Quantum Mechanics Core Proficiency Exam
January 2018

January 27, 2018

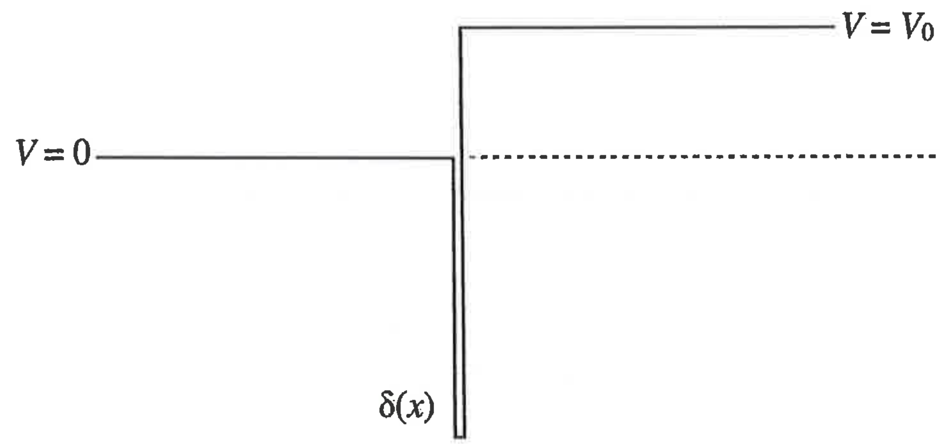
Instructions

The exam consists of two longer questions and two shorter questions. You are required to attempt to solve all four problems. You have two hours to work on the solutions. You are allowed one textbook of your choice, one math reference, and a calculator.

Problem 1

Consider a one-dimensional step-function potential plus an attractive δ-function potential at its edge

$$V(x) = V_0\Theta(x) - (\hbar^2 g/2m)\delta(x)$$



(a) A wavefunction $A\,e^{ikx}$ with A=1, representing particles of energy $E > V_0$, is incident from the left. Find the complete wavefunction in each region of x in terms of k and q, where

$$k = \frac{\sqrt{2mE}}{\hbar} \qquad q = \frac{\sqrt{2m(E - V_0)}}{\hbar}$$

(b) Now consider energies $E < 0$. In terms of s and t, where

$$s = \frac{\sqrt{-2mE}}{\hbar} \qquad t = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

find an equation that could be solved for allowed bound state energies E (you do not need to solve the equation).

Problem 2

The pion comes in three states: two are electrically charged ($\pi^{\pm}$) and the third is electrically neutral (π^0). The π^0 has a mass of $m = 134.9766\,\mathrm{MeV/c^2}$ and consists of a quark and anti-quark pair. Quarks are fermions, each with spin $s^{quark} = \frac{1}{2}\hbar$, while the pions are bosons with spin $s^{pion} = 0\hbar$ and are spin singlets.

(a) Write the spin wavefunction for the π^0 in terms of the spin wavefunctions of the constituent quark and anti-quark. Show your work and reasoning for your final answer.

(b) If π^0 scattering were described by the following potential, find the total cross-section for that process using the Born Approximation in the low-energy limit. The potential is

$$V = V_0 e^{-r^2/a^2},$$

where a is a constant that has units of distance and represents the physical size of the target.

(c) What is the maximum kinetic energy a π^0 can have such that the low-energy Born Approximation is valid? Write your answer in terms of a and m.

Problem 3

The harmonic oscillator potential $V = Kx^2/2$ yields a wavefunction of $\psi(x) = Be(-\alpha x^2/2)$, where $\alpha = mK/h^2$. Assuming an electron mass, normalize this wavefunction in terms of K.

Problem 4

Answer each of the following questions:

(a) What is the condition on two operators Q_1 and Q_2 for them to share the same eigenfunctions?

(b) What is the condition on an operator Q_1 for it to be Hermitian?

(c) Why are the operators corresponding to physical observables chosen to be Hermitian?

(d) What is the precise physical meaning (in experiments) of $\langle Q_1 \rangle$ and the eigenvalues of Q_1?

(e) What is the condition on two operators Q_1 and Q_2 for them to have an uncertainty principle (both observables cannot be in definite states simultaneously)?