High precision QCD Tools for the LHC

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SMU Seminar

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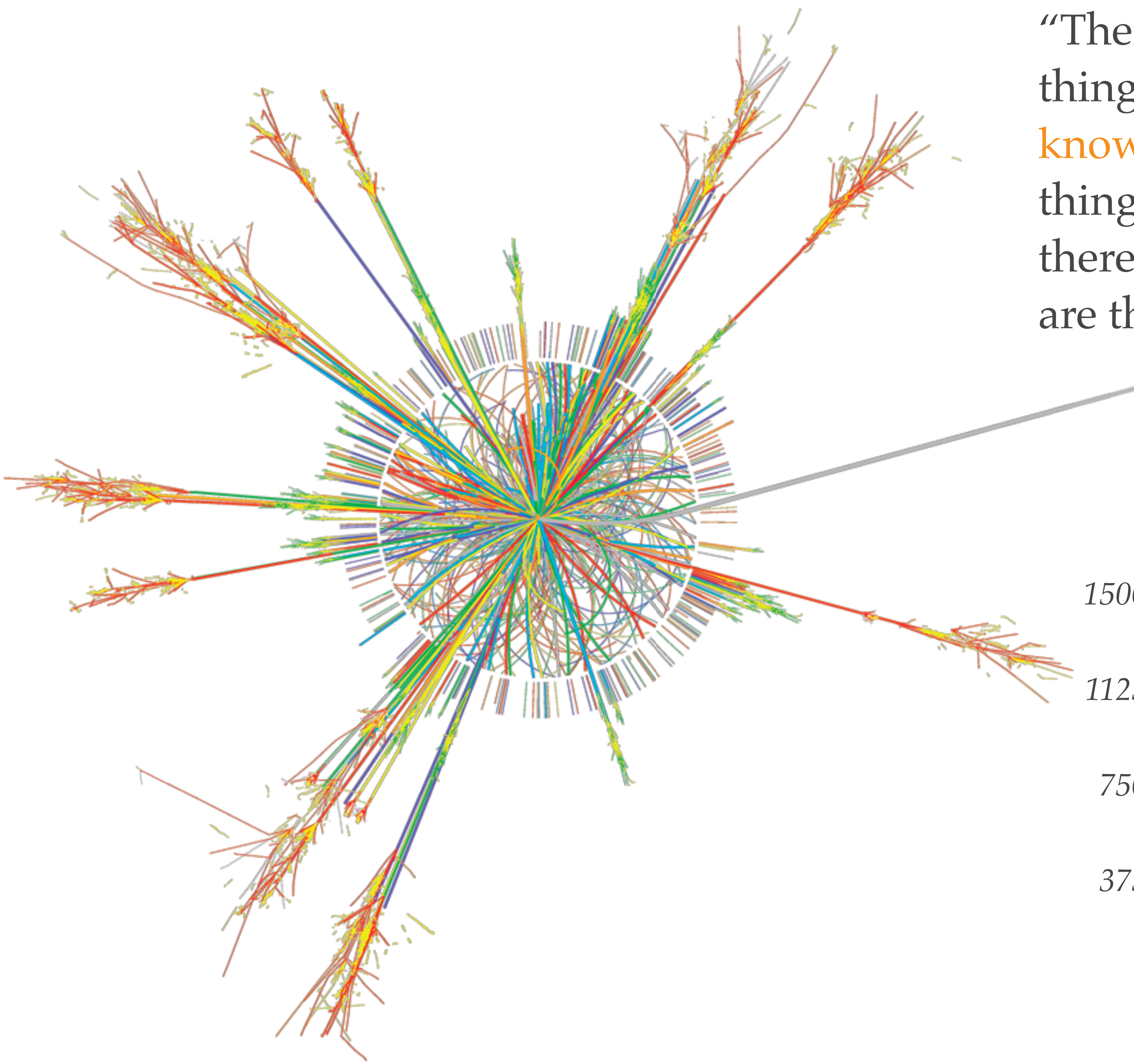
Introduction

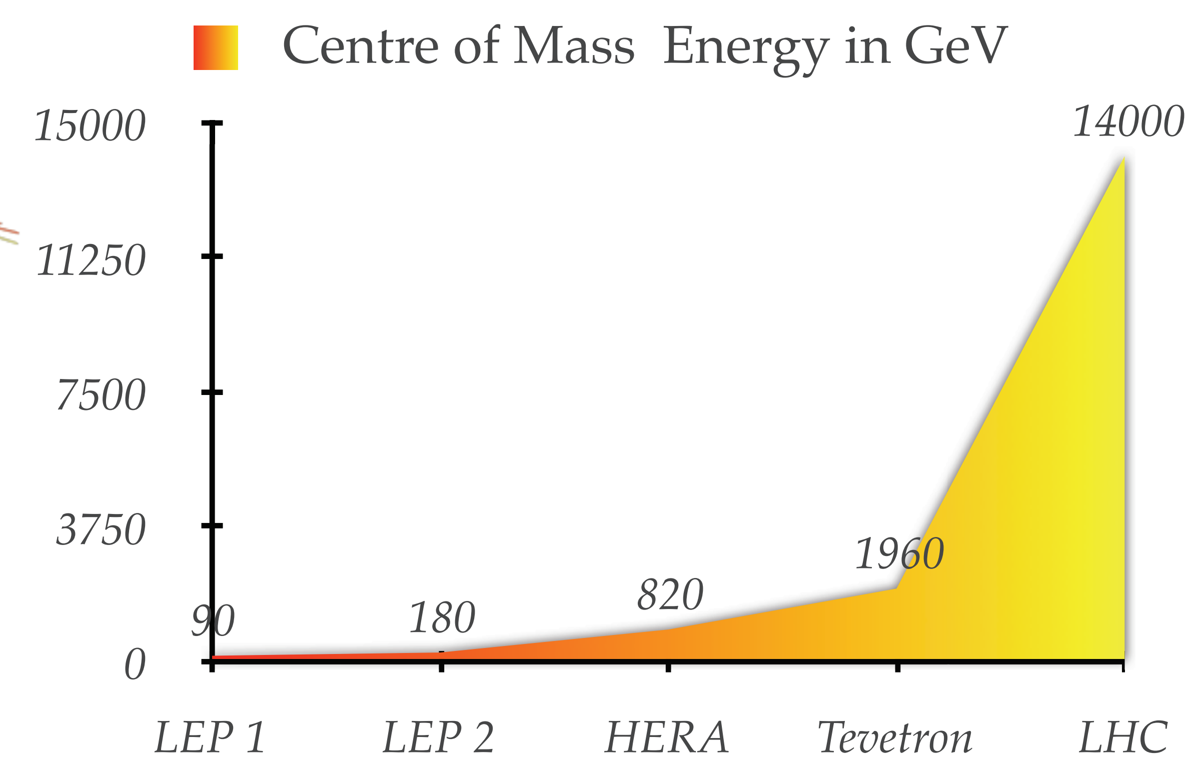
The expectations for LHC physics can be sorted into three categories:



Picture: ATLAS simulation

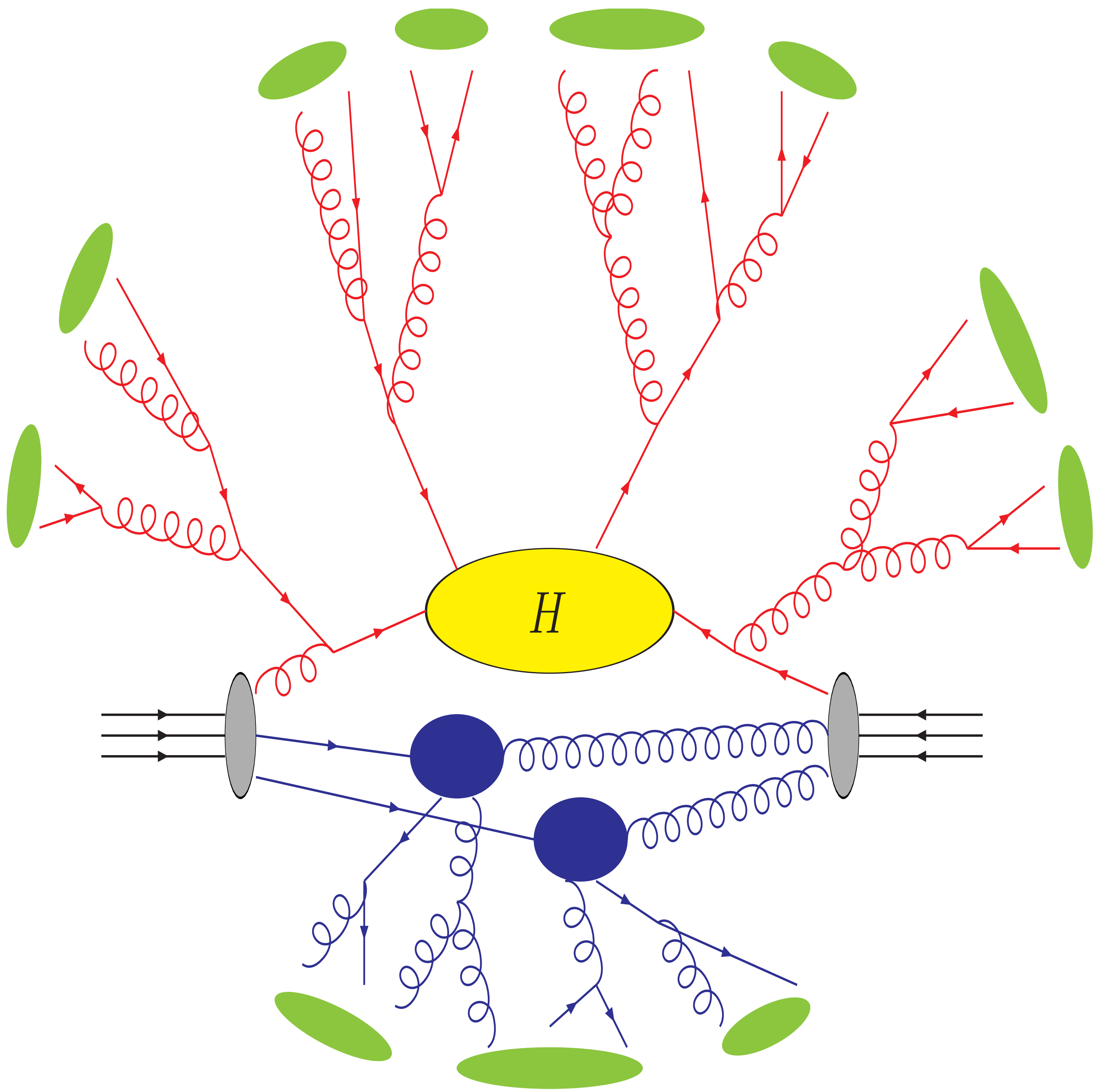
"There are known knowns. These are things we know that we know. There are known unknowns. That is to say, there are things that we know we don't know. But there are also unknown unknowns. There are things we don't know we don't know."

D. Rumsfeld



Introduction

From theory point of view this event looks very complicated



1. Incoming hadron *(gray bubbles)*
   ➡ Parton distribution function

2. Hard part of the process *(yellow bubble)*
   ➡ Matrix element calculation, cross sections at LO, NLO, NNLO level

3. Radiations *(red graphs)*
   ➡ Parton shower calculation
   ➡ Matching to the hard part

4. Underlying event *(blue graphs)*
   ➡ Models based on multiple interaction

5. Hardonization *(green bubbles)*
   ➡ Universal models

Born Level Calculation

$$\sigma[F_J] = \int_m d\Gamma^{(m)}(\{p\}_m)|\mathcal{M}(\{p\}_m)|^2 F_J(\{p\}_m)$$

- ✓ Easy to calculate, no IR singularities. Several matrix element generators are available (Alpgen, Helac, MadGraph, Sherpa)
- ✗ Strong dependence on the unphysical scales (renormalization and factorization scales)
- ✗ Exclusive quantities suffer on large logarithms
- ✗ Every jet is represented by a single parton
- ✗ No quantum corrections
- ✗ No hadronization

NLO Level Calculation

$$\sigma_{\mathrm{NLO}} = \int_N d\sigma^B + \int_{N+1} \left[d\sigma^R - d\sigma^A\right]_{\epsilon=0} + \int_N \left[d\sigma^V + \int_1 d\sigma^A\right]_{\epsilon=0}$$

$$d\sigma^A \sim d\Gamma(\{p\}_{N+1}) \underbrace{V \otimes |\mathcal{M}(\{\tilde{p}\}_N)|^2}_{\text{Based on soft collinear factorization}} F_J(\{\tilde{p}\}_N)$$

- ✓ Includes quantum corrections, in most of the cases it significantly reduces the unphysical scale dependences
- ✓ One of the jets consists of two partons (still very poor)
- ✓ Hard to calculate, the most complicated available processes are $2 \rightarrow 3$ (NLOJET++[1], MCFM, PHOX,...)
- ✗ Exclusive quantities suffer on large logarithms
- ✗ No hadronization

[1] http://nagyz.web.cern.ch/nagyz/Site/NLOJET++/NLOJET++.html

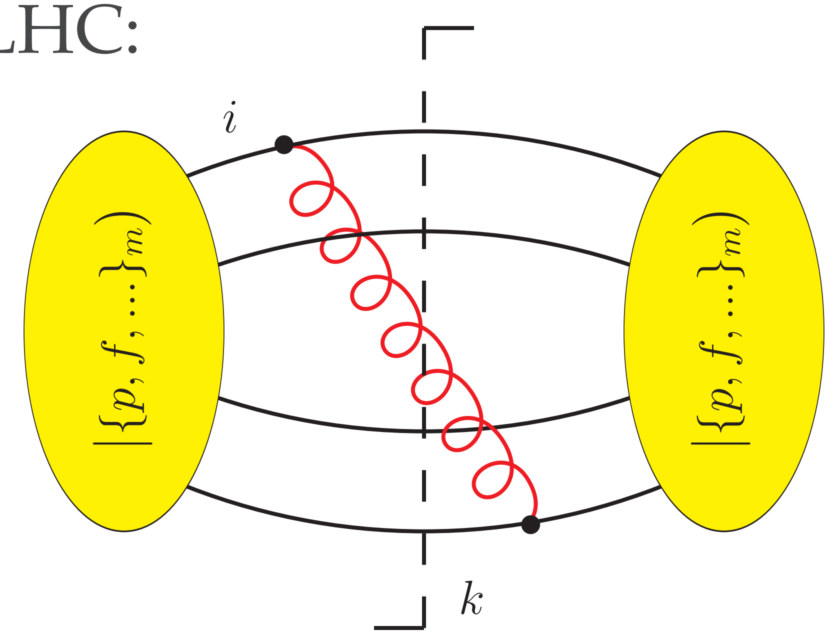
Experimenter's NLO Wish List

Single boson	Diboson	Triboson	Heavy Flavor
Run II Monte Carlo Workshop, April 2001 (Almost 7 years to the day and yet not a single calculation finished!)			
$V+\leq 5$jets $V+bb+\leq 3$jets $V+cc+\leq 3$jets	$VV+\leq 5$jets $VV+bb+\leq 3$jets $VV+cc+\leq 3$jets $WZ+\leq 5$jets $WZ+bb+\leq 3$jets $WZ+cc+\leq 3$jets $W\gamma+\leq 3$jets $Z\gamma+\leq 3$jets	$WWW+\leq 3$jets $WWW+bb+\leq 3$jets $WWW+cc+\leq 3$jets $Z\gamma\gamma+\leq 3$jets $WZZ+\leq 3$jets $ZZZ+\leq 3$jets	$tt+\leq 3$jets $bb+\leq 3$jets $tt+V+\leq 2$jets $tt+H+\leq 2$jets $tb+\leq 2$jets
Les Houches Workshop 2005			
$V+3$jets $H+2$jets	$VV+\leq 2$jets $VV+bb$	ZZZ	$tt+2$jets $tt+bb$
$V\in\{W,Z,\gamma\}$			

Why are these calculations so hard?

NLO: Real part

The Catani-Seymour method is widely used in NLO calculations to deal with IR singularities. We might need a new technique for the LHC:



Catani-Seymour scheme is very nice and clever, but ....

✗ The number of the subtraction terms is $(N+1)^2 (N+4)/2$.

➡ Too many counterterms can kill the numerical stability in the high multiplicity processes.

✗ Matching to the shower is possible only at "classical level"

➡ Some artificial color structure that works in NLO calculations but makes trouble in the parton shower.

We have derived a new scheme from our shower algorithm

✓ The number of the subtraction terms is $(N+1)(N+4)/2$

✓ Fully exclusive in color and spin space

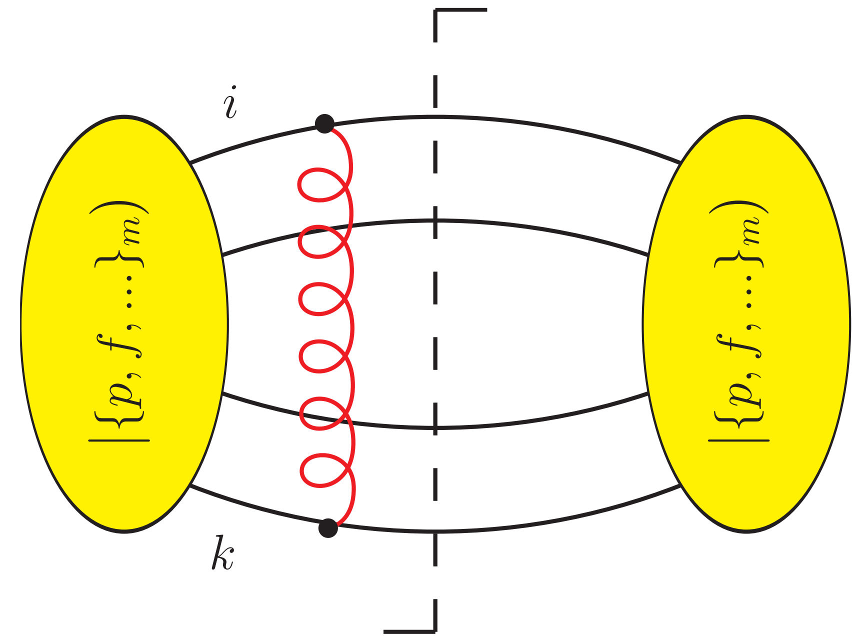
✓ No "artificial" color structure

NLO: Virtual part

The 1-loop matrix elements has complicated algebraic and analytic structure.

$$\mathcal{M}_m^{(1)} = \sum_{\text{graphs}} \underbrace{\int \frac{d^d l}{(2\pi)^d}} N(l) \prod_{i=1}^{n} \frac{1}{(l-Q_i)^2 - m_i^2 + \mathrm{i}0}$$

The challenge is to perform the loop integral.



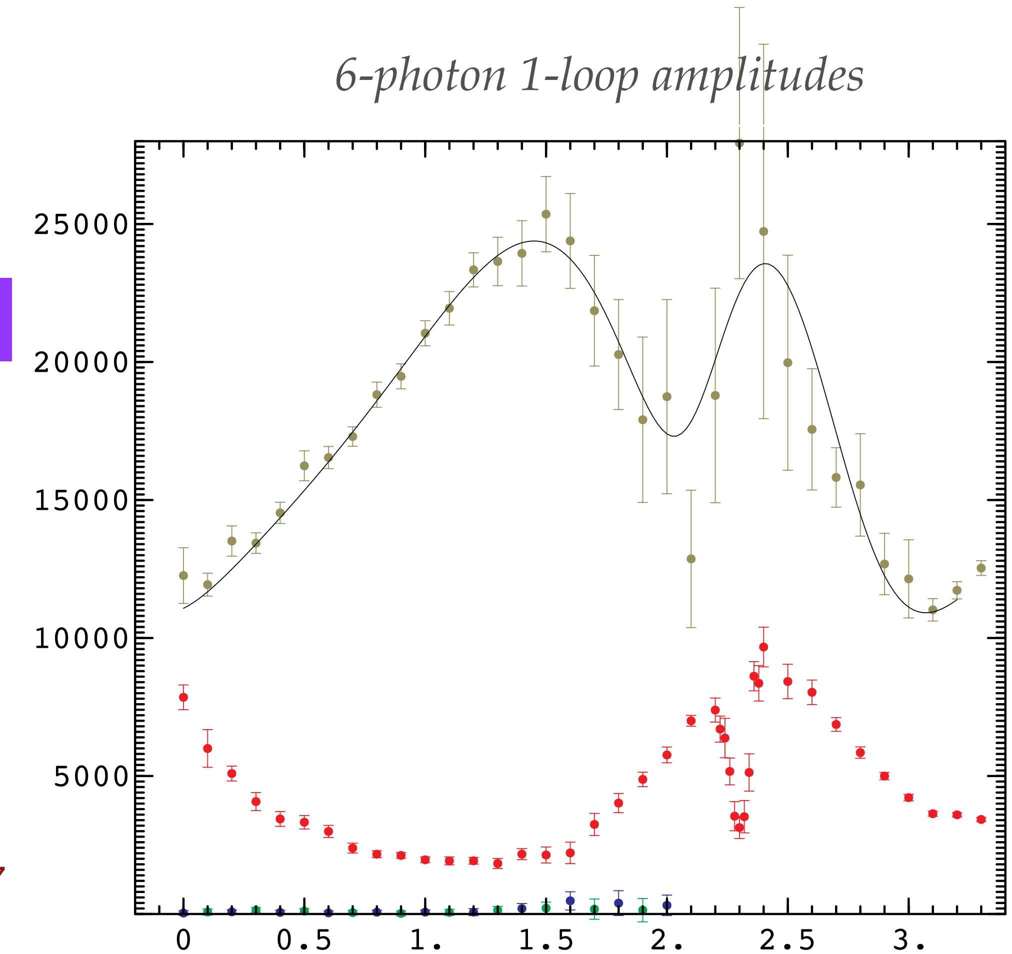
Analytic methods

- Master integrals, recursion relations, twistors, ...
- One loop integral is parametric integral.

Ossola, Papadopoulos, Pittau: **Nucl.Phys. B763**:147-169,2007

Giele, Kunszt, Melnikov: **arXiv**:0801.2237 [hep-ph]

6-photon 1-loop amplitudes



Pure numerical method

- Universal subtraction at integrand level
- Using Feynman parameters to perform the integral

ZN and D. Soper: **JHEP** 0309:055,2003

ZN and D. Soper: **Phys.Rev. D74**:093006,2006

- This method was used to calculate ZZZ, ttZ and NLO and gg→H at 2-loop level

Lazopoulos, Melnikov, Petriello: **Phys.Rev.D76**:014001,2007

Lazopoulos, Melnikov, Petriello: **arXiv**:0709.4044 [hep-ph]

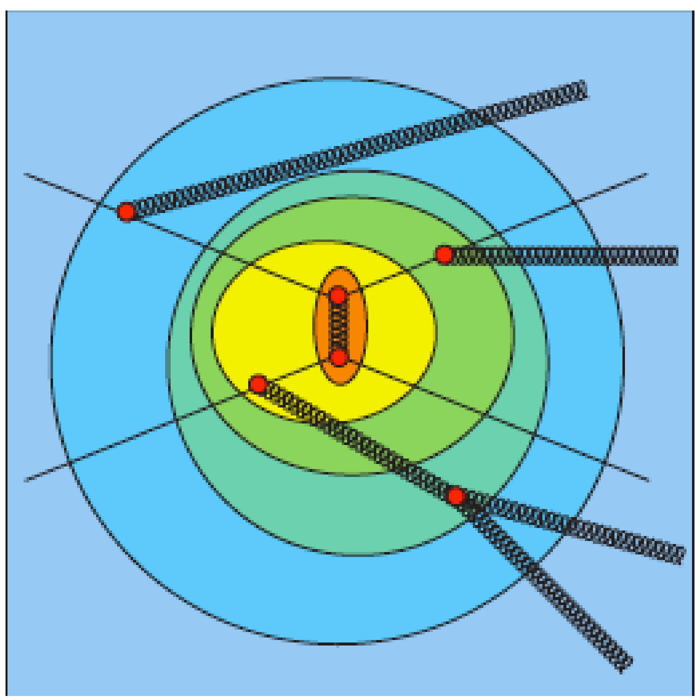
Anastasiou, Beerli, Daleo: **JHEP** 0705:071,2007

LO Parton Shower

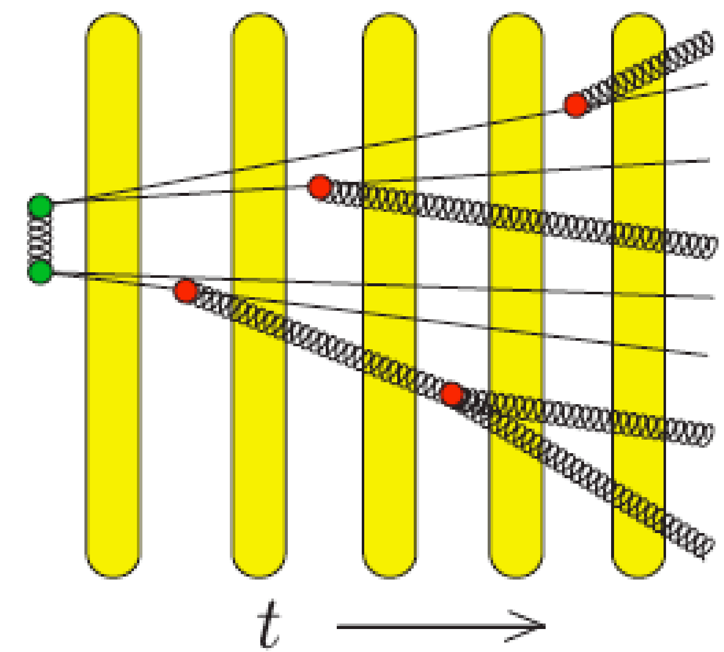
- ✓ It is an iterative algorithm. Arbitrary number of partons.
- ✓ Based on the universal soft and collinear factorization property of the QCD matrix elements. (This is the basic approximation and should be the only.)
- ✓ Matched to the hadronization models (which is universal effect).
- ✓ In the best cases it resumes the leading large logarithms properly.
- ✗ Needs more, rather non systematic approximations. *(See next slides!)*
- ✗ Only leading order splitting kernels are involved, we can expect strong dependence on the unphysical scales.
- ✗ The only exact matrix element in the calculations is 2→2 like at Born level.
- ✗ Positive unweighted events. I think it is a misleading concept.

Shower from Inside Out

Think of shower branching as developing in a “time” that goes from most virtual to least virtual.



Real time picture

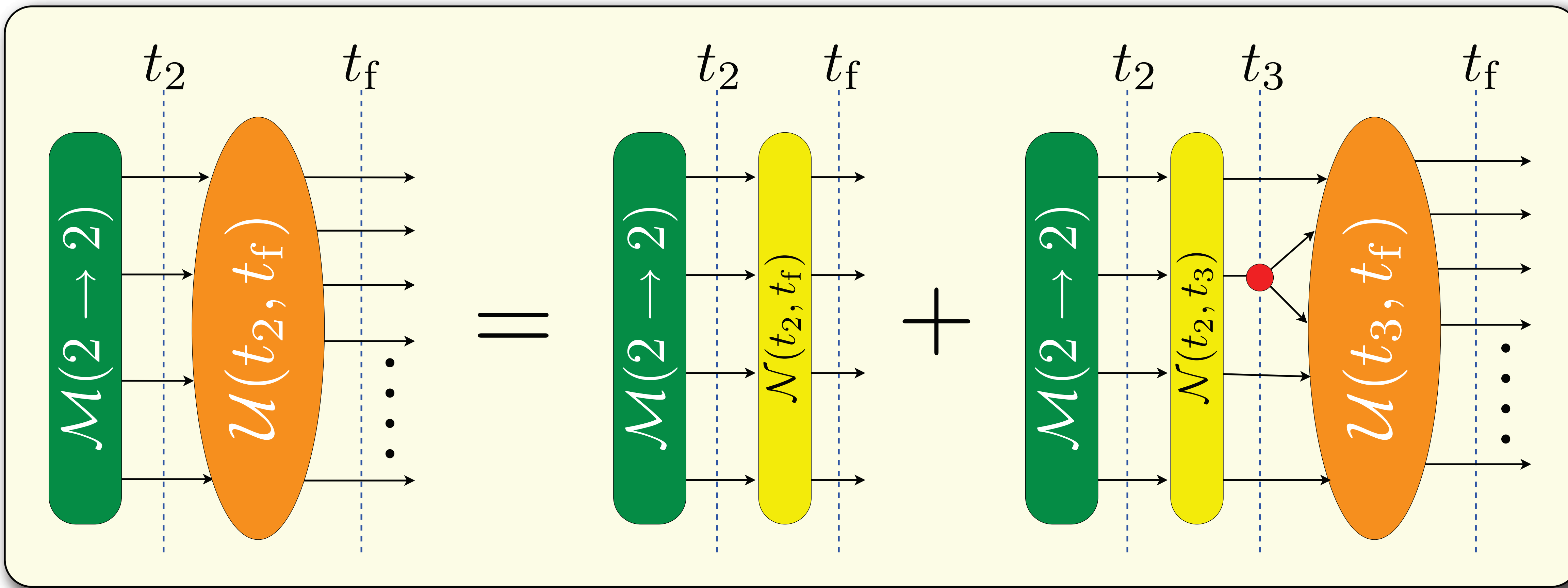


Shower time picture

Thus shower time proceeds backward in physical time for initial state radiation.

Iterative Algorithm

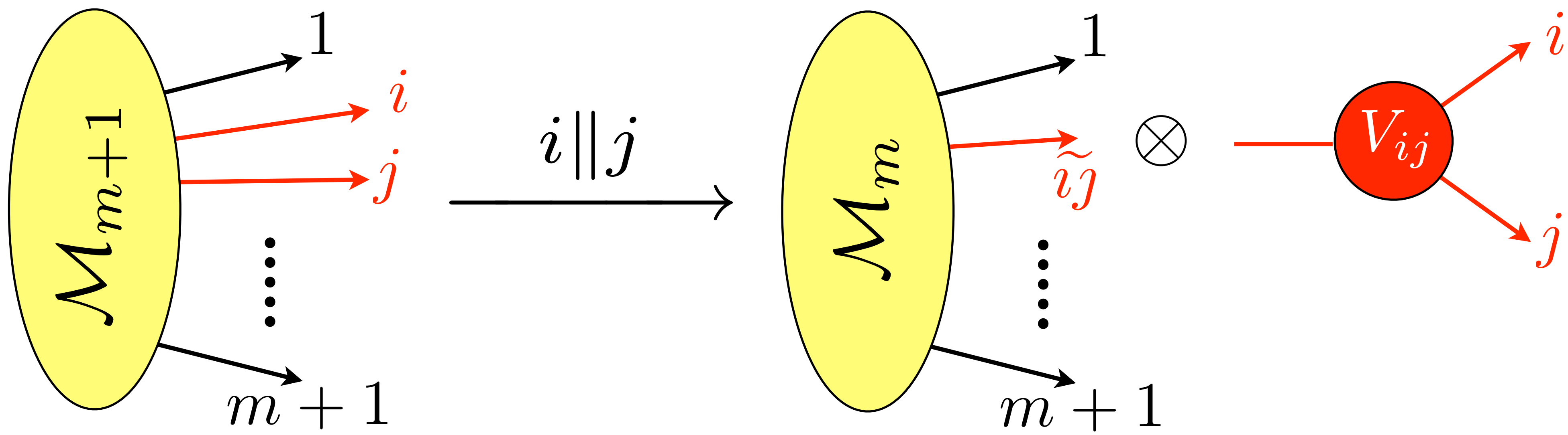
The parton shower evolution starts from the simplest hard configuration, that is usually 2→2 like.



$$\mathcal{U}(t_f, t_2)|\mathcal{M}_2) = \underbrace{\mathcal{N}(t_f, t_2)|\mathcal{M}_2)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_f} dt_3\, \mathcal{U}(t_f, t_3)\mathcal{H}(t_3)\mathcal{N}(t_3, t_2)|\mathcal{M}_2)}^{\text{“Something happens”}}$$

Collinear Approximation

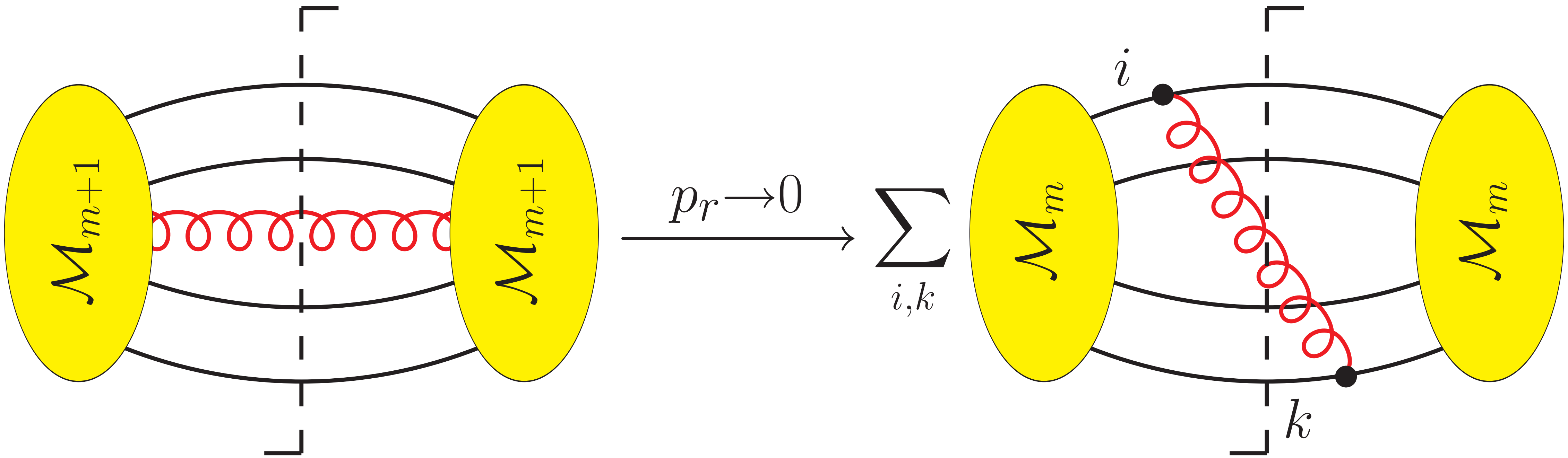
The QCD matrix elements have universal factorization property when two external partons become collinear



- Produces leading and next-to-leading logarithms.
- It is diagonal color, no color correlations.
- The gluon splitting is not diagonal in spin.
- The spin correlations are not really complicated but one can use average spin *as extra approximation*.

Soft Approximation

The QCD matrix elements have universal factorization property when an external gluon becomes soft



- Soft contributions produce next-to-leading logarithms.
- No spin correlation.
- Soft gluon connects everywhere and the color structure is not diagonal; *quantum interferences*.
- Does it spoil the independent evolution picture? Yes, it does, but ...

Color Coherence

There are three way to deal with the soft gluon color interferences:

1. The soft gluon contributions are cancelled in the wide angle region. One can apply angular ordering (Herwig/Herwig++) or impose angular ordering by angular veto (old Phytia). This is an extra approximation, especially for massive quarks. In the massive quark case the color coherence breaks down.

2. One can do leading color approximation. In the large Nc limit the soft gluon is radiated from a color dipole. The leading color contributions are diagonal in color space, thus no technical complication with colors. (Ariadne, new Phytia, Vincia)

3. No extra approximation, treat the soft gluon as it is. Full color correlations.

ZN and D. Soper: **JHEP:** 0709 114,2007

Facts you should beware of

The shower is derived from QCD but you cannot use the shower cross sections as QCD prediction.

- ✗ Very crude approximation in the phase space. Angular ordered shower doesn't cover the whole phase space (dead cone).
  - ➡ In every step of the shower the phase space should be exact, every parton should be onshell.

- ✗ The independent emission picture is valid only in the strict collinear limit. The color correlations are not considered properly even at leading color level.
  - ➡ Color and spin correlation must be considered systematically. We should work with exact color and spin correlations.

- ✗ They are not defined systematically *e.g.*: angular ordering at NLO level??? Even the kinematics of the color dipole model is inconsistent at higher order.
  - ➡ The core algorithm shouldn't depend on the level of the calculation.

Facts you should beware of

The shower is derived from QCD but you cannot use the shower cross sections as QCD prediction.

Cross sections at $\sqrt{s} = 1960$ GeV, with structure functions, in nanobarns, $p_T > 10\,\mathrm{GeV}$ $|\eta| < 2.0$.

Process	σ_0 : Normal	σ_1 : Large Nc component	$\frac{\sigma_1 - \sigma_0}{\sigma_0}$
ud→W+g	0.1029(5)D+01	0.1158(5)D+01	13%
ud→W+gg	0.1018(8)D+00	0.1283(10)D+00	26%
ud→W+ggg	0.1119(17)D-01	0.1564(22)D-01	40%
ud→W+gggg	0.1339(36)D-02	0.2838(71)D-02	120%

Results were calculated by HELAC

✗ They are not defined systematically e.g.: angular ordering at NLO level??? Even the kinematics of the color dipole model is inconsistent at higher order.

➡ The core algorithm shouldn't depend on the level of the calculation.

Classical Parton Shower

- The parton shower relies on the universal soft and collinear factorization of the QCD matrix elements. It is universal property and true at all order. This should be the only approximation ...

... but we have some further approximations:

- ✗ Interference diagrams are treated approximately with the angular ordering
- ✗ Color treatment is valid in the $N_c \to \infty$ limit
- ✗ Spin treatment is usually approximated.
- ✗ Usually very crude approximation in the phase space

→ Parton shower as classical statistical mechanics

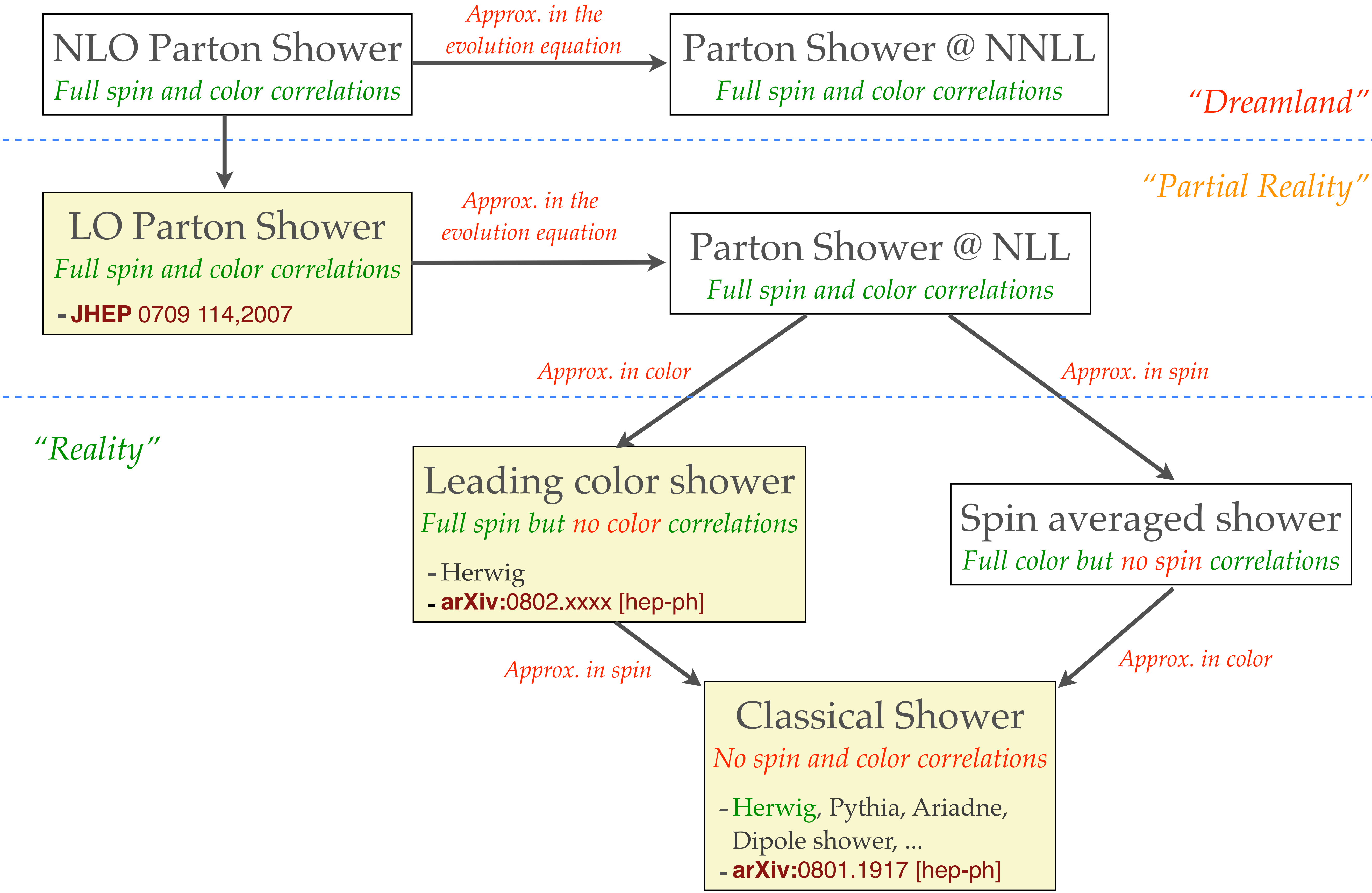
“Quantum Parton Shower”

- The parton shower relies on the universal soft and collinear factorization of the QCD matrix elements. It is universal property and true at all order. This should be the only approximation ...

Parton shower as Quantum statistical mechanics

ZN and D. Soper: **JHEP** 0510:024,2005
arXiv:0801.1917 [hep-ph]

Shower Family Tree



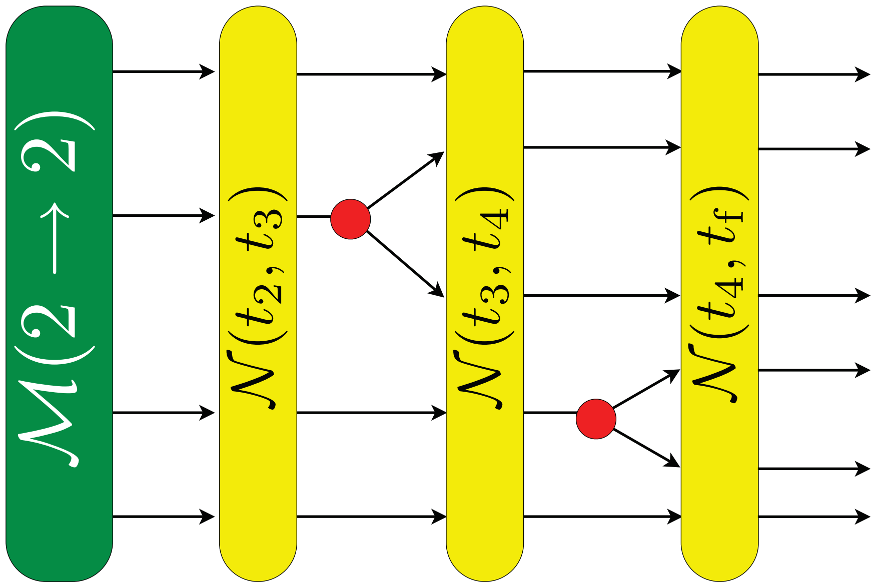
LO Matching Schemes

There are two algorithm available in the literature for LO matching:

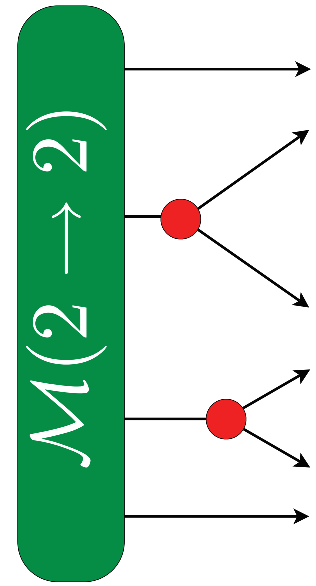
- **CKKW-L algorithm**: Reweighting Born matrix elements with Sudakov factors
  S. Catani, R. Kuhn, F. Krauss, B. Webber: **JHEP** 0111:063,2001
  L. Lönnblad: **JHEP** 0205:046,2002

- **MLM algorithm**: Reweighting shower contributions with Born level matrix elements
  M. Mangano
  M. Mangano , M. Moretti, F. Piccinini, M. Treccani: **JHEP** 0701:013,2007

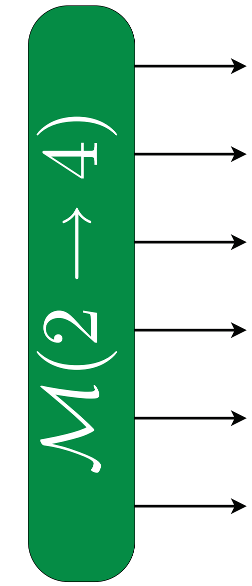
Deficiency of Shower



Standard shower contribution



Small pT approximation

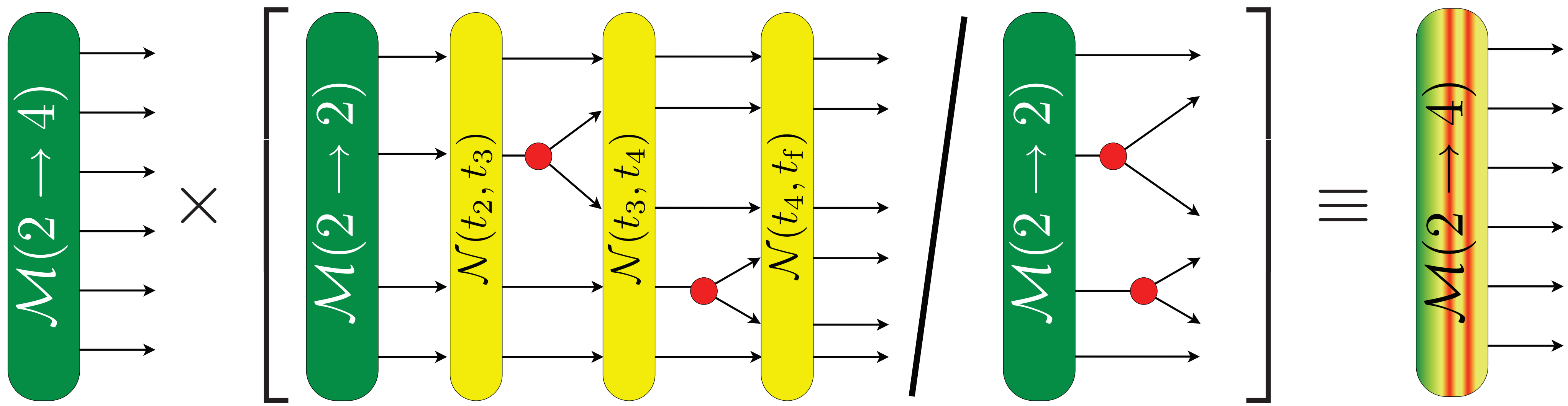


$|\mathcal{M}(2 \to 4)|^2$

- The shower approximation relies on the small pT splittings.
- May be the exact matrix element would be better.
- But that lacks the Sudakov exponents.

Rewieght the exact matrix elements with Sudakov exponents

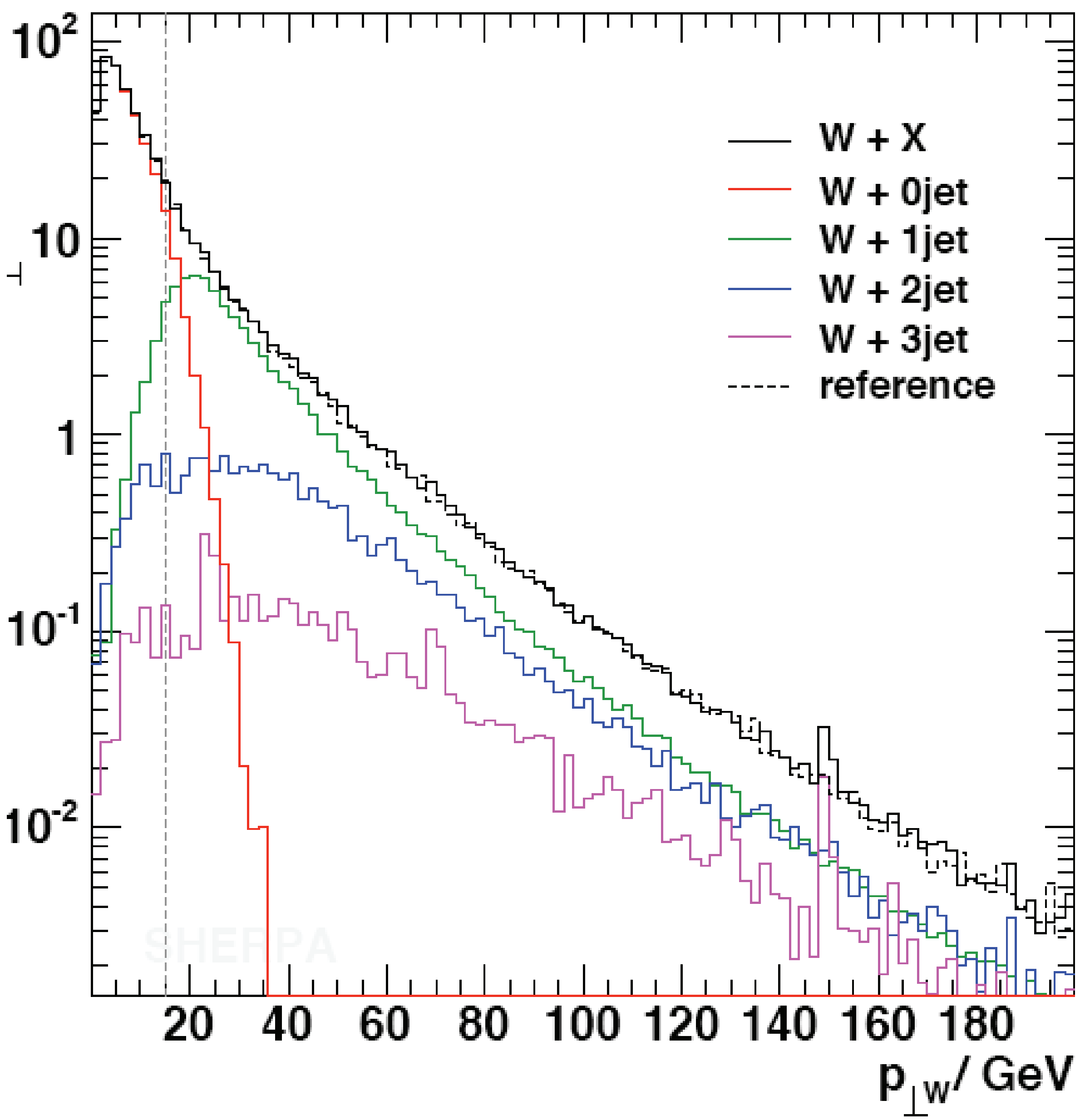
Improved weighting



Rewieghting factor

- This is the essential part of the CKKW matching procedure.
- In general there are many ways to get from $2 \to 2$ configuration to $2 \to m$ configuration.
- CKKW use the kT algorithm to find a unique history to define the Sudakov reweighting.
- The unique history requires to introduce matching scale.

LO Matching Schemes



- ✓ The CKKW-L algorithm is implemented in Sherpa and in Ariadne.
- ✓ It is certainly an improvement.
- ✗ Only normalized cross section can be calculated.
- ✗ The result could strongly depend on the matching scale.
  - ➡ It would be nice NOT to use matching scale.
- ✗ Matching scale dependence cancelled at NLL level but only in *e+e-* annihilation.
- ✗ No matching at quantum level.
- ✗ It is still LO order calculation thus the scale dependence is large.
  - ➡ The algorithm can be generalized at NLO level.

ZN and D. Soper: **JHEP** 0510:024,2005

NLO Matching Schemes

There are several algorithm available in the literature for NLO matching:

- MC@NLO: Avoiding double counting by introducing extra subtraction terms.

  S. Frixione and B. Webber: **JHEP** 0206:029,2002
  S. Frixione, P. Nason and B. Webber: **JHEP** 0308:007,2003

- KS approach: The main idea is to include the first step of the shower in NLO calculation and then start the shower from this configuration.

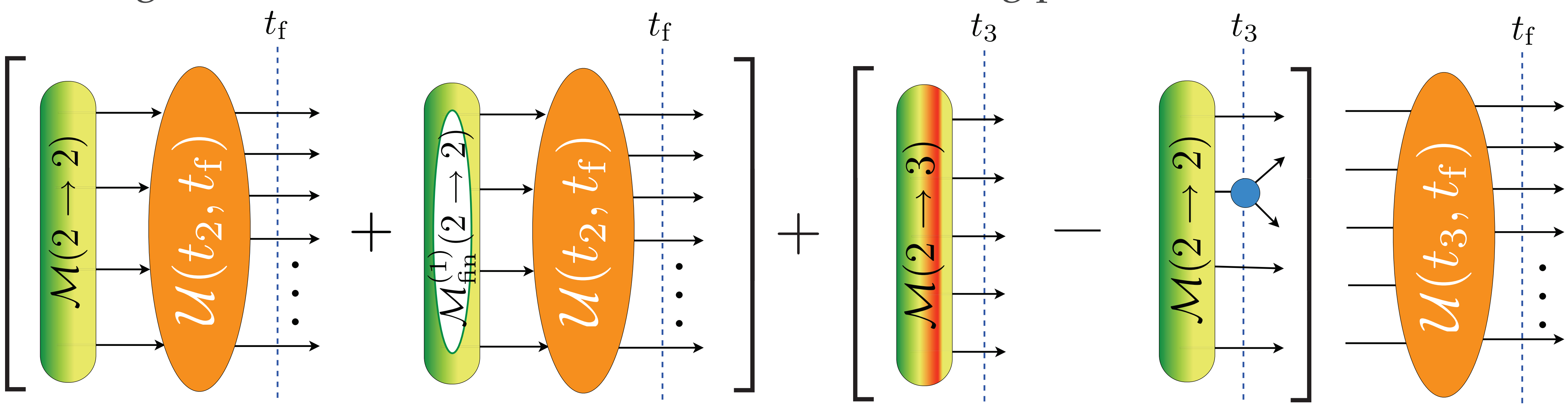
  M. Krämer and D. Soper: **Phys.Rev.** D69:054019,2004
  P. Nason: **JHEP** 0411:040,2004

- "CKKW@NLO" Combines the KS approach and the CKKW matching procedure.

  ZN and D. Soper: **JHEP** 0510:024,2005
  Giele, Kosower, Skands: **arXiv**:0707.3652 [hep-ph]

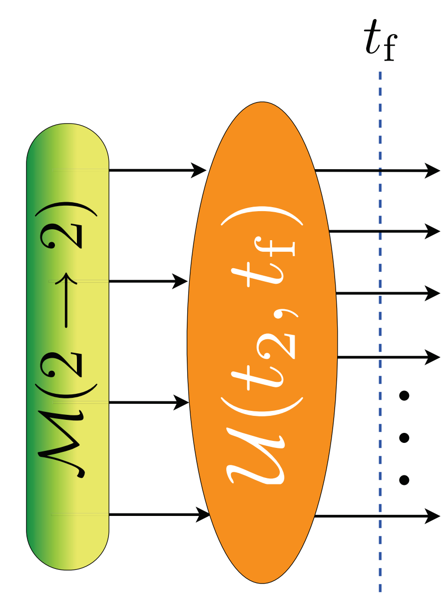
MC@NLO

It might be a good idea to illustrate the MC@NLO matching procedure:



- ✓ Several simple processes are implemented in the MC@NLO framework.
- ✗ The MC@NLO is worked out for HERWIG. If you want to use it with PYTHIA you have to redo the MC subtraction.
- ✗ MC@NLO is defined only for the simplest processes, like *2→0,1,(2)* processes.
- ✗ No quantum correlations.

$$+\int_1 \left[dV_{\text{MC}} - dV\right]$$

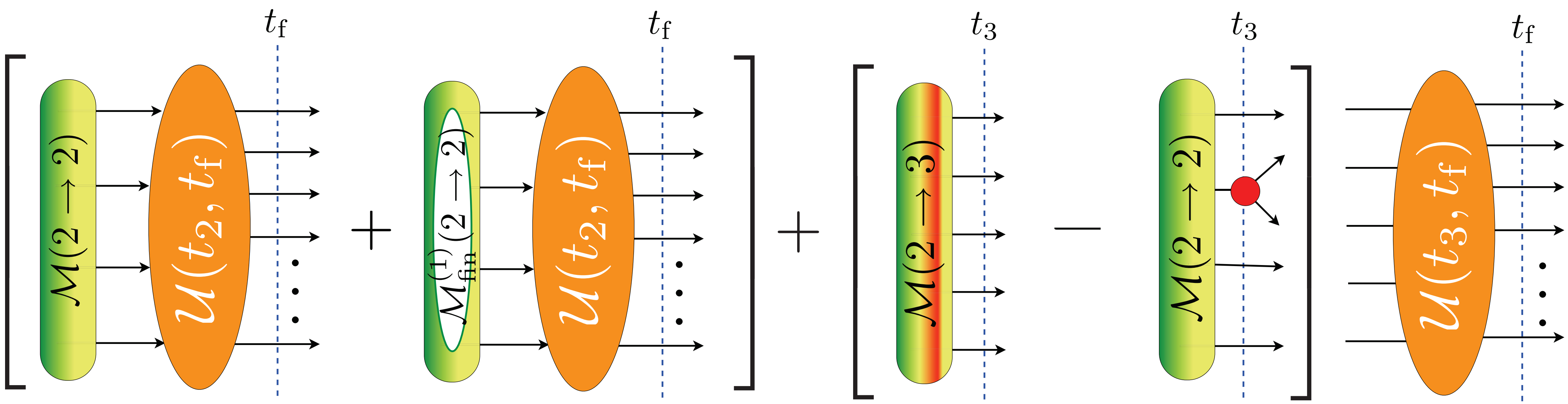


Obvious step to choose

$$dV_{\text{MC}} = dV$$

Other approaches

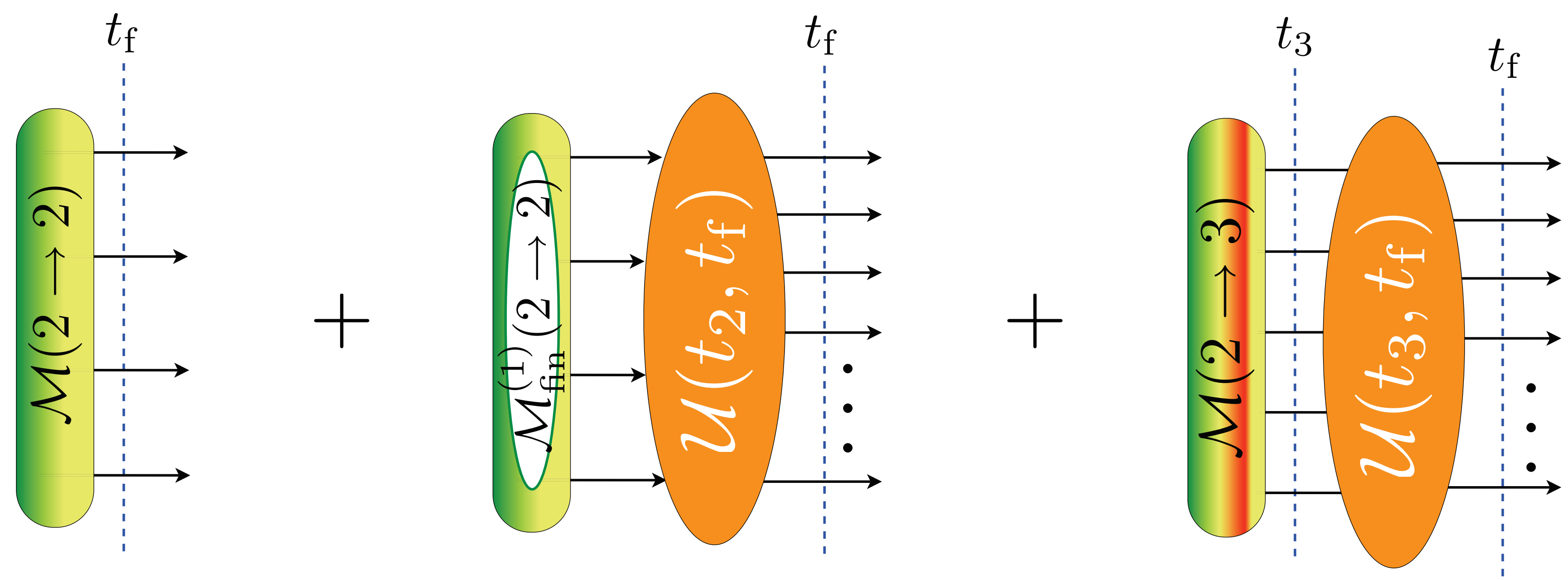
At least the first step of the shower is done with the NLO splitting functions.



- ✓ This matching works with any shower algorithm.
- ✗ Several proposal but NO implementation in a general purpose program so far.
- ✗ No quantum correlations. Matching only in the momentum and flavor space.
- ✗ It is usually defined only for the simplest processes, like *2→ 0,1,2* processes.
- ✓ To apply for other processes one has to combine the NLO matching with the CKKW algorithm. *ZN and D. Soper:* **JHEP** 0510:024,2005

Quantum level NLO matching

Including the quantum correlations (color and spin) properly the structure of the shower with NLO matching is simpler (no subtraction).



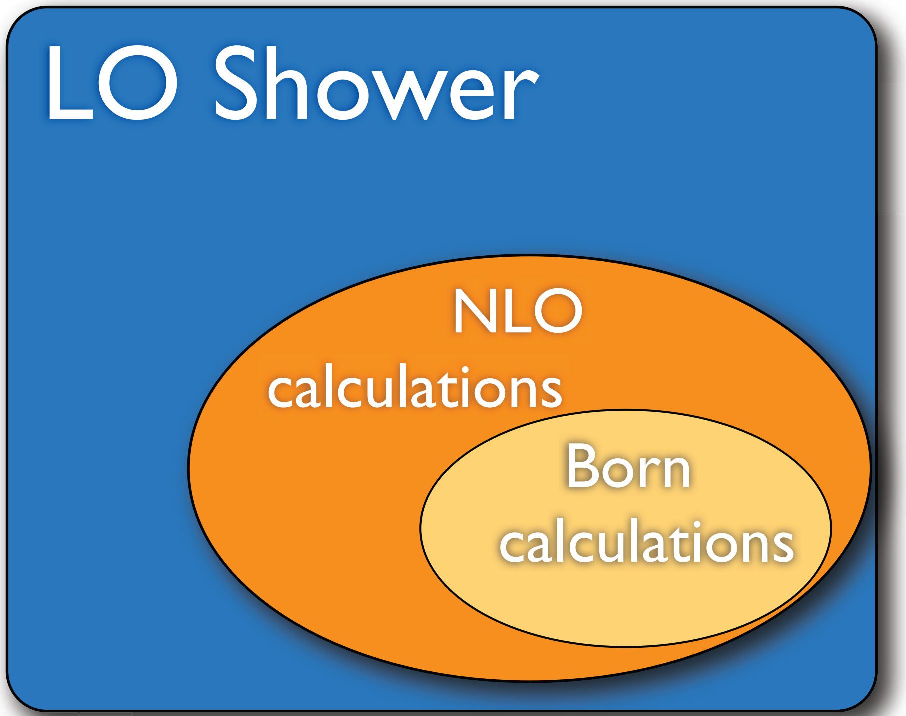
- ✓ This matching requires shower with quantum interference.
- ✓ All the quantum correlations are included.
- ✓ Systematically defined for any process.
- ✗ No algorithm worked out, No implementation.

ZN and D. Soper: **hep-ph/**0601021
http://cern.ch/nagyz

Conclusions

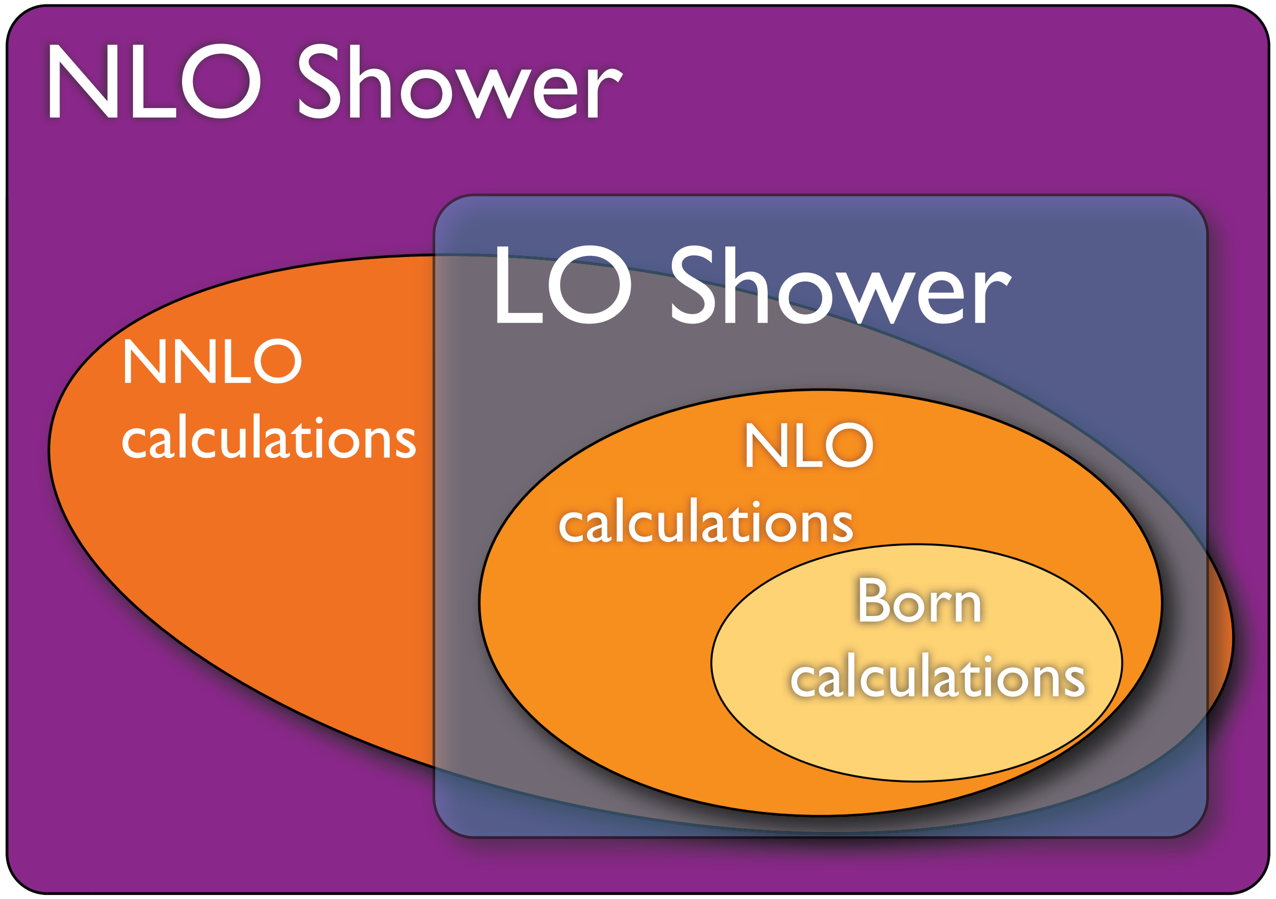
Instead of having defined LO, NLO and shower calculation separately and patching the gap between them by matching schemes

we should define a new shower concept that can naturally cooperate with NLO calculations



Conclusions

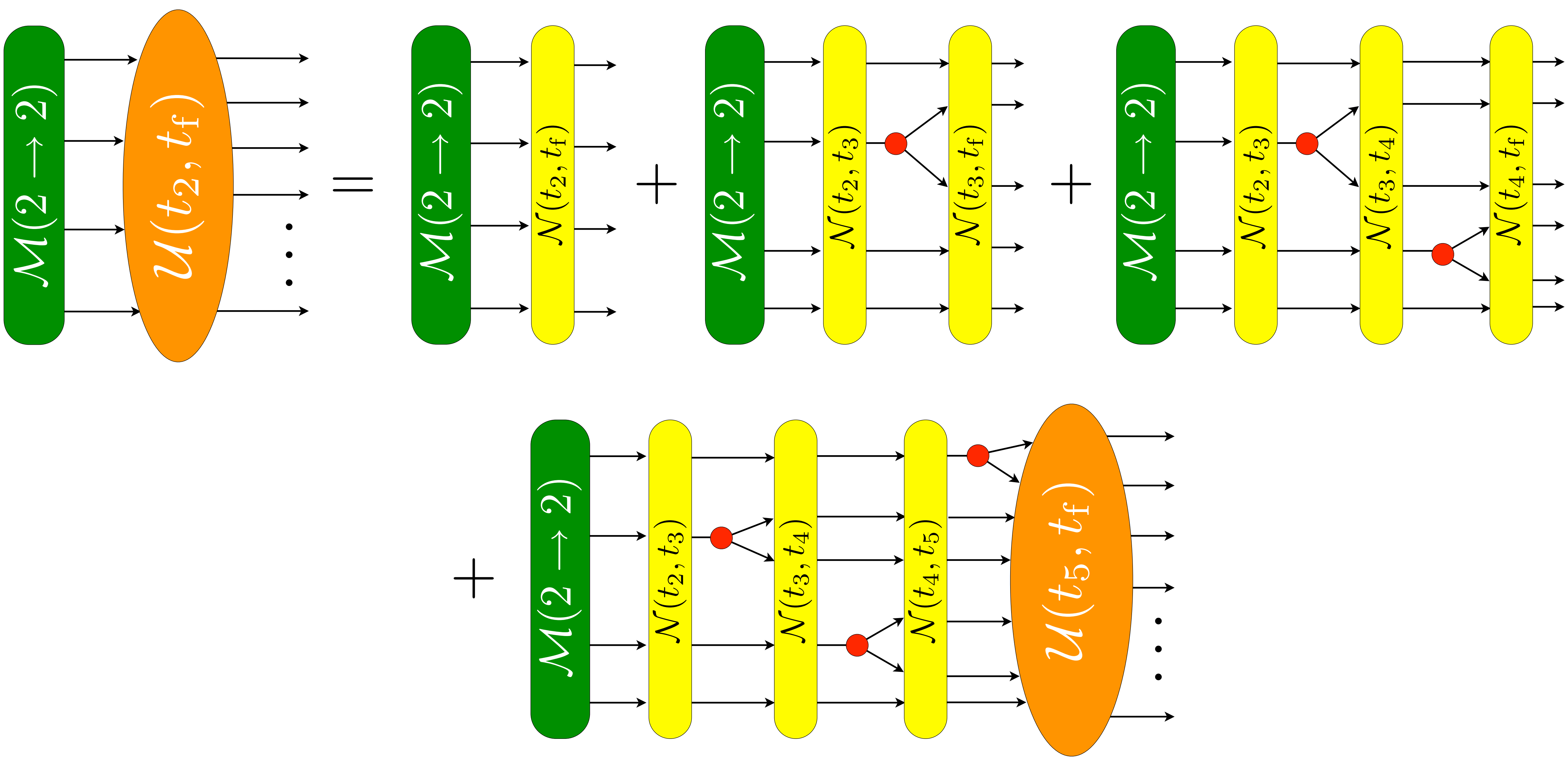
Or, one can be more ambitious and define this framework at NLO level.



Backup Slides

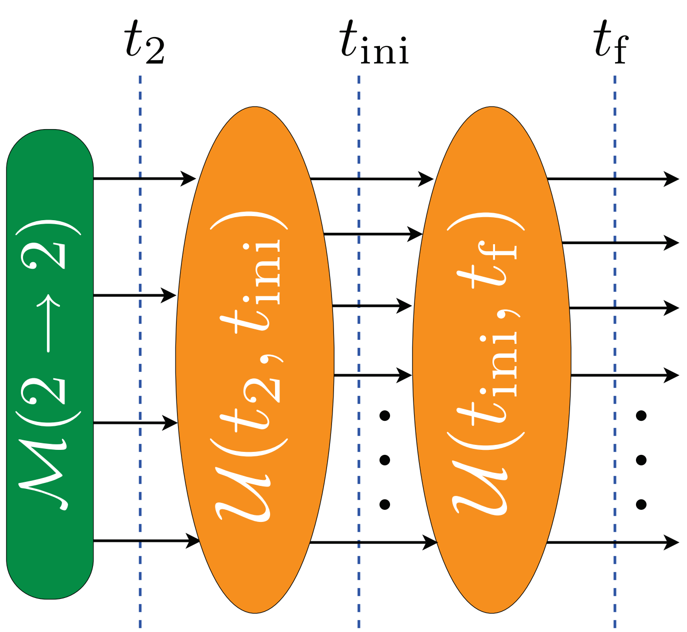
Shower Cross Section

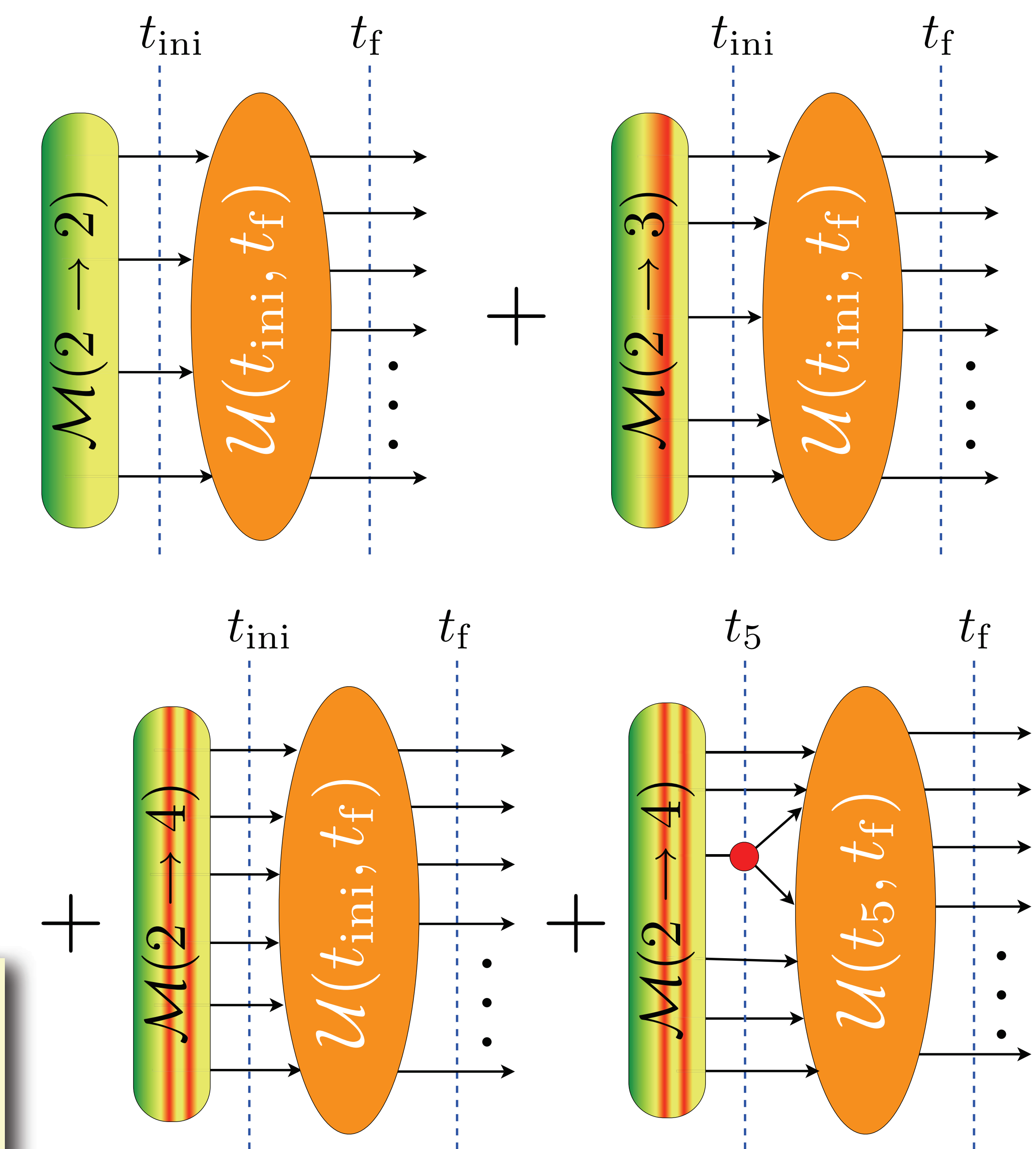
Iterating the evolution twice, then we have



CKKW Algorithm

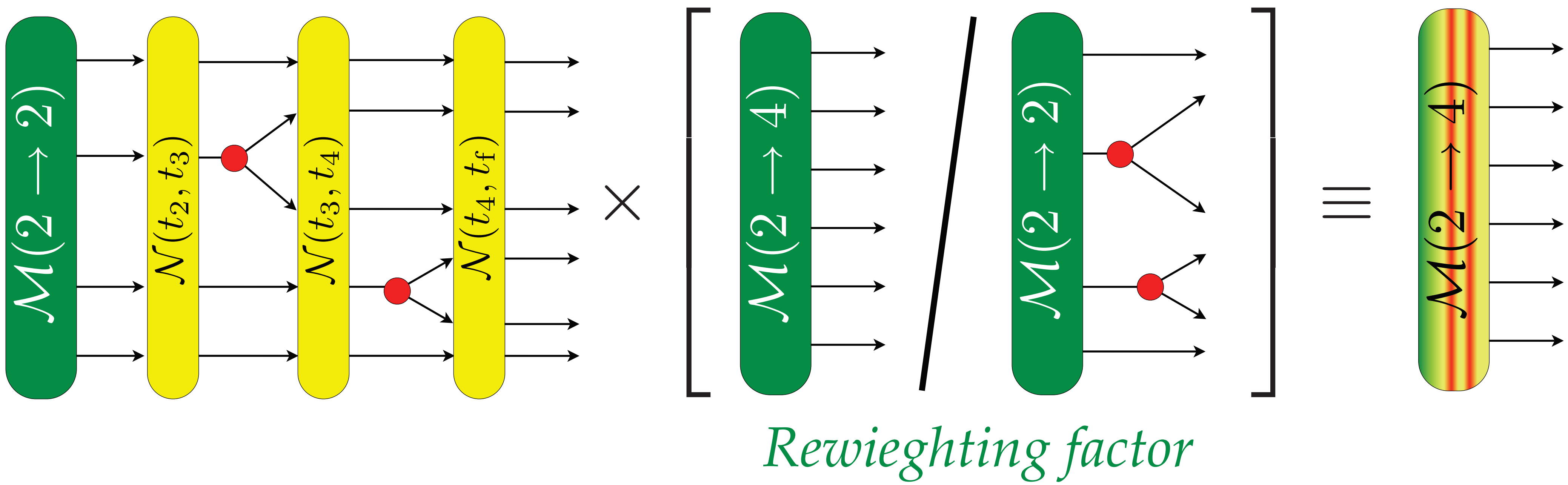
CKKW break the evolution into $0 < t < t_{\rm ini}$ and $t_{\rm ini} < t < t_{\rm f}$





- CKKW use improved weighting for $0 < t < t_{\rm ini}$
- For $t_{\rm ini} < t < t_{\rm f}$ they have standard shower (in Herwig and old Phytia case transverse momentum veto is needed)
- They use the kT algorithm and NLL Sudakov factors to do the reweighting.

MLM Algorithm



- This is the essential part of the MLM matching procedure.
- MLM algorithm use the cone jet finding algorithm to define the ratio
- No analytic Sudakov factors, it use the native Sudakov of the underlying parton shower.
- Matching parameters: $p_{T_{\min}}, \eta_{\max}, R_{\min}$

NLO Calculation

The NLO fix order calculations can be organized by the following way

$$\sigma_{\mathrm{NLO}} = \int_m \left[d\sigma^B + d\sigma^V + d\sigma^B \otimes \int_1 dV\right] F_{\mathrm{J}}^{(m)} + \int_{m+1} \left[d\sigma^{\mathrm{R}} F_{\mathrm{J}}^{(m+1)} - dV \otimes d\sigma^{\mathrm{B}} F_{\mathrm{J}}^{(m)}\right]$$

The born ($d\sigma^B$) and the real ($d\sigma^R$) are based on the *m* and *m+1* parton matrix elements, respectively and $d\sigma^V$ is the contribution of the virtual graphs. The approximated *m+1* parton matrix element has universal structure

$$\boxed{d\sigma^{\mathrm{R}} \approx dV \otimes d\sigma^{\mathrm{B}}}$$

It has the same singularity structure as $d\sigma^R$

MC@NLO

The naive way doesn't work when we want to match the shower to NLO calculation. It leads to double counting. Frixione and Webber managed in the following way:

$$\sigma_{\mathrm{MC}} = \int_m \left[d\sigma^B + d\sigma^V + d\sigma^B \otimes \int_1 dV\right] I_{\mathrm{MC}}^{(2\to m)}$$
$$+ \int_{m+1} \left[d\sigma^R - dV_{\mathrm{MC}} \otimes d\sigma^B\right] I_{\mathrm{MC}}^{(2\to m+1)}$$
$$+ \int_1 \left[dV_{\mathrm{MC}} - dV\right] \otimes \int_m d\sigma^B I_{\mathrm{MC}}^{(2\to m)}$$

here m=0,1,2 only!

The dV_{MC} term is extracted from the underlaying shower algorithm and it is subtracted and added back in different way. The function $I_{\mathrm{MC}}^{(2\to m)}$ and $I_{\mathrm{MC}}^{(2\to m+1)}$ are the interface to the shower.

$$\boxed{I_{\mathrm{MC}}^{(2\to m)} \sim \mathcal{U}(t_{\mathrm{f}}, t_2) \quad \text{and} \quad I_{\mathrm{MC}}^{(2\to m+1)} \sim \mathcal{U}(t_{\mathrm{f}}, t_3)\Delta(t_3, t_2)}$$

With these choices one can avoid double counting.