Combination of CDF and DØ Results on the Mass of the Top Quark

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Abstract

We summarize the top-quark mass measurements from the CDF and DØ experiments at Fermilab. We combine published Run-I (1992-1996) measurements with the most recent preliminary Run-II (2001-present) measurements using up to 2.8 fb$^{-1}$ of data. Taking correlated uncertainties properly into account the resulting preliminary world average mass of the top quark is $M_t = 172.4 \pm 0.7$ (stat.) $\pm 1.0$ (syst.) GeV/$c^2$, assuming Gaussian systematic uncertainties. Adding in quadrature yields a total uncertainty of 1.2 GeV/$c^2$, corresponding to a relative precision of 0.7% on the top-quark mass.
1 Introduction

The experiments CDF and DØ, taking data at the Tevatron proton-antiproton collider located at the Fermi National Accelerator Laboratory, have made several direct experimental measurements of the top-quark pole mass, $M_t$. The pioneering measurements were based on about 100 pb$^{-1}$ of Run-I (1992-1996) data [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and include results from the $t\bar{t} \rightarrow qq'bqq'$ (all-j), the $t\bar{t} \rightarrow l\nuqq'bb'$ (l+j), and the $t\bar{t} \rightarrow l^+\nu b\ell^-\nu b$ (di-l) decay channels$^2$. The Run-II measurements summarized here are the most recent results in the l+j, di-l, and all-j channels using 1.9 – 2.8 fb$^{-1}$ of data and improved analysis techniques [13, 14, 15, 16, 17, 18, 19, 20].

This note reports the world average top-quark mass obtained by combining five published Run-I measurements [2, 3, 5, 7, 10, 11] with four preliminary Run-II CDF results [13, 14, 15, 16] and three preliminary Run-II DØ results [17, 18, 19, 20]. The combination takes into account the statistical and systematic uncertainties and their correlations using the method of references [21, 22] and supersedes previous combinations [23, 24, 25, 26, 27, 28].

The input measurements and error categories used in the combination are detailed in Sections 2 and 3, respectively. The correlations used in the combination are discussed in Section 4 and the resulting world average top-quark mass is given in Section 5. A summary and outlook are presented in Section 6.

2 Input Measurements

For this combination twelve measurements of $M_t$ are used: five published Run-I results, and seven preliminary Run-II results, all reported in Table 1. In general, the Run-I measurements all have relatively large statistical uncertainties and their systematic uncertainty is dominated by the total jet energy scale (JES) uncertainty. In Run-II both CDF and DØ take advantage of the larger $t\bar{t}$ samples available and employ new analysis techniques to reduce both these uncertainties. In particular the JES is constrained using an in-situ calibration based on the invariant mass of $W \rightarrow qq'$ decays in the l+j and all-j channels. The Run-II DØ analysis in the l+j channel constrains the response of light-quark jets using the in-situ $W \rightarrow qq'$ decays. Residual JES uncertainties associated with $\eta$ and $p_T$ dependencies as well as uncertainties specific to the response of $b$-jets are treated separately. Similarly, the Run-II CDF analyses in the l+j and all-j channels also constrain the JES using the in-situ $W \rightarrow qq'$ decays. Small residual JES uncertainties arising from $\eta$ and $p_T$ dependencies and the modeling of $b$-jets are included in separate error categories. The Run-II CDF di-l measurement uses a JES determined

$^2$Here $\ell = e$ or $\mu$. Decay channels with explicit tau lepton identification are presently under study and are not yet used for measurements of the top-quark mass.
### Table 1: Summary of the measurements used to determine the world average $M_t$. All numbers are in GeV/$c^2$. The error categories and their correlations are described in the text. The total systematic uncertainty and the total uncertainty are obtained by adding the relevant contributions in quadrature.

<table>
<thead>
<tr>
<th></th>
<th>Run-I published</th>
<th>Run-II preliminary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CDF</td>
<td>DØ</td>
</tr>
<tr>
<td></td>
<td>all-j  l+j  di-l</td>
<td>1+j  di-l</td>
</tr>
<tr>
<td><strong>Result</strong></td>
<td>186.0  176.1  167.4</td>
<td>180.1  168.4</td>
</tr>
<tr>
<td><strong>iJES</strong></td>
<td>0.0   0.0   0.0</td>
<td>0.0   0.0</td>
</tr>
<tr>
<td><strong>aJES</strong></td>
<td>0.0   0.0   0.0</td>
<td>0.0   0.0</td>
</tr>
<tr>
<td><strong>bJES</strong></td>
<td>0.6   0.6   0.8</td>
<td>0.7   0.7</td>
</tr>
<tr>
<td><strong>cJES</strong></td>
<td>3.0   2.7   2.6</td>
<td>2.0   2.0</td>
</tr>
<tr>
<td><strong>dJES</strong></td>
<td>0.3   0.7   0.6</td>
<td>0.0   0.0</td>
</tr>
<tr>
<td><strong>rJES</strong></td>
<td>4.0   3.4   2.7</td>
<td>2.5   1.1</td>
</tr>
<tr>
<td><strong>lepPt</strong></td>
<td>0.0   0.0   0.0</td>
<td>0.0   0.0</td>
</tr>
<tr>
<td><strong>Signal</strong></td>
<td>1.8   2.6   2.8</td>
<td>1.1   1.8</td>
</tr>
<tr>
<td><strong>BG</strong></td>
<td>1.7   1.3   0.3</td>
<td>1.0   1.1</td>
</tr>
<tr>
<td><strong>Fit</strong></td>
<td>0.6   0.0   0.7</td>
<td>0.6   1.1</td>
</tr>
<tr>
<td><strong>MC</strong></td>
<td>0.8   0.1   0.6</td>
<td>0.0   0.0</td>
</tr>
<tr>
<td><strong>UN/MI</strong></td>
<td>0.0   0.0   0.0</td>
<td>1.3   1.3</td>
</tr>
<tr>
<td><strong>Syst.</strong></td>
<td>5.7   5.3   4.9</td>
<td>3.9   3.6</td>
</tr>
<tr>
<td><strong>Stat.</strong></td>
<td>10.0  5.1  10.3</td>
<td>3.6   12.3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>11.5  7.3   11.4</td>
<td>5.3   12.8</td>
</tr>
</tbody>
</table>

from external calibration samples. Some parts of the associated uncertainty are correlated with the Run-I JES uncertainty as noted below.

A new analysis technique from CDF is included (trk). This measurement uses both the mean decay-length from B-tagged jets and the mean lepton transverse momentum to determine the top-quark mass in $l+j$ candidate events. While the statistical sensitivity is not as good as the more traditional methods, this technique has the advantage that since it uses primarily tracking information, it is almost entirely independent of JES uncertainties. As the statistics of this sample continue to grow, this method could offer a nice cross-check of the top-quark mass that’s largely independent of the dominant JES systematic uncertainty which plagues the other measurements. The statistical correlation between an earlier version of the trk analysis and a traditional Run-II CDF $l+j$ measurement was studied using Monte Carlo signal-plus-
background pseudo-experiments which correctly account for the sample overlap and was found to be consistent with zero (to within < 1%) independent of the assumed top-quark mass.

The two DØ Run-II lepton+jets results [17, 18] are derived from Run-IIa and Run-IIb datasets, respectively, and are labelled as such. The DØ Run-II dilepton result is itself a combination of two results using different techniques but partially overlapping dilepton data sets [19, 20].

Table 1 also lists the uncertainties of the results, sub-divided into the categories described in the next Section. The correlations between the inputs are described in Section 4.

3 Error Categories

We employ the same error categories as used for the previous world average [28], plus one new category (lepPt). They include a detailed breakdown of the various sources of uncertainty and aim to lump together sources of systematic uncertainty that share the same or similar origin. For example, the “Signal” category discussed below includes the uncertainties from ISR, FSR, and PDF—all of which affect the modeling of the $t\bar{t}$ signal. Additional categories are included in order to accommodate specific types of correlations. For example, the jet energy scale (JES) uncertainty is sub-divided into several components in order to more accurately accommodate our best estimate of the relevant correlations. Each error category is discussed below.

**Statistical:** The statistical uncertainty associated with the $M_t$ determination.

**iJES:** That part of the JES uncertainty which originates from in-situ calibration procedures and is uncorrelated among the measurements. In the combination reported here it corresponds to the statistical uncertainty associated with the JES determination using the $W \rightarrow qq'\ell$ invariant mass in the CDF Run-II l+j and all-h measurements and DØ Run-IIa and Run-IIb l+j measurements. Residual JES uncertainties, which arise from effects not considered in the in-situ calibration, are included in other categories.

**aJES:** That part of the JES uncertainty which originates from differences in detector $e/h$ response between $b$-jets and light-quark jets. It is specific to the DØ Run-II measurements and is taken to be uncorrelated with the DØ Run-I and CDF measurements.

