Electrostatics

1. Charge → force:
   - Like sign charges repel, unlike sign charges attract.
   - Coulomb's Law: direction and magnitude of force between two point charges.

$$F = k_0 \frac{q_1 q_2}{r^2}$$

2. Force → electric field:
   - Introduced to explain interaction over a distance.
   - Field lines: start from positive charge, end at negative charge.
   - More often used as:

$$\vec{E} \equiv \frac{\vec{F}}{q_{test}}$$

$$\vec{F} = q\vec{E}$$

3. Force → charge:
   - Force moves charge, does work on charge.
   - Electric potential.

$$U(\mathrm{A} \rightarrow \mathrm{B}) = -\int_A^B \vec{F} \cdot d\vec{r}$$

4. Electric field → charge:
   - Electric flux:
   - Electric field originated from charge: Gauss' Law.

$$\Phi_E \equiv \int \vec{E} \cdot d\vec{S}$$

$$\Phi_E = \oint_{enclosed\ surface} \vec{E} \cdot d\vec{S} = \frac{\sum q}{\varepsilon_0}$$

Direct Current

1. Moving charge → current:
   - Electric field inside a conductor keeps the charges moving.
   - Current direction is defined as the electric field direction.

$$I \equiv \frac{dq}{dt}$$

2. Charge → electric potential:
   - Accumulated charge generates potential: capacitance.
   - Parallel plate capacitor.
   - Energy stored in a capacitor.

$$C \equiv \frac{Q}{V}$$

$$C = \varepsilon_0 \frac{A}{d}$$

$$PE = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

3. Current → resistance:
   - Ohm's law defines resistance.
   - Resistance as a function of the wire.
   - Resistance as a linear function of temperature.

$$V = IR$$

$$R = \rho \frac{L}{A}$$

$$R(T_2) = R(T_1) \cdot [1 + \alpha(T_2 - T_1)]$$

4. Power
   - General formula.
   - With a resistor only (combined with Ohm's law).

$$P = VI$$

$$P = VI = \frac{V^2}{R} = I^2 R$$

Circuits

1. Resistors network:
   - Connection in series, current the same, voltage divided.
   - Connection in parallel, voltage the same, current divided.
   - Questions with power: nominal power v.s. actual power.
2. Capacitors network:
   - Connection in series, the same charge, voltage divided.
   - Connection in parallel, the same voltage, charge divided.
3. Kirchhoff's rules
   - Loop:
   - Junction:
   - Follow the steps.
4. RC circuit
   - Charging.
   - Discharging.
   - The time constant.

$$R_{eff} = R_1 + R_2 + \ldots + R_n$$

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$$

$$C_{eff} = C_1 + C_2 + \ldots + C_n$$

$$\sum V\Big|_{loop} = 0 \qquad \sum I\Big|_{junction} = 0$$

$$I = \frac{emf}{R} e^{-\frac{t}{\tau}} \qquad I = \frac{Q}{RC} e^{-\frac{t}{\tau}} \qquad \tau = RC$$

Magnetism and Induction

1. Current (moving charge) generates magnetic field.
   - The Biot-Savart law and Ampere's law.
   - B-field of a straight wire, at the center of a wire loop, inside a solenoid and the right-hand rules.

$$B = \frac{\mu_0 I}{2\pi R} \quad B = \frac{\mu_0 I}{2R} \quad B = \mu_0 I n$$

2. Force on a charge from the E and B fields:
   - The Lorentz force law.

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

3. Induction of emf from changing magnetic flux.
   - Motion emf.
   - Faraday's law, Lenz law.

$$emf = LvB \quad emf = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

4. Self-induction and the inductor
   - Self-induction
   - Magnetic energy in an inductor
   - Inductor and the LR circuit.
   - The LC circuit.

$$emf_{self} = -L\frac{dI}{dt} \quad PE = \frac{1}{2}LI^2$$

$$I = \frac{emf}{R}(1 - e^{-\frac{t}{\tau}}) \quad I = I_0 e^{-\frac{t}{\tau}} \quad \tau = \frac{L}{R}$$

$$I = I_{MAX}\sin(\omega t) \quad \omega^2 \equiv \frac{1}{LC}$$

Alternating Current and the circuits

1. AC
   - AC voltage power supply.
   - Average voltage, current and power.

$$V = V_{MAX}\sin(\omega t)$$

$$V_{MAX} = \sqrt{2}V_{rms}$$ $$I_{MAX} = \sqrt{2}I_{rms}$$ $$P_{MAX} = 2P_{rms}$$

2. R, C and L in AC circuit
   - R, Ohm's law applies.
   - C, current leads voltage by 90^o
   - L, voltage leads current by 90^o

3. RLC in AC circuit
   - Impedance Z
   - Phase angle φ

$$I = I_{MAX}\sin(\omega t + \varphi) = \frac{V_{MAX}}{Z}\sin(\omega t + \varphi)$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$ $$\tan\varphi = \frac{X_L - X_C}{R}$$

   - Resonance frequency

$$X_L = X_C,\ \omega_0^2 = \frac{1}{LC}$$

   - Power consumed in an AC circuit.

$$p_{rms} = I_{rms}^2 R = I_{rms}V_{rms}\cos\varphi$$

Optics -- reflection

1. The law of reflection
   - Angles in optics are always measured with respect to the normal of the interface.

$$\theta_r = \theta_i$$

2. Planary mirror
   - The virtual image has the same distance to the mirror as the object does.
   - The magnification is 1.
3. Spherical mirors
   - Converging and diverging mirrors.
   - Focal length and the radius.
   - Mirror equation and the sign conventions.
   - The magnification.
   - The 3-ray diagram.
   - Multi-mirror problems.

$$f = (-)\frac{R}{2}$$

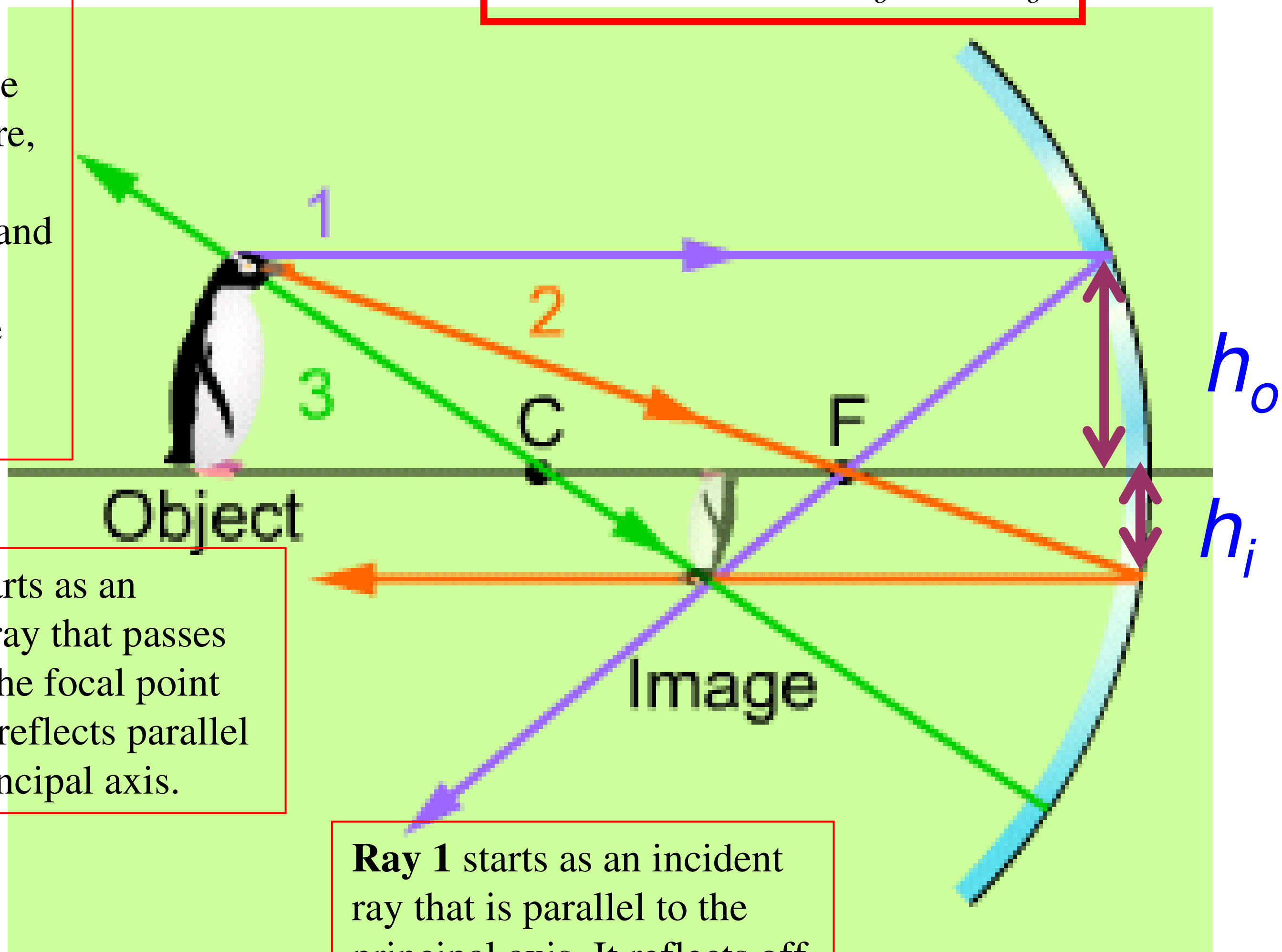
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$m \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

The 3-ray diagram:

Magnification $M \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

Ray 3 begins as an incident ray that passes through the center of curvature, strikes the mirror perpendicularly, and reflects back, moving along the same line as the incident ray.



Ray 2 starts as an incident ray that passes through the focal point and then reflects parallel to the principal axis.

Ray 1 starts as an incident ray that is parallel to the principal axis. It reflects off the mirror and passes through the focal point after it reflects.

Image up-side-down, smaller, real

- The 3-ray diagram, convex mirror:

