

Statistical Mechanics and Thermodynamics
August 2013

You may refer to a statistical mechanics text of your choice, but no other notes or references.

1. Consider a system A with a single spin-1/2 particle having a magnetic moment μ , and a system A' consisting of three spin-1/2 particles, also with moment μ . These systems are in the same magnetic field H (along z), are in thermal contact with each other, and in equilibrium. The combined system $A + A'$ is isolated. You observe that when the particle in A is spin up, two spins in A' are up, one is down. Compute the probability P_+ that the particle in A is spin up. Repeat for the case where you observe two down and one up in A' when A is up.

Choose one of the following two problems:

2. A vertical cylinder contains ν moles of an ideal gas and is closed off by a piston of mass M and area A . The molar specific heat c_V (at constant volume) of the gas is a constant independent of temperature. The heat capacities of the piston and cylinder are negligibly small and any frictional forces between the piston and the cylinder walls can be neglected. The whole system is thermally insulated. Initially, the piston is clamped in position so that the gas has a volume V_0 and a temperature T_0 . The piston is now released and, after some oscillations, comes to rest in a final equilibrium configuration where the volume of the gas is larger.
 - (a) Does the temperature of the gas increase, decrease, or remain the same? Explain.
 - (b) Does the entropy of the gas increase, decrease, or remain the same? Explain.
 - (c) Calculate the final temperature of the gas in terms of T_0 , V_0 , ν , M , A , c_V , the acceleration due to gravity g , and the gas constant R .
 - (d) Calculate the change in entropy in terms of these same constants.
3. The tension of a rubber band in equilibrium is given by

$$t = AT \left(\frac{x}{l_0} - \frac{l_0^2}{x^2} \right),$$

where t is tension, T is absolute temperature, x is the length of the band, l_0 is the length of the band when $t = 0$, and A is a constant. When $x = l_0$, the heat capacity $C_x(x, T)$ is observed to equal the constant K .

- (a) Give the fundamental thermodynamic relation between changes in the entropy, energy, and length of the system (dS , dE , and dx).

- (b) Find as functions of T and x :
- $\left(\frac{\partial E}{\partial x}\right)_T$ where E is internal energy
 - $\left(\frac{\partial C_x}{\partial x}\right)_T$
 - $C_x(x, T)$
 - $E(x, T)$
 - $S(x, T)$ where S is entropy.
- (c) The band is stretched adiabatically from $x = l_0$ to $x = 1.5l_0$. Its initial temperature was T_0 . What is its final temperature?

Choose one of the following two problems:

4. Consider a system of N identical but distinguishable particles, each of which has two energy levels with energy 0 or $\epsilon > 0$. The upper energy level has a g -fold degeneracy while the lower level is non-degenerate. The total energy of the system is E .
- Using the microcanonical ensemble, find the occupation numbers n_+ and n_0 in terms of the temperature of the system (where n_+ corresponds to the upper level and n_0 to the lower one.)
 - Consider the case $g = 2$. If the system has energy $E = 0.75N\epsilon$ and is brought into contact with a bath at constant temperature $T = 500^\circ\text{K}$, in what direction does heat flow and why?
5. The potential energy between atoms of a hydrogen molecule can be modelled by means of the Morse potential

$$V(r) = V_0 \left[\exp\left(\frac{-2(r - r_0)}{a}\right) - 2 \exp\left(\frac{-(r - r_0)}{a}\right) \right] \quad (1)$$

where $V_0 = 7 \times 10^{-12} \text{erg}$, $r_0 = 8 \times 10^{-9} \text{cm}$ and $a = 5 \times 10^{-9} \text{cm}$.

- Find the lowest angular frequency of rotational motion and the frequency of small-amplitude vibrations.
- Estimate the temperatures T_{rot} and T_{vib} at which rotations and vibrations respectively begin to contribute significantly to the internal energy.