**bJES:** That part of the JES uncertainty which originates from uncertainties specific to the modeling of $b$-jets and which is correlated across all measurements. For both CDF and DØ this includes uncertainties arising from variations in the semi-leptonic branching fraction, $b$-fragmentation modeling, and differences in the color flow between $b$-jets and light-quark jets. These were determined from Run-II studies but back-propagated to the
Run-I measurements, whose rJES uncertainties (see below) were then corrected in order to keep the total JES uncertainty constant.

cJES: That part of the JES uncertainty which originates from modeling uncertainties correlated across all measurements. Specifically it includes the modeling uncertainties associated with light-quark fragmentation and out-of-cone corrections.

dJES: That part of the JES uncertainty which originates from limitations in the calibration data samples used and which is correlated between measurements within the same data-taking period, such as Run I or Run II, but not between experiments. For CDF this corresponds to uncertainties associated with the $\eta$-dependent JES corrections which are estimated using di-jet data events. For DØ this includes uncertainties in the calorimeter response to light-quark jets, and $\eta$- and $p_T$-dependent uncertainties constrained using Run-II $\gamma$+jet data samples.

rJES: The remaining part of the JES uncertainty which is correlated between all measurements of the same experiment independent of data-taking period, but is uncorrelated between experiments. For CDF, this is dominated by uncertainties in the calorimeter response to light-quark jets, and also includes small uncertainties associated with the multiple interaction and underlying event corrections.

lepPt: The systematic uncertainty arising from uncertainties in the scale of lepton transverse momentum measurements. This is an important uncertainty for CDF’s track-based measurement. It was not considered as a source of systematic uncertainty in the Run-I measurements or in measurements at DØ.

Signal: The systematic uncertainty arising from uncertainties in the modeling of the $t\bar{t}$ signal which is correlated across all measurements. This includes uncertainties from variations in the ISR, FSR, and PDF descriptions used to generate the $t\bar{t}$ Monte Carlo samples that calibrate each method. It also includes small uncertainties associated with biases associated with the identification of $b$-jets.

Background: The systematic uncertainty arising from uncertainties in modeling the dominant background sources and correlated across all measurements in the same channel. These include uncertainties on the background composition and shape. In particular uncertainties associated with the modeling of the QCD multi-jet background (all-j and l+j), uncertainties associated with the modeling of the Drell-Yan background (di-l), and uncertainties associated with variations of the fragmentation scale used to model W+jets background (all channels) are included.

Fit: The systematic uncertainty arising from any source specific to a particular fit method, including the finite Monte Carlo statistics available to calibrate each method.

Monte Carlo: The systematic uncertainty associated with variations of the physics model used to calibrate the fit methods and correlated across all measurements. For CDF it includes variations observed when substituting PYTHIA [29, 30, 31] (Run I and Run II)
or ISAJET [32] (Run I) for HERWIG [33, 34] when modeling the $t\bar{t}$ signal. Similar variations are included for the DØ Run-I measurements. The DØ Run-II measurements use ALPGEN [35] to model the $t\bar{t}$ signal and the variations considered are included in the Signal category above.

**UN/MI:** This is specific to DØ and includes the uncertainty arising from uranium noise in the DØ calorimeter and from the multiple interaction corrections to the JES. For DØ Run-I these uncertainties were sizable, while for Run-II owing to the shorter integration time and in-situ JES determination, these uncertainties are negligible.

These categories represent the current preliminary understanding of the various sources of uncertainty and their correlations. We expect these to evolve as we continue to probe each method’s sensitivity to the various systematic sources with ever improving precision. Variations in the assignment of uncertainties to the error categories, in the back-propagation of the bJES uncertainties to Run-I measurements, in the approximations made to symmetrize the uncertainties used in the combination, and in the assumed magnitude of the correlations all negligibly effect ($\ll 0.1$GeV/$c^2$) the combined $M_t$ and total uncertainty.

### 4 Correlations

The following correlations are used when making the combination:

- The uncertainties in the Statistical, Fit, and iJES categories are taken to be uncorrelated among the measurements.

- The uncertainties in the aJES and dJES categories are taken to be 100% correlated among all Run-I and all Run-II measurements on the same experiment, but uncorrelated between Run I and Run II and uncorrelated between the experiments.

- The uncertainties in the rJES and UN/MI categories are taken to be 100% correlated among all measurements on the same experiment.

- The uncertainties in the Background category are taken to be 100% correlated among all measurements in the same channel.

- The uncertainties in the bJES, cJES, Signal, and Generator categories are taken to be 100% correlated among all measurements.

Using the inputs from Table 1 and the correlations specified here, the resulting matrix of total correlation co-efficients is given in Table 2.
Table 2: The resulting matrix of total correlation coefficients used to determine the world average top quark mass.

