Combination of the DØ top quark mass measurements

The DØ Collaboration
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We present a combination of the top quark mass measurements performed by the DØ experiment in the dilepton (ℓℓ) and lepton plus jets (ℓ+jets) channels. The combined result yields:

\[ m_{\text{top}} = 174.2 \pm 0.9 \text{ (stat)} \pm 1.5 \text{ (syst)} \text{ GeV or} \]
\[ m_{\text{top}} = 174.2 \pm 1.7 \text{ GeV}. \]

Result for winter 2009 conferences
I. INTRODUCTION

The combination of top quark mass measurements presented in this note includes the best DO measurements from Run I of the Tevatron in the dilepton [1] and $\ell$+jets [2] channels, and the most recent Run II measurements in the dilepton [3, 4] and the $\ell$+jets [5, 6] channels with 3.6 fb$^{-1}$ data set.

II. METHOD AND INPUTS

We use the BLUE [7] method to perform the combination of the top quark mass measurements. We follow the same procedure, use the same classes of uncertainties and the same package as is used to compute the world average top quark mass [8].

Since the last combination of summer 2008 CDF and DØ collaborations worked together on the review of the systematic uncertainties and establishing common procedures of their evaluation where possible. Both CDF and DØ experiments added an uncertainty coming from the color reconnection modeling in the $tt$ event generation. The DØ experiment included the uncertainties associated with the initial and final state radiation modeling following the method used by CDF [9] and evaluated uncertainties from the different hadronization models and higher order corrections to the $tt$ matrix element calculation. Table I summarizes the top quark mass measurements that enter the combination with the corresponding statistical and systematic uncertainties. Definition of uncertainties follows Ref. [8]. These categories represent the current preliminary understanding of the various sources of uncertainty and their correlations. We expect these to evolve as we continue to probe each method’s sensitivity to the various systematic sources with ever improving precision.

We have back-propagated the uncertainties associated with hadronization model, higher order corrections, color reconnection modeling, lepton momentum scale and a part of jet energy calibration uncertainty associated with the difference in $\gamma$+jets and $tt$ sample composition to the published $\ell$+jets Run IIa measurement. Inclusion of more uncertainties resulted in the same size of the total uncertainty on the combined mass as in summer 2008 despite the decrease of statistical uncertainties.

For the dilepton combination, we use Matrix Element results in the $e\mu$ channel for RunIIa and RunIIb [4] and Template methods results in the Run IIa $ee$, $e\mu$, and $\ell$+track channels [3]. The latter approach is itself a combination of neutrino weighting and matrix weighting approaches, where the correlations between results have been accounted for. The $e\mu$ channel have been constructed to be orthogonal to $ee$, $e\mu$, and $\ell$+track channels. We propagated the uncertainties of the electron muon Matrix Element analysis coming from hadronization and underlying event, color reconnection and lepton momentum scale to the template results, while removing the underlying event uncertainty obtained by comparing different PYTHIA tunes from the latter to avoid double counting.

The top quark mass measurement in the $\ell$+jets channel has been performed separately using Run IIa [5] and Run IIb [6] data yielding:

$$m_{\text{top}}^{\ell+jets} = 171.5 \pm 1.8 \text{ (stat+JES) } \pm 1.4 \text{ (syst) GeV (Run IIa)}$$

$$m_{\text{top}}^{\ell+jets} = 174.8 \pm 1.3 \text{ (stat+JES) } \pm 1.4 \text{ (syst) GeV (Run IIb)},$$

respectively. Both analyses use Matrix Element method with the in-situ jet energy calibration. In addition, the measurements take advantage of the external jet energy scale calibration to achieve the best precision. Therefore, to correctly take into account correlations between different sources of systematic uncertainties, the JES uncertainty has to be split into two parts: coming from the in-situ calibration and from the external JES. To perform such breakdown we used three mass measurements performed under different conditions (systematics uncertainties are not shown):

- with fixed JES:
  $$m_1 = 173.05 \pm 1.45 \text{ (stat) (RunIIa)}, \quad m_1 = 176.48 \pm 1.02 \text{ (stat) (RunIIb)}$$

