly above some surface point A. Compute where the mass M lands relative to point A. Give both the *magnitude* and *direction* of the displacement. This may be a function of Ω, the gravitational acceleration g, and the colatitude ψ where the colatitude is the complementary angle of the latitude; i.e., $\psi = 0$ at the north pole, $\psi = \pi/2$ at the equator, and $\psi = \pi$ at the south pole. (Hint: check that your answer makes sense at these three points.) (You may use approximations as you deem necessary; in particular, you may neglect terms of order Ω^2 and assume g is a constant over the range of the fall.)

Statistical Mechanics and Thermodynamics
December 2008

You may use Reif or another text of your choice, but no other notes or references.

1. Consider two distinguishable marbles both of mass m which may be found on (but not between) any step of a staircase, with the steps a distance h apart. Consider the potential due to gravity, but ignore their kinetic energy, or any internal energy in either the marbles or stairs. (The marbles can, however, bounce each other from step to step.)

(a) The system (marbles and stairs) is isolated, in equilibrium, and known to have energy $4mgh$ (with high precision) relative to its minimum. Enumerate the possible configurations and give the probability of each.

(b) Compute the average and standard deviation for the height of the first marble.

(c) The system is now placed in thermal equilibrium with a reservoir at temperature T. Give the probability distribution for possible configurations in this case.

(d) Compute the average height for the first marble in this case.

(e) Give an expression for the average energy in this case as a function of T when T is large. Explain your answer. At what T should this become accurate? (It's possible to answer this without further calculation.)

Choose one of the following two problems:

2. A particular stretched steel spring tends to contract as it is heated, and is observed to obey an equation of state

$$F = aT^2(L - L_0) \qquad (1)$$

where F is the tension, L_0 the unstretched length, and a is a constant.

(a) Write a relation expressing the change dE in mean energy of the spring in terms of changes in entropy dS and length dL.

(b) Calculate the change in entropy $\Delta S = S(L) - S(0)$ and the change in mean energy $\Delta E = E(L) - E(0)$ of the spring when it is stretched at a constant temperature T_0 from a length L_0 to a length L.

(c) Calculate the work W done and heat Q absorbed by the spring as it is stretched at this constant temperature from a length L_0 to a length L.

3. SMU physicists have discovered a new material that they named Malaisium. The internal energy of Malaisium satisfies the following relation:

$$E(S, V, N) = \frac{aS^5}{V^2 N^2}$$

where S is the entropy, V is the volume, and N is the particle number. The numerical value of the constant a is 2.00×10^{48} in MKS units.

(a) Derive the analog of the ideal gas law for this system - an equation of state relating pressure p, temperature T, N and V.

(b) How much work is required to expand isothermally 3 moles of Malaisium from $V_i = 2$ m^3 to $V_f = 3$ m^3 at a temperature $T = 300$ K? Recall one mole is $N_0 = 6.02 \times 10^{23}$.

(c) How much work is required to expand adiabatically 3 moles of Malaisium from $V_i = 2$ m^3 to $V_f = 3$ m^3?

(d) At a pressure of $p = 10^5$ Pa, a quantity of Malaisium is placed in a sealed chamber. The chamber is thermally insulated so that no heat is exchanged between sample and environment. Additional pressure is applied and the change in volume is recorded. What is the measured compressibility $\kappa = -V^{-1}\frac{\partial V}{\partial p}$? (Note that Pa stands for pascals, the MKS unit for pressure.)

Choose one of the following two problems:

4. Consider a chain with $N \gg 1$ massless links of length ℓ that can be oriented freely in three directions with respect to the link above: left, right, or down, as shown in the figure. Suppose that the upper end of the chain is fixed, a constant force F is applied to the lower end of the chain, and the system is in thermodynamic equilibrium at temperature T.

   (a) Give the partition function Z first for one link, then for the entire chain.

   (b) What is the mean vertical extension, $\overline{L_z}$, of the chain?

   (c) What is the mean internal energy, $\overline{E}$, of the chain?

   (d) Consider the high and low temperature limits, $F\ell \ll k_B T$ and $F\ell \gg k_B T$, respectively. Solve for $\overline{L_z}$, $\overline{E}$, and give the entropy S of the chain in these limits. Explain the result for S.
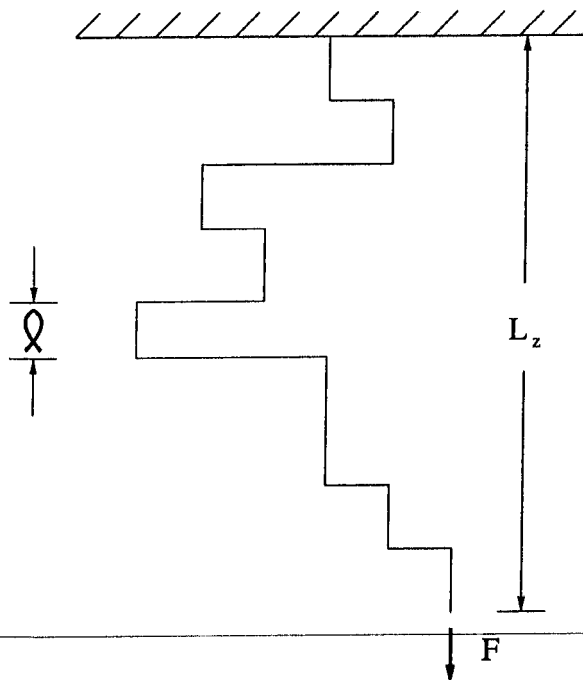
5. A brick of mass M sits on the floor, in thermal equilibrium with the room at temperature T.

   (a) Using entropy considerations, give the probability of finding the brick at a height between L and $2L$ the next time you look. You may treat the brick as a point mass; that is, ignore its own internal energy and contribution to entropy.

   (b) Use the Boltzmann distribution for the same system to give the probability of finding the brick at a particular position and momentum. Compute the partition function and the average height.

Quantum Mechanics PhD Qualifying Exam
Spring 2009

Please answer two questions from section A and two questions from section B. You may use one math reference and one Quantum Mechanics textbook. Under no circumtances are you allowed to use the internet. All work must be your own.

SECTION A

1. Suppose that the hypothetical wavefunction $\psi(x) = A/(a^2+x^2)$, $(-\infty \leq x \leq \infty)$ is a solution of the Schrodinger equation for some potential $V(x)$ with energy E.

   (a) Find the normalization constant A in terms of a.

   (b) Find the potential $V(x)$.

   (c) Evaluate $\Delta x \Delta p$ and verify if the uncertainty principle is satisfied.

   Useful identities,

   $$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}(x/a)$$

   $$\int \frac{dx}{(a^2 + x^2)^n} = \frac{1}{2a^2(n-1)} \left[\frac{x}{(a^2 + x^2)^{n-1}} + (2n-3) \int \frac{dx}{(a^2 + x^2)^{n-1}} \right]$$

2. The Hydrogen Atom.

   (a) Determine $<r>$ and $<1/r>$ for a hydrogen atom in the $(n, l, m) = (2, 1, 0)$ state.

   (b) Verify that the most probable value of, r, for the position of an electron in this state is $4a_o$, where a_o is the Bohr radius.

   (c) Find the probability of finding the electron between 3.9 and 4.1 Bohr radii from the nucleus.
   You may find this identity useful,

   $$\int_0^\infty x^n e^{-x/a} \, dx = n! \, a^{n+1}$$

3. An electron is in the spin state,

   $$\chi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

   (a) Normalize this state vector.

   (b) Find the expectation values of S_x, S_y, and S_z.

   (c) Find the "uncertainties" σ_{S_x}, σ_{S_y}, and σ_{S_z}. (Note: These sigmas are the standard deviations, not Pauli matrices!)

   (d) Find the probability of observing (i) $S_z = \hbar/2$ and (ii) $S_y^2 = \hbar^2/4$.

SECTION B

1. Transition Rates in Hydrogen.

   (a) Show that in the dipole aproximation for the decay of a $3d$ state of hydrogen atom to the $2p$ level, if the initial state has $m = 2$, the only possible final state is $m = 1$.

   (b) Compute the decay rate for this the transition in part (a).

2. This problem concerns the hydrogen atom in a constant external electric field. When placed in an external field the atom will deform and develop an induced dipole moment which contributes an extra interaction energy term to the hamiltonian of the form,

$$H' = -e\vec{r} \cdot \vec{\mathcal{E}},$$

where we take the external electric field $\mathcal{E}$ to be along the z-axis.

   (a) Show that the first order energy correction to the ground state energy of hydrogen due to this pertubation is zero.

   (b) For the first excited state show that,

$$< l', m'|H'|l, m >= 0$$

   unless $l' = l \pm 1$ and $m' = m$. Hint: Use parity arguments and the fact that L_z commutes with z.

   (c) Find the second order corrections to the Hydrogen ground state energy.

   (d) Find the linear combination of the $n = 2$ states which are eigenkets of H' corresponding to the two non-zero eigenvalues found in the previous part. Note that they are not parity eigenkets because H' is not parity invariant.

   (e) The derivative of the energy with respect to the magnitude of the electric field is the induced dipole moment. Obtain the induced dipole moment.

3. Consider a state of three spin one particles (three bosons).

   (a) Enumerate all the possible values for the total angular momentum for the combination of these three states. Some values may be repeated, please indicate how many times. There will be a total of 27 possible states with distinct values of j and m, please count to make sure you have all of them.

   (b) Show that the totally antisymmetric combination of the three angular momentum states, $|l_1 = 1, m_1 >$, $|l_2 = 1, m_2 >$, and $|l_3 = 1, m_3 >$, such that $m = m_1 + m_2 + m_3 = 0$ corresponds to $j = 0$.

   (c) Show that the totally symmetric combination of the three angular momentum states, $|l_1 = 1, m_1 >$, $|l_2 = 1, m_2 >$, and $|l_3 = 1, m_3 >$, such that $m = m_1 + m_2 + m_3 = 0$ corresponds to $j = 3$

   (d) Explain how these results show that a spin zero particle at rest cannot decay into three spin 1 particles. This is known as Yang's theorem in particle physics.

Your name:

Electricity and magnetism

Qualifying Ph. D. exam

June 15, 2009

Instructions. Please solve any three out of the four following problems. You are allowed to use one textbook of your choice, one math reference, and a calculator. Figures for each problem are shown on a separate page. **In each solution, state clearly the system of units you are using.**

1. A cylindrical conductor of radius a and length l has two cylindrical cavities of diameter slightly smaller than a ($a - \epsilon$, where ϵ can be ignored) through its entire length. The origin of the reference frame coincides with the geometrical center of the cylinder. The z axis is directed along the cylinder's main axis. A uniform steady current I flows through the conductor.

    (a) What is the current density j?

    (b) Find the magnetic field $\vec{B}$ in the xy plane ($z = 0$) at a moderate radial distance s ($a < s \ll l$) from the axis of the conductor. Sketch your choice of the observation point, directions of x and y axes, direction of $\vec{B}$, and other relevant information in the figure

    (c) Find the magnetic field $\vec{B}$ at *large* distance r ($r \gg a$, $r \gg l$), not necessarily in the xy plane

    (d) Does the r dependence of $\vec{B}$ at large r agree with the multipole expansion and differential Ampere's law? If not, explain why.

2. A coaxial cable consists of a solid cylindrical conductor of radius s_1, surrounded by a cylindrical conducting tube of inner radius s_2 and outer radius s_3. An insulating layer with a relative electric permittivity ϵ_r and relative magnetic permeability μ_r separates the inner core and outer shell.

    (a) Sketch and briefly describe the distribution of charge throughout the cable's cross section when the inner core carries a uniform line charge density λ.

    (b) Find C, the capacitance of the cable per unit length

    (c) A current I flows in one direction through the inner core and returns through the outer shell. Describe the current distribution in the cross section if I is direct; if I alternates with high frequency.

    (d) Characteristic impedance Z of the cable in a lossless condition (assuming no resistance of the conductors and infinite resistance of the insulator) is defined as $Z = \sqrt{L/C}$, where L and C are inductance and capacitance of the cable per unit length. Find Z for a high-frequency AC current, in a common situation when magnetic flux exists mostly in the insulating layer.

1

3. A small dielectric sphere S with time-dependent uniform polarization $\vec{P} = \{0, 0, P_0 \cos\omega t\}$ is suspended at height z above the origin of the coordinate system. The radius R of the sphere is much smaller than z in all cases.

   (a) What is the net dipole moment of the sphere?

   (b) How does the wavelength λ of electromagnetic radiation depend on angular frequency ω of dipole oscillation?

   (c) Find $\vec{E}$ at the origin when $\lambda^2 \gg z^2 + s^2$ (long wavelengths). How is this dynamical regime called?

   (d) Briefly describe dependence of $\vec{E}$ in the xy plane when $\lambda^2 \ll z^2 + s^2$ (short wavelengths). How is this dynamical regime called?

   (e) Is there a non-zero flux of electromagnetic energy I_f through the xy plane in case (c)? In case (d)?

   (f) The average Pointing vector $\langle \vec{S} \rangle$ at distance $\vec{r}$ from S is given by

$$\langle \vec{S} \rangle(\vec{r}) = \kappa \frac{\sin^2 \theta}{r^3} \vec{r},$$

   where κ is a constant. Compute the total radiation energy striking the entire xy plane per unit time. You may need an integral

$$\int_0^\infty \frac{s^3 ds}{(s^2 + z^2)^{5/2}} = \frac{2}{3z}.$$

4. A particle collider accelerates electrons along a circular ring until they acquire energy $E = 0.1 \cdot 10^{12}$ electronvolt $= 1.6 \cdot 10^{-8}$ J. The electrons are kept in the ring by a vertical magnetic field $B = 0.1$ T.

   (a) Estimate the needed radius R of the ring, given the cyclotron formula $p = eB\,R$ for the relativistic momentum p and electric charge e of the electron. The electron's rest energy can be neglected.

   (b) Assume that $\vec{B}$ is produced by a long solenoid with length $a = 4\pi$ m, radius $r = 10$ cm, and $N = 10$ windings. Compute the solenoid's operating current I, inductance L, and stored magnetic energy W.

   (c) Under normal operation, the material of the magnet is superconducting (has zero resistance). However, under a "quench" emergency, it suddenly acquires resistance $R = 0.1\,\Omega$, which leads to dissipation of stored energy W through Joule's heating. Imagine that a normally operating magnet switches into the "quench" mode at time $t = 0$. Write down and solve a differential equation describing time dependence of the current $I_q(t)$ during the quench in terms of R, L, and initial condition $I_q(0) = I$. How long does it take I_q to decrease by a factor of 10? What is the heating power at $t = 0$?