Quantum Mechanics Qualifier
August, Summer 2011

NAME: _____ DATE: _____

PART II

This part consist of three problems. You may select two of the three. If you attempt all three please clearly indicate which two you want graded. Otherwise the first two problems will be graded. You may use one Qunatum MEchanics reference book and one MAthematics reference book.

1. The Hamiltonian for the Helium atom has the form,

$$H = -\frac{\hbar^2}{2m_e}(\nabla_1^2 + \nabla_2^2) - \frac{2e^2}{r_1} - \frac{2e^2}{r_2} + \frac{2e^2}{r_{12}}.$$

   Where $\vec{r}_1$ and $\vec{r}_2$ are the positions vectors of the two electrons and $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$. Unless otherwise stated ignore relativistic corrections and spin effects.

   (a) Treating the last term as a pertubation find the first order energy corrections to the state $1s^2$.

   (b) Find the first order correction to the state $1s^2 2p^1$. Note that you will have a two-fold degeneracy in this case. The degeneracy is split at first order. Find the energy corrections and the "good" states.

   (c) Taking into account spin list the four distinct $1s^1p^1$ states indicating the values of the total angular momentum j and total spin s.

   (d) Assuming that the dipole transitions dominate draw a diagram indicating the allowed transitions between the states $1s^2$, $1s^1 2s^1$, and $1s^2 2p^1$. Note that for $1s^1 2s^1$ there are two distinct states distinguihed by the total spin and for the $1s^1 2p^1$ states there are four states as found in part (c). Please clearly state your criteria for picking the allowed transitions.

   The following identity may be useful,

$$\frac{1}{r_{12}} = \frac{1}{r_<} \sum_{l,m_l} \left(\frac{4\pi}{2l+1}\right) \left(\frac{r_>}{r_<}\right)^l Y_{lm_l}(\theta_1, \phi_1) Y_{lm_l}(\theta_2, \phi_2),$$

   where $r_<$ and $r_>$ are the smaller and greater of r_1, r_2 respectively.

2. Use the Born approximation to find the differential scattering cross section for for particles of mass m and incident energy E scattering off a potential given by,

$$V(r) = \begin{cases} V_o, & 0 < r < a \\ 0, & r > a \end{cases}$$

   Please show explicit E and θ dependence.

3. Determine the cross section for emission of a photoelectron ejected when linearly polarized monochromatic light of frequency ω is incident on a complex atom. Simulate the initial state of the atomic electron as the ground state of an isotropic three dimensional harmonic oscillator. Assume the final state to be a plane wave.