- with floating JES without a prior:
  $$m_2 = 170.59 \pm 2.15 \text{ (stat+in-situ) (RunIIa)}, \quad m_2 = 174.54 \pm 1.43 \text{ (stat+in-situ) (RunIIb)}$$

- with floating JES and a prior:
  $$m = 171.50 \pm 1.78 \text{ (stat+in-situ/prior) (RunIIa)}, \quad m = 174.75 \pm 1.28 \text{ (stat+in-situ/prior) (RunIIb)}.$$

The last line corresponds to the actual results.
From the first two measurements we obtain a relative statistical uncertainty (0.84% for RunIIa and 0.58% for RunIIb) and a relative uncertainty due to in-situ calibration (0.94% for RunIIa and 0.58% for RunIIb). These translate into 1.44 GeV and 1.01 GeV statistical uncertainty for RunIIa and RunIIb respectively and 1.61 GeV and 1.02 GeV in-situ uncertainty for RunIIa and RunIIb respectively for the third measurement. The usage of the prior reduces the latter. To extract the uncertainty solely due to prior, i.e., due to external JES, we use the BLUE method iteratively and combine the third measurement with the statistical and in-situ uncertainties mentioned above with a pseudo-measurement that has the same central value and statistical uncertainty, zero uncertainty from in-situ calibration and unknown uncertainty from the external JES. We take statistical uncertainties fully correlated in the combination, so that the statistical uncertainty of the combined result is the same as for inputs. We stop the iterative process when the combined in-situ and prior uncertainty on the resulting mass becomes equal to the measured one. We obtain that the uncertainty from the prior, if no in-situ calibration is used, would be 1.37 GeV for RunIIa and 1.24 GeV for RunIIb (see Table II). The output of the BLUE combination code provides a breakdown of the total measured uncertainty of 1.78 GeV for RunIIa and 1.28 GeV for RunIIb into a statistical component (1.44 GeV for RunIIa and 1.01 GeV for RunIIb), uncertainty from in-situ calibration (0.68 GeV for RunIIa and 0.61 GeV for RunIIb) and uncertainty from the prior (0.80 GeV for RunIIa and 0.50 GeV for RunIIb). The latter is included in dJES and the former corresponds to iJES entries in Table I.

In general, the assignment of the uncertainties to different categories is driven by the two factors: the nature of the source and by the proper treatment of correlations in the Tevatron mass combination. For the latter reason uncertainties coming from the multijet background modeling estimated using data were moved from the "Background" category which is correlated between CDF and D0 within the same channel to the "Fit" category which is uncorrelated between the two experiments.

<table>
<thead>
<tr>
<th>Run I</th>
<th>Run II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ℓ+jets</td>
</tr>
<tr>
<td>lumi</td>
<td>130 pb⁻¹</td>
</tr>
<tr>
<td>top quark mass</td>
<td>180.1 GeV</td>
</tr>
<tr>
<td>tJES</td>
<td>0.0</td>
</tr>
<tr>
<td>aJES</td>
<td>0.0</td>
</tr>
<tr>
<td>bJES</td>
<td>0.7</td>
</tr>
<tr>
<td>cJES</td>
<td>2.0</td>
</tr>
<tr>
<td>rJES</td>
<td>0.0</td>
</tr>
<tr>
<td>lept</td>
<td>2.5</td>
</tr>
<tr>
<td>Signal</td>
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</tr>
<tr>
<td>MC</td>
<td>1.1</td>
</tr>
<tr>
<td>UN/MI</td>
<td>0.0</td>
</tr>
<tr>
<td>Background</td>
<td>1.3</td>
</tr>
<tr>
<td>Fit</td>
<td>1.0</td>
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<tr>
<td>CR</td>
<td>0.6</td>
</tr>
<tr>
<td>MHI</td>
<td>0.0</td>
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<tr>
<td>systematic</td>
<td>3.9</td>
</tr>
<tr>
<td>statistical</td>
<td>3.6</td>
</tr>
<tr>
<td>total</td>
<td>5.3</td>
</tr>
</tbody>
</table>

TABLE I: Inputs to the D0 top quark mass combination. Uncertainties are in GeV.

<table>
<thead>
<tr>
<th>Run IIa</th>
<th>mean</th>
<th>statistical</th>
<th>in-situ</th>
<th>prior</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>input 1</td>
<td>171.50</td>
<td>1.44</td>
<td>1.61</td>
<td>0</td>
<td>2.16</td>
</tr>
<tr>
<td>input 2</td>
<td>171.50</td>
<td>1.44</td>
<td>0</td>
<td>1.37</td>
<td>2.00</td>
</tr>
<tr>
<td>output</td>
<td>171.50</td>
<td>1.44</td>
<td>0.68</td>
<td>0.80</td>
<td>1.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run IIb</th>
<th>mean</th>
<th>statistical</th>
<th>in-situ</th>
<th>prior</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>input 1</td>
<td>174.75</td>
<td>1.31</td>
<td>1.02</td>
<td>0</td>
<td>1.43</td>
</tr>
<tr>
<td>input 2</td>
<td>174.75</td>
<td>1.31</td>
<td>0</td>
<td>1.24</td>
<td>1.60</td>
</tr>
<tr>
<td>output</td>
<td>174.75</td>
<td>1.31</td>
<td>0.61</td>
<td>0.50</td>
<td>1.28</td>
</tr>
</tbody>
</table>

TABLE II: Inputs and output of the BLUE combination code used for splitting of prior and in-situ calibration uncertainties in ℓ+jets channel for RunIIa and RunIIb. All numbers are in GeV.
TABLE III: Summary of correlations between different sources of uncertainties. Within each category same symbol indicates that uncertainties are taken as 100% correlated, no symbol indicates no correlation.

III. RESULTS

Combining the top quark mass measurements performed using Run IIa [5] and Run IIb data [6] in $\ell$+jets channel we obtain:

\[ m_{\text{top}}^{\ell+\text{jets}} = 173.7 \pm 0.8 \text{ (stat)} \pm 1.6 \text{ (syst)} \text{ GeV or} \]
\[ m_{\text{top}}^{\ell+\text{jets}} = 173.7 \pm 1.8 \text{ GeV}. \]

The $\chi^2$ for combination is 2.5 for 1 degree of freedom. The probability to get this or larger value of the $\chi^2$ is 11.2%.

Combined top quark mass measurement in dilepton channel using up to 3.6 fb$^{-1}$ of data in ee, $e\mu$, $\mu\mu$ and lepton+track channels is:

\[ m_{\text{top}}^{\ell\ell} = 174.7 \pm 2.9 \text{ (stat)} \pm 2.4 \text{ (syst)} \text{ GeV or} \]
\[ m_{\text{top}}^{\ell\ell} = 174.7 \pm 3.8 \text{ GeV}. \]

Combination of all Run I and Run II measurements yields:

\[ m_{\text{top}} = 174.2 \pm 0.9 \text{ (stat)} \pm 1.5 \text{ (syst)} \text{ GeV or} \]
\[ m_{\text{top}} = 174.2 \pm 1.7 \text{ GeV}. \]

Table IV summarizes the pulls and weights of individual measurements. The $\chi^2$ for combination is 4.8 for 6 degrees of freedom. The probability to get this or larger value of the $\chi^2$ is 57.2%. Breakdown of uncertainties on the combined result is shown in Tab.V.

The weights of the $e\mu$ measurements are negative. In general, this situation can occur if the correlation between two measurements is larger than the ratio of their total uncertainties. This is indeed the case here. In these instances the less precise measurement will usually acquire a negative weight. While a weight of zero means that a particular input is effectively ignored in the combination, a negative weight means that it affects the resulting central value and helps reduce the total uncertainty. See reference [10] for further discussion of negative weights.

TABLE IV: Summary of weights of the individual measurements.
<table>
<thead>
<tr>
<th></th>
<th>$\ell^+\text{jets}$ Run II</th>
<th>$\ell\ell$ Run II</th>
<th>all channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>top quark mass</td>
<td>173.7 GeV</td>
<td>174.7</td>
<td>174.2 GeV</td>
</tr>
<tr>
<td>iJES</td>
<td>0.47</td>
<td>0.00</td>
<td>0.44</td>
</tr>
<tr>
<td>aJES</td>
<td>0.91</td>
<td>1.32</td>
<td>0.80</td>
</tr>
<tr>
<td>bJES</td>
<td>0.07</td>
<td>0.26</td>
<td>0.12</td>
</tr>
<tr>
<td>cJES</td>
<td>0.00</td>
<td>0.00</td>
<td>0.18</td>
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<tr>
<td>dJES</td>
<td>0.84</td>
<td>1.46</td>
<td>0.74</td>
</tr>
<tr>
<td>rJES</td>
<td>0.00</td>
<td>0.00</td>
<td>0.22</td>
</tr>
<tr>
<td>Leptpt</td>
<td>0.18</td>
<td>0.32</td>
<td>0.16</td>
</tr>
<tr>
<td>Signal</td>
<td>0.45</td>
<td>0.65</td>
<td>0.51</td>
</tr>
<tr>
<td>MC</td>
<td>0.58</td>
<td>1.00</td>
<td>0.51</td>
</tr>
<tr>
<td>UN/MI</td>
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<td>0.00</td>
<td>0.12</td>
</tr>
<tr>
<td>Background</td>
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<td>0.08</td>
<td>0.16</td>
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<td>0.51</td>
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<tr>
<td>CR</td>
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<tr>
<td>MHI</td>
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<td>0.00</td>
<td>0.05</td>
</tr>
<tr>
<td>systematic</td>
<td>1.60</td>
<td>2.43</td>
<td>1.50</td>
</tr>
<tr>
<td>statistical</td>
<td>0.83</td>
<td>2.92</td>
<td>0.85</td>
</tr>
<tr>
<td>total</td>
<td>1.80</td>
<td>3.80</td>
<td>1.72</td>
</tr>
</tbody>
</table>

TABLE V: Breakdown of systematic uncertainties on the combined Run II top quark mass measurement in the $\ell^+\text{jets}$ channel and for the complete DØ combination. Uncertainties are in GeV.

Fig. 1 shows a summary of measurements used for DØ combination along with the DØ combination result, the world average and the top quark mass extracted from the cross section measurement [11].

[8] CDF and DØ Collaborations, “Combination of CDF and DØ Results on the Mass of the Top quark”, DØ note 5899-CONF
<table>
<thead>
<tr>
<th>Category</th>
<th>Mass (GeV)</th>
<th>Error 1 STD</th>
<th>Error 2 STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run I Dileptons</td>
<td>168.4 ± 12.3 ± 3.6</td>
<td>± 12.8 GeV</td>
<td></td>
</tr>
<tr>
<td>Run I Lepton+jets</td>
<td>180.1 ± 3.6 ± 3.9</td>
<td>± 5.3 GeV</td>
<td></td>
</tr>
<tr>
<td>Run II Dileptons *</td>
<td>174.7 ± 2.9 ± 2.4</td>
<td>± 3.8 GeV</td>
<td></td>
</tr>
<tr>
<td>Run II Lepton+jets *</td>
<td>173.7 ± 0.8 ± 1.6</td>
<td>± 1.8 GeV</td>
<td></td>
</tr>
<tr>
<td>DØ combined (March 2009)</td>
<td>174.2 ± 0.9 ± 1.5</td>
<td>± 1.7 GeV</td>
<td></td>
</tr>
<tr>
<td>World average (March 2009)</td>
<td>173.1 ± 0.6 ± 1.1</td>
<td>± 1.3 GeV</td>
<td></td>
</tr>
<tr>
<td>Run II σ(l+jets, ll,l+c) *</td>
<td>169.1 ± 5.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIG. 1: A summary of the top quark mass measurements used for DØ combination along with the DØ combination result, the world average top quark mass and the top quark mass extracted from the cross section measurement.