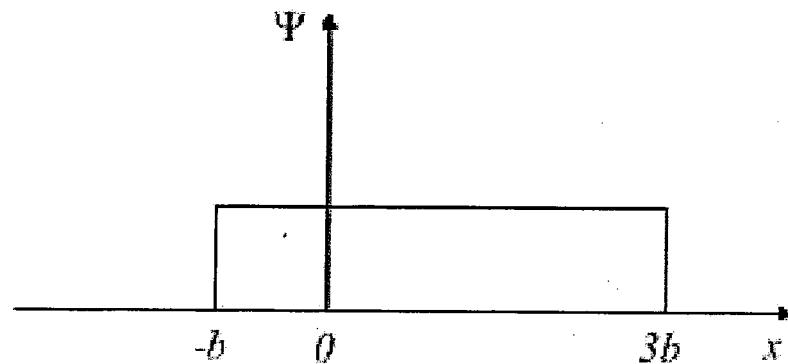


1.

The state of a free particle is described by the following wave function:

$$\psi(x) = \begin{cases} 0 & \text{for } x < -b \\ A & \text{for } -b \leq x \leq 3b \\ 0 & \text{for } x > 3b \end{cases}$$



- (a). (5 points) Find A using the normalization condition. (You may choose the phase convention such that A is real.)
- (b). (5 points) What is the probability of finding the particle within the interval $[0, b]$?
- (c). (10 points) Calculate $\langle x \rangle$ and $\langle x^2 \rangle$ for this state.
- (d). (10 points) Calculate the momentum probability density.

2.

Infinite potential well

$$V(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq a \\ \infty & \text{elsewhere} \end{cases}$$

$$E_n = \frac{\hbar^2 \pi^2}{2ma^2} n^2$$

$$u_n(x) = \sqrt{\frac{2}{a}} \sin(n\pi \frac{x}{a})$$

The wavefunction is specified at $t=0$ to be

$$\psi(x, t=0) = C(3 \sin 2kx - 2 \sin 3kx), \text{ where } k = \pi/a.$$

- (5 points) Determine the normalization coefficient C .
- (5 points) Expand the wavefunction at the initial time $\psi(x, t=0)$ in terms of eigenfunctions of the infinite box, i.e. determine the expansion coefficients c_n . The eigenfunctions are given in the formula section of this exam.
- (5 points) Write down $\psi(x, t)$ at an arbitrary later time t .
- (7 points) If a measurement of the particle's energy at time t is performed, what will be the possible outcomes, and with what probability will those values be measured? What is the average energy $\langle E \rangle$ of the particle in the box? Is $\langle E \rangle$ changed by the measurement?
- (8 points) Calculate the probability current $J(x, t=0)$ at the initial time. Does it depend on the complex phase that you have chosen for C , and why (not)? Will the value of J remain unchanged at later times? Express the fact that the particle does not leave the box as a mathematical condition on J .

3.

An operator \hat{A} is defined as $\hat{A} = a\hat{x} + ib\hat{p}$, where a, b are real numbers.

- a) (8 points) What is the Hermitian adjoint operator \hat{A}^\dagger ?
- b) (12 points) Calculate the commutators $[\hat{A}, \hat{A}]$, $[\hat{A}, \hat{x}]$ and $[\hat{A}, \hat{p}]$.
- c) (8 points) By comparing to eigenvalues of the energy in the simple harmonic oscillator potential (which you may look up), find the eigenvalues of the operator $\hat{A}^\dagger \hat{A}$

Quantum Mechanics Qualifier
Fall 2013

NAME: _____

DATE: _____

PART II

This part consists of three problems. You are required to answer the first problem and may select either of the remaining two problems. If you attempt all three please clearly indicate which two you want graded. Otherwise the first two problems will be graded. You may use one Quantum Mechanics reference book and one Mathematics reference book.

1. Consider the following scattering process,

$$\pi^- + p \rightarrow K^0 + \Lambda,$$

where the π^- , p , K^0 , and Λ are particles of spin, 0, 1/2, 0, and 3/2 respectively. In the CM where we take the beam direction as the z -axis the initial two particle state can be characterized by two possible states,

$$\begin{aligned}\psi_1 &= \sum_l a_l Y_l^0 | \tfrac{1}{2} \tfrac{1}{2} \rangle \\ \psi_2 &= \sum_l a_l Y_l^0 | \tfrac{1}{2} -\tfrac{1}{2} \rangle.\end{aligned}$$

For simplicity assume only the $l = 1$ states contribute to this sum.

- (a) List the possible values for the total angular momentum in the initial state
- (b) List the possible values of orbital angular momentum in the final state
- (c) Write the form of the final state wave function for each of two possible initial states indicated above. Hint: For each of the two cases you should have twelve independent terms.
- (d) Show that if we place a detector at angles $\theta = 0, \pi$ we will detect a pure state, $|J_\Lambda \tfrac{1}{2}\rangle$.
- (e) The Λ is then observed to decay via,

$$\Lambda \rightarrow p + \pi$$

Show that if parity is not conserved the final state can finally be written in the form,

$$\psi_1^f = s_p \left(\sqrt{\frac{2}{3}} Y_1^0 | \tfrac{1}{2} \tfrac{1}{2} \rangle + \sqrt{\frac{1}{3}} Y_1^1 | \tfrac{1}{2} -\tfrac{1}{2} \rangle \right) + s_D \left(-\sqrt{\frac{2}{5}} Y_2^0 | \tfrac{1}{2} \tfrac{1}{2} \rangle + \sqrt{\frac{3}{5}} Y_2^1 | \tfrac{1}{2} -\tfrac{1}{2} \rangle \right)$$

Assume that, $|s_p|^2 + |s_D|^2 = 1$.

- (f) Finally show that for an initial unpolarized proton the angular distribution of the final products of the Λ decay is given by,

$$\frac{dN}{d\Omega} = \frac{1}{2} (1 + 3x^2),$$

where $x = \cos \theta$ and the distribution has been normalized to one.

2. The hyperfine structure of the hydrogen atom in the presence of an external magnetic field, B , can be studied by considering the following interaction Hamiltonian,

$$H = B\mu_e \sigma_{ez} + W \vec{\sigma}_e \cdot \vec{\sigma}_p$$

where the $\vec{\sigma}$ are the usual Pauli matrices, and W is a constant.

- (a) Find the eigenstates and eigenvalues of H in the case where $B = 0$.
 - (b) Find the energy for general B .
 - (c) Sketch the energy values labeling each curve by the corresponding angular momentum.
3. Suppose the nuclear force has a potential of the form,

$$V(r) = V_o e^{-\alpha r}$$

with $V_o = 50$ MeV, $\alpha = 1/R_o$, and $R_o = 1.3A^{1/3}$ Fermis.

- (a) Calculate the elastic differential cross section in the Born approximation.
- (b) Give an estimate of how good the approximation is in this case.
- (c) For a target nucleus with $A = 3$ and an incident nucleon with $KE = 300$ MeV calculate the total elastic cross section.