The measurements are combined using a program implementing a numerical $\chi^2$ minimization as well as the analytic BLUE method [21, 22]. The two methods used are mathematically equivalent, and are also equivalent to the method used in an older combination [36], and give identical results for the combination. In addition, the BLUE method yields the decomposition of the error on the average in terms of the error categories specified for the input measurements [22].

## 5 Results

The combined value for the top-quark mass is:

$$M_t = 172.4 \pm 1.2 \text{ GeV}/c^2,$$

with a $\chi^2$ of 6.9 for 11 degrees of freedom, which corresponds to a probability of 81%, indicating good agreement among all the input measurements. The total uncertainty can be sub-divided into the contributions from the various error categories as: Statistical ($\pm 0.7$), total JES ($\pm 0.8$), Lepton scale ($\pm 0.1$), Signal ($\pm 0.3$), Background ($\pm 0.3$), Fit ($\pm 0.1$), Monte Carlo ($\pm 0.3$), and
Table 3: The pull and weight for each of the inputs used to determine the world average mass of the top quark. See Reference [21] for a discussion of negative weights.

UN/MI \((\pm 0.02)\), for a total Systematic \((\pm 1.0)\), where all numbers are in units of \(\text{GeV}/c^{2}\). The pull and weight for each of the inputs are listed in Table 3. The input measurements and the resulting world average mass of the top quark are summarized in Figure 1.

The weights of many of the Run-I measurements are negative. In general, this situation can occur if the correlation between two measurements is larger than the ratio of their total uncertainties. This is indeed the case here. In these instances the less precise measurement will usually acquire a negative weight. While a weight of zero means that a particular input is effectively ignored in the combination, a negative weight means that it affects the resulting central value and helps reduce the total uncertainty. See reference [21] for further discussion of negative weights.

Although the \(\chi^{2}\) from the combination of all measurements indicates that there is good agreement among them, and no input has an anomalously large pull, it is still interesting to also fit for the top-quark mass in the all-j, 1+j, and di-l channels separately. We use the same methodology, inputs, error categories, and correlations as described above, but fit for the three physical observables, \(M_{t\text{all-j}}\), \(M_{t\text{1+j}}\), and \(M_{t\text{di-l}}\). The results of this combination are shown in Table 4 and have \(\chi^{2}\) of 4.9 for 9 degrees of freedom, which corresponds to a probability of 84%. These results differ from a naive combination, where only the measurements in a given channel contribute to the \(M_{t}\) determination in that channel, since the combination here fully accounts for all correlations, including those which cross-correlate the different channels. Using the results of Table 4 we calculate the chi-squared consistency between any two channels, including all correlations, as \(\chi^{2}(\text{dil} - \text{lj}) = 0.08\), \(\chi^{2}(\text{lj} - \text{allj}) = 1.7\), and \(\chi^{2}(\text{allj} - \text{dil}) = 1.9\). These correspond to chi-squared probabilities of 78%, 19%, and 17%, respectively, and indicate that the determinations of \(M_{t}\) from the three channels are consistent with one another.

### 6 Summary

A preliminary combination of measurements of the mass of the top quark from the Tevatron experiments CDF and D\(\O\) is presented. The combination includes five published Run-I mea-
Figure 1: A summary of the input measurements and resulting world average mass of the top quark.
Table 4: Summary of the combination of the 12 measurements by CDF and DØ in terms of three physical quantities, the mass of the top quark in the all-jets, lepton+jets, and di-lepton channels.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (GeV/c²)</th>
<th>Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{t}^{\text{all-j}}$</td>
<td>177.5 ± 4.0</td>
<td>1.00</td>
</tr>
<tr>
<td>$M_{t}^{\text{l+j}}$</td>
<td>172.2 ± 1.2</td>
<td>0.14 1.00</td>
</tr>
<tr>
<td>$M_{t}^{\text{di-l}}$</td>
<td>171.5 ± 2.6</td>
<td>0.17 0.32 1.00</td>
</tr>
</tbody>
</table>

The mass of the top quark is now known with a relative precision of 0.7%, limited by the systematic uncertainties, which are dominated by the jet energy scale uncertainty. This systematic is expected to improve as larger data sets are collected since new analysis techniques constrain the jet energy scale using in-situ $W \rightarrow q\bar{q}'$ decays. It can be reasonably expected that with the full Run-II data set the top-quark mass will be known to better than 0.7%. To reach this level of precision further work is required to determine more accurately the various correlations present, and to understand more precisely the $b$-jet modeling, Signal, and Background uncertainties which may limit the sensitivity at larger data sets. Limitations of the Monte Carlo generators used to calibrate each fit method may also become important as the precision reaches the $\sim 1$ GeV/c² level and will warrant further study in the near future.
References


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