Name:____________________

Qualifying Exam:
Quantum Mechanics
Fall, 2014

Instructions:

There are two sections in this exam. Each section contains 3 problems. Choose 2 problems from each section. You should show all work. Indicate clearly which 2 problems in each section you wish to have graded. Attach your work to the back of this exam (e.g. by stapling).

Section A – answer two out of three questions

1)

a)

In a region of space, a particle with mass m and with zero energy has a time-independent wave

$$\psi(x) = Axe^{-x^2/L^2}$$

where A and L are constants.

•Determine the potential energy $U(x)$ of the particle.

b)

An electron is described by the wave function

$$\psi(x) = \begin{cases} 0 & \text{for } x < 0 \\ Ce^{-x}(1 - e^{-x}) & \text{for } x > 0, \end{cases} \tag{58}$$

where x is in nm and C is a constant.

•Determine the value of C that normalizes $\psi(x)$.

•Where is the electron most likely to be found? That is, for what value of x is the probability of finding the electron the largest?

•Calculate the average position $\langle x \rangle$ for the electron. Compare this result with the most likely position, and comment on the difference.

2)

A particle in a potential well $U(x)$ is initially in a state whose wavefunction $\Psi(x, 0)$ is an equal-weight superposition of the ground state and first excited state wavefunctions:

$$\Psi(x, 0) = C[\psi_1(x) + \psi_2(x)] \tag{76}$$

•Show that the value $C = 1/\sqrt{2}$ normalizes $\Psi(x, 0)$, assuming that ψ_1 and ψ_2 are themselves normalized.

•Determine $\Psi(x, t)$ at any later time t.

•Show that the average energy $\langle E \rangle$ for $\Psi(x, t)$ is the arithmetic mean of the ground and first excited state energies E_1 and E_2, that is $\langle E \rangle = (E_1 + E_2)/2$.

•Determine the uncertainty ΔE of energy for $\Psi(x, t)$.

•Determine the average position $\langle x(t) \rangle$ of a particle with nonstationary state wave function $\Psi(x, t)$

3)

A particle of mass m and positive charge q, m o ving in one dimen-sion, is subject to a uniform electric field $\mathcal{E}[\Theta(x) - \Theta(-x)]$; see Fig. 42.

(a) Consider a trial wave function $\psi(x) \propto e^{-\alpha|x|}$ and estimate the ground-state energy by minimizing the expectation value of the energy.

(b) Obtain an estimate of the ground-state energy by applying the Bohr–Sommerfeld WKB quantization rule.

(c) Compare the size of results to a) and b)

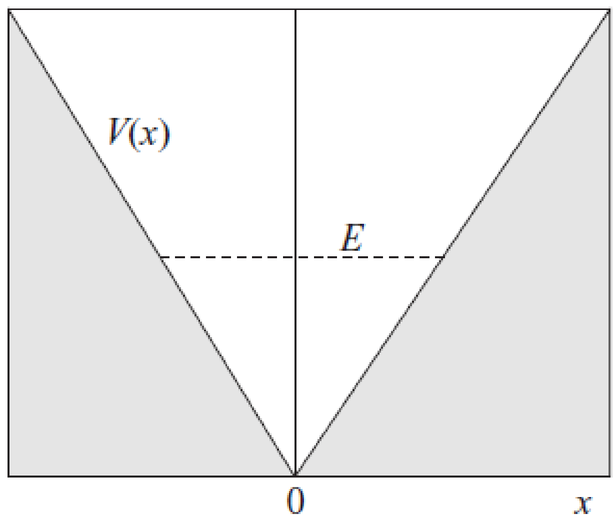

Fig. 42 Particle in an electric field with energy E.

Section B

August 22, 2014

Choose two of the following three problems

1. Consider a spherically symmetric system of three, spin-$\frac{1}{2}$ particles. Each particle's individual total spin state, $s_i = \frac{1}{2}$, can either be projected positively or negatively (m_i) along the z-axis and the individual particle state can be written as $|s_i, m_i\rangle = |\frac{1}{2}, \pm\frac{1}{2}\rangle$; this can be written more simply as $|\pm\rangle$. The particles have been prepared in a state of total orbital angular momentum $\ell = 0$, but with spin angular momentum, s. This state, $|s, m\rangle$, is given by:

$$|s, m\rangle = \frac{1}{\sqrt{3}} \left(|--+\rangle + |-+-\rangle + |+--\rangle\right),$$

where for instance $|--+\rangle = |-\rangle|-\rangle|+\rangle$ (a short-hand for writing the individual particle states in one ket).

   (a) Use the spin operators to calculate the total z-axis spin projection quantum number, m, of this state.

   (b) Use the spin operators to calculate the total spin quantum number, s, of this state.

   (c) Use the spin operators to calculate all other states with the same total spin, s, as the above state.

2. The time-independent wave function of a particle is given by:

$$\psi(r, \theta, \phi) = -\frac{1}{2}\sqrt{\frac{3}{48\pi a_0^5}}\, r\, e^{-r/(2a_0)} \sin\theta\, e^{i\phi},$$

where a_0 is the Bohr Radius.

   (a) Using the angular momentum operator, calculate the z-projection quantum number, m_ℓ, of the total orbital angular momentum of this state.

   (b) Using the angular momentum operator, calculate the total orbital angular momentum quantum number, ℓ, of this state.

   (c) Calculate the overlap of the operator

$$z = r \sin\theta \cos\phi$$

   between the $(n, \ell, m) = (2, 1, 1)$ state of the Hydrogen atom wave function and the ground state, $(1, 0, 0)$.

3. Consider the nonrelativistic scattering of an electron of mass m and momentum k through an angle θ. The electron scatters via the spin-dependent potential

$$V = e^{-\mu r^2}\left[A + B\vec{\sigma} \cdot \vec{r}\right]$$

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices and (μ, A, B) are constants. The angle of the scatter is θ, the polar angle with respect to the initial flight direction. Assume that the initial spin of the electron of polarized along its direction of motion (Note: ignore the fact that the potential is explicitly Parity-violating).

(a) Calculate the matrix element-squared of this potential, $|\langle\psi_f|V(\vec{r})|\psi_i\rangle|^2$, where ψ_f and ψ_i are the final-state and initial-state wave functions, respectively. Sum over final-state spins when computing this matrix element-squared.

(b) Calculate the differential scattering cross-section, in the Born approximation, for the electron scattering via this potential.