Qualifying Exam: Quantum Mechanics Fall, 2016

Name:______________________________

Instructions: There are two sections in this exam. Each section contains 3 problems. Choose 2 problems from each section. You should show all work. Indicate clearly which 2 problems in each section you wish to have graded. Attach your work to the back of this exam (e.g. by stapling).

Section A

August 20, 2016

Choose two of the following three problems.

1. **ESTIMATING THE GROUND STATE ENERGY [25 Points].** Consider the potential:

$$V(x) = \frac{1}{2}kx^4.$$

You are given the following trial wave function for a particle subjected to this potential well:

$$\psi(x) = Ae^{-bx^2}.$$

   (a) [5 Points] Show how to normalize this trial wave function.

   (b) [10 Points] Using the normalized trial wave function, estimate the ground-state energy of the given potential.

   (c) [10 Points] Find the value of b that minimizes your estimated ground-state energy.

2. **GROUND STATE OF AN OSCILLATING POTENTIAL WELL [25 Points].** Consider an infinite potential well whose potential is described as follows:

$$V(x) = \begin{cases} \infty & (x < 0) \\ V_0 \sin(\frac{\pi}{L}x) & (0 \leq x \leq L) \\ \infty & (x > L) \end{cases}$$

Estimate the ground-state energy of a particle of mass, m, in this potential well, for the condition that $E > V_0$ and assuming that the variation of the potential is slow compared to the wavelength of the particle.

3. **THE UNCERTAINTY PRINCIPLE AND A PARTICLE IN A BOX [25 Points].** Given the solution to the time-dependent particle-in-a-box problem,

$$\psi(x,t) = \begin{cases} \sqrt{\frac{2}{L}} \sin(\frac{n\pi}{L}x) e^{-\frac{i\pi^2 \hbar n^2}{8L^2 m}t} & 0 < x < L \\ 0 & otherwise \end{cases}$$

for a particle of mass, m, in a state labeled by quantum number, n, show that this solution satisfies the Heisenberg Uncertainty Principle for the case $0 < x < L$.

Section B

Choose two of the following three problems.

Problem 4

A system of two particles with spins $s_1 = \frac{3}{2}$ and $s_2 = \frac{1}{2}$ is described by the approximate Hamiltonian $H = \alpha\, \mathbf{S}_1 \cdot \mathbf{S}_2$, with α a given constant. The system is initially (at $t = 0$) in the following eigenstate of $\mathbf{S}_1^2,\ \mathbf{S}_2^2,\ S_{1z},\ S_{2z}$:

$$\left|\tfrac{3}{2}\ \tfrac{1}{2};\ \tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$$

Find the state of the system at times $t > 0$. What is the probability of finding the system in the state $\left|\frac{3}{2}\ \frac{3}{2};\ \frac{1}{2}\ -\frac{1}{2}\right\rangle$?

Problem 5

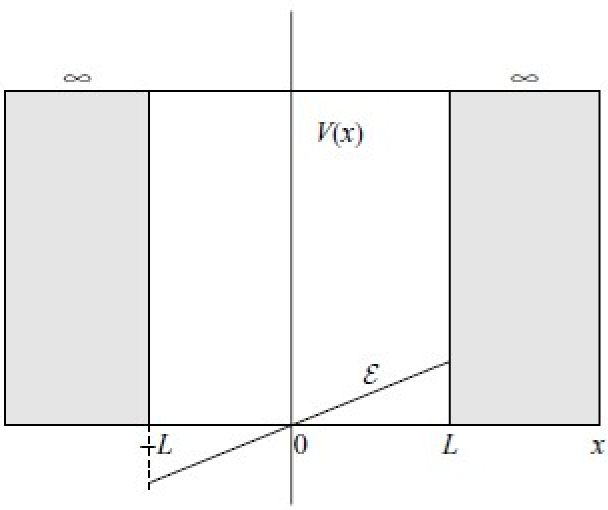

Fig. 40 Linear perturbation in an infinite square well.

A particle of mass m and charge q moves in one dimension between the impenetrable walls of an infinite square-well potential

$$V(x) = \begin{cases} 0, & |x| < L \\ \infty, & |x| > L \end{cases}$$

(a) Consider a weak uniform electric field of strength $\mathcal{E}$ that acts on the particle; see Fig. 40. Calculate the first non-trivial correction to the particle's ground-state energy.[11] What is the probability of finding the particle in the first excited state?

(b) Consider now the case of a time-dependent electric field of the form $\mathcal{E}(t) = \mathcal{E}_0 \Theta(t) e^{-t/\tau}$. Calculate the transition probability from the ground state of the system to the first excited state in first-order time-dependent perturbation theory for times $t \gg \tau$.

[11] You may use the sum

$$\sum_{n=1}^{\infty} \left[\frac{1}{(4n^2-1)^3} + \frac{4}{(4n^2-1)^4} + \frac{4}{(4n^2-1)^5} \right] = \frac{1}{2} - \frac{\pi^2}{64}\left(\frac{7}{4} + \frac{\pi^2}{12} \right)$$

Problem 6

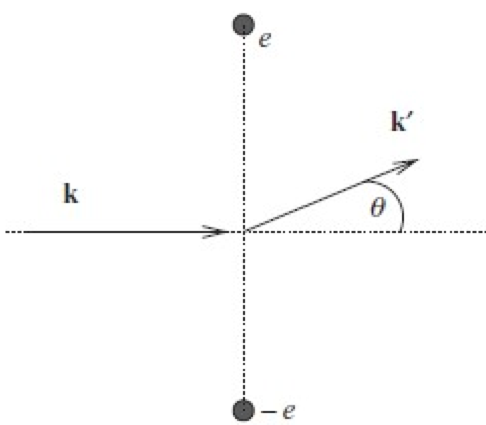

Fig. 45 Scattering from a dipole.

Consider an electric dipole consisting of two electric charges e and $-e$ at a mutual distance $2a$. Consider also a particle of charge e and mass m with an incident wave vector $\mathbf{k}$ perpendicular to the direction of the dipole; see Fig. 45.

(a) Calculate the scattering amplitude in the Born approximation. Find the directions at which the differential cross section is maximal.

(b) Consider a different system with a target consisting of two arbitrary charges q_1 and q_2 similarly placed. Calculate again the scattering amplitude and the directions of maximal scattering.