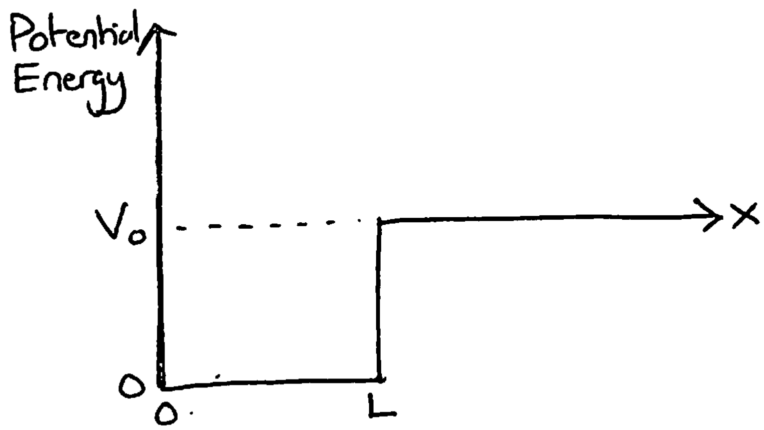
QM 2011

Part I

(1) Set up and solve the Schrodinger equation for a particle of mass m moving in the semi-infinite potential well shown below. Give the normalized wavefunctions in terms of energy eigenvalues $E < V_0$. Sketch how one could determine the allowed values of E by a graphical method. Discuss what range of values of $\frac{L\sqrt{2mV_0}}{\hbar}$ allow at least one boundstate to exist.



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(2) Consider the radial analogue of the harmonic oscillator potential

$$V(r) = \left(\frac{K}{2}\right) r^2 \qquad (0 \leq r < \infty)$$

- Writing $\Psi(r,\theta,\phi) = R(r)\,\Theta(\theta)\,\Phi(\phi)$, find the differential equations satisfied by Θ, Φ, and $U(\rho) = r\mathbf{R}(r)$, $\rho = \alpha r$, $\alpha = (mk/\hbar^2)^{1/4}$.
- Consider the behavior of solutions $U(\rho)$, $\rho \to \infty$. Show that $U(\rho) \sim e^{-\beta\rho^2}$ as $\rho \to \infty$ for suitable choice of β.
- Consider the behavior of solutions $U(\rho)$, $\rho \to 0$. Show that $U(\rho) \sim \rho^{\gamma}$ as $\rho \to 0$ for suitable choice of γ.
- Hence find the differential equation satisfied by $F(\rho)$ where $U(\rho) = F(\rho)\rho^{\gamma}e^{-\beta\rho^2}$ (This equation is the Kummer-Laplace equation).

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3. $$L_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right) \qquad p_x = -i\hbar\frac{d}{dx}$$

$$L_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right) \qquad p_y = -i\hbar\frac{d}{dy}$$

$$L_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) \qquad p_z = -i\hbar\frac{d}{dz}$$

Prove the following using these definitions!

- $[L_z, L_x] = i\hbar L_y$
- $[L_x, p_x] = 0$
- $[L_x, p_y] = i\hbar p_z$
- $[L_x, p_z] = -i\hbar p_y$
- $[L_x, x] = 0$
- $[L_x, y] = i\hbar z$
- $[L_x, z] = -i\hbar y$
- $[p_x, x] = -i\hbar$
- $[p_y, x] = [p_z, x] = 0$

By permuting coordinates, write down all the results for commutators of any pair of position, momentum, and angular momentum operators $\{x, y, z, p_x, p_y, p_z, L_x, L_y, L_z\}$

NOTE There are $\frac{9!}{7!2!} = 36$ results.