Statistical Mechanics and Thermodynamics
August 2016

You may refer to a statistical mechanics text of your choice, but no other notes or references.

1. Consider a system A with two spin-1 particles at fixed sites each having a magnetic moment μ. The system is in a magnetic field H pointing along the z direction, is thermally isolated, in equilibrium, and has a total energy of exactly zero. (The only contribution to energy is due to the magnetic moment coupled to the magnetic field. You may ignore kinetic energy.)

   (1) Determine the probability that the first particle has $s_z = +1\hbar$.

   (2) This system is brought into thermal contact with a second system B with a single spin-1 particle which originally had $s_z = +1\hbar$ in the same magnetic field. After reaching equilibrium, what is the new probability that the first particle in A has $s_z = +1\hbar$?

   (3) What are the new average energies E_A and E_B of A and B, and the heat Q_A absorbed by A in part (2)?

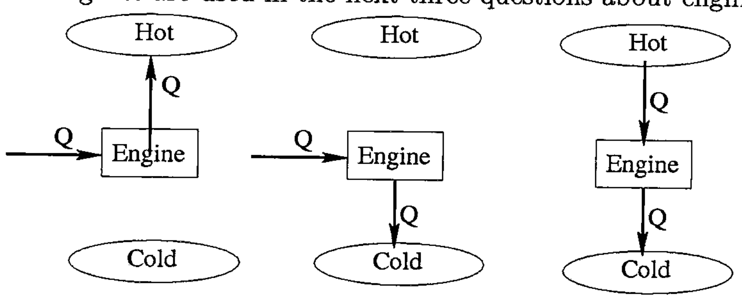
2. True or false? If false, give a counterexample.

   (a) ______ All reversible processes are isentropic.

   (b) ______ All processes with $\Delta Q = 0$ are reversible.

   (c) ______ All processes with $\Delta Q = 0$ are isentropic.

   The figures are used in the next three questions about engines and heat reservoirs.



   (d) ______ It is possible to build a machine on which one can perform work and dump all of that energy into the hot reservoir without using the cold reservoir at all.

   (e) ______ It is possible to build a machine on which one can perform work and dump all of that energy into the cold reservoir without using the hot reservoir at all.

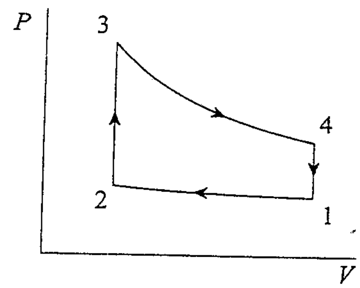
   (f) ______ It is possible for heat to flow from the hot reservoir to the cold reservoir with no work at all being done.

   (g) ______ For a system of non-interacting particles (though possibly interacting separately with an external field), the internal energy is always proportional to the absolute temperature.

   (h) ______ If the average velocity of a gas molecule is zero, then the average speed is also zero.

Answer two of the following three problems.

3. The cycle of a highly idealized gasoline engine can be approximated by the Otto cycle, illustrated. The paths $1 \to 2$ and $3 \to 4$ are adiabatic compression and expansion, respectively; $2 \to 3$ and $4 \to 1$ are constant-volume processes. Treat the working medium as an ideal gas with constant $\gamma = c_p/c_v$.



(a) Compute the efficiency of this cycle for $\gamma = 1.4$ and compression ratio $r = V_i/V_f = 10$.

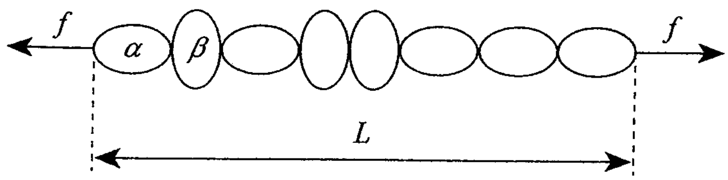
(b) Calculate the work done on the gas in the compression process $1 \to 2$, assuming initial volume $V_i = 2\text{L}$ (liters) and $p_i = 1$ atm.

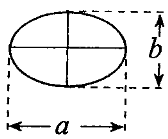
4. A brick of mass M sits on the floor, in thermal equilibrium with its surroundings which are at temperature T and act as a reservoir.

(a) Using entropy considerations (for example, the relation between heat and change in entropy), give the probability of finding the brick at a height between L and $2L$ the next time you look. You may treat the brick as a point mass; that is, ignore its own internal energy and contribution to entropy.

(b) Use the Boltzmann distribution for the brick to give the probability of finding it at a particular position and momentum. Give the partition function for the brick and use it to compute its average height.

5. Consider a one-dimensional chain consisting of N molecules which exist in two configurations, α and β, with corresponding energies ε_α and ε_β, and lengths a and b. The chain is subject to a constant tensile force f.





(a) Write the partition function Z_N for the system. (Account for the potential energy associated with f in addition to the individual energies ε_α and ε_β. You could think of the work involved in stretching each molecule to its current length.)

(b) Calculate the average length $\langle L \rangle$ as a function of f and the temperature T.

(c) Assume that $\varepsilon_\alpha > \varepsilon_\beta$ and $a > b$. Give the average length $\langle L \rangle$ in the absence of the tensile force $f = 0$. What are the high- and low-temperature limits, and what is the characteristic temperature at which the changeover between the two limits occurs?