Quantum Mechanics Qualifying Exam

PART II

20 August 2010

Please select two of the following three problems. You may use one Quantum Mechanics reference and one Math reference.

1. Consider scattering from a simple potential:

$$V(x) = \begin{cases} V_o & \text{for } r = |\vec{x}| < R \\ 0 & \text{for } r > R. \end{cases}$$

In the low-energy limit, we might only look at S-wave $l = 0$ scattering. However, in the high energy limit, we expect scattering in other partial waves to become significant. For simplicity, let us here consider scattering on a hard sphere, $V_o \to \infty$.

(a) For a hard sphere potential, calculate the total cross section in partial wave l. Obtain an exact result, i.e., dont take the high-energy limit. You may quote your answer in terms of the spherical Bessel functions.

(b) Find a simple expression for the phase shift δ_l in the high energy limit ($kR \gg l$). Keep terms up to O(1) in your result.

(c) Determine the total cross section (including all partial waves) in the high energy limit, $kR \to \infty$.

2. Suppose the nuclear force has a potential of the form:

$$V(r) = V_o e^{-\alpha r},$$

with $V_o = 50$ MeV, $\alpha = 1/R_o$, $R_o = 1.3A^{\frac{1}{3}}$ fermis.

(a) Compare the accuracy of Born approximation used in this case to when used in the square well potential.

(b) Calculate the elastic differntial cross section in the Born approximation.

(c) For a target nucleus with $A = 3$ and an incident nucleon with kinetic energy of 300 MeV calculate the total elastic cross section (in the Born approximation).

3. The scattering amplitude for neutron-proton scattering is given by

$$f(\theta) = \langle \xi_f | (A + B\,\sigma^p \cdot \sigma^n) | \xi_i \rangle$$

where A and B are constants, the σ's are Pauli matrices, and $|\xi_i\rangle$, $|\xi_f\rangle$ are the initial and final spin states of the system: $|\xi_{i,f}\rangle = |+_p+_n\rangle,\ |+_p-_n\rangle,\ |-_p+_n\rangle,\ |-_p-_n\rangle$.

(a) Calculate the scattering amplitude for each of the 16 possibilities,

(b) Find the differential cross section for scattering of $|+\rangle_n \rightarrow |+\rangle_n$ and $|+\rangle_n \rightarrow |-\rangle_n$ when the spin of the emergent proton is not measured by the detector,

(c) Find the cross section for scattering in the $p - n$ spin states, $|singlet\rangle \rightarrow |singlet\rangle$, $|triplet\rangle \rightarrow |triplet\rangle$, and $|singlet\rangle \rightarrow |triplet\rangle$.

Section A

[redacted]

PART 1 SOLNS

1 [redacted] A particle is represented (at time $t = 0$) by the wave function

$$\Psi(x, 0) = \begin{cases} A(a^2 - x^2), & \text{if } -a \leq x \leq +a, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the normalization constant A.

(b) What is the expectation value of x (at time $t = 0$)?

(c) What is the expectation value of p (at time $t = 0$)? (Note that you *cannot* get it from $p = md\langle x \rangle/dt$. Why not?)

(d) Find the expectation value of x^2.

(e) Find the expectation value of p^2.

(f) Find the uncertainty in x (σ_x).

(g) Find the uncertainty in p (σ_p).

(h) Check that your results are consistent with the uncertainty principle.

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2 [redacted] The Hamiltonian for a certain three-level system is represented by the matrix

$$\mathbf{H} = \hbar\omega \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

Two other observables, A and B, are represented by the matrices

$$\mathbf{A} = \lambda \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \mathbf{B} = \mu \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

where ω, λ, and μ are positive real numbers.

(a) Find the eigenvalues and (normalized) eigenvectors of **H**, **A**, and **B**.

(b) Suppose the system starts out in the generic state

$$|\mathcal{S}(0)\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix},$$

with $|c_1|^2 + |c_2|^2 + |c_3|^2 = 1$. Find the expectation values (at $t = 0$) of H, A, and B.

(c) What is $|\mathcal{S}(t)\rangle$? If you measured the energy of this state (at time t), what values might you get, and what is the probability of each? Answer the same questions for A and for B.

(OVER)

3) A particle of mass m is placed in a finite spherical potential well:

$$V(r) = -V_0 \text{ for } r < a$$
$$\text{zero otherwise}$$

Find the form of the ground state wavefunction by solving the radial equation for zero angular momentum $l = 0$ and show graphically that there is no bound state if $V_0 a^2 < h^2/32m$.
When the particle is only just bound, $V_0 a^2 \rightarrow h^2/32m$, find the most probable radius in terms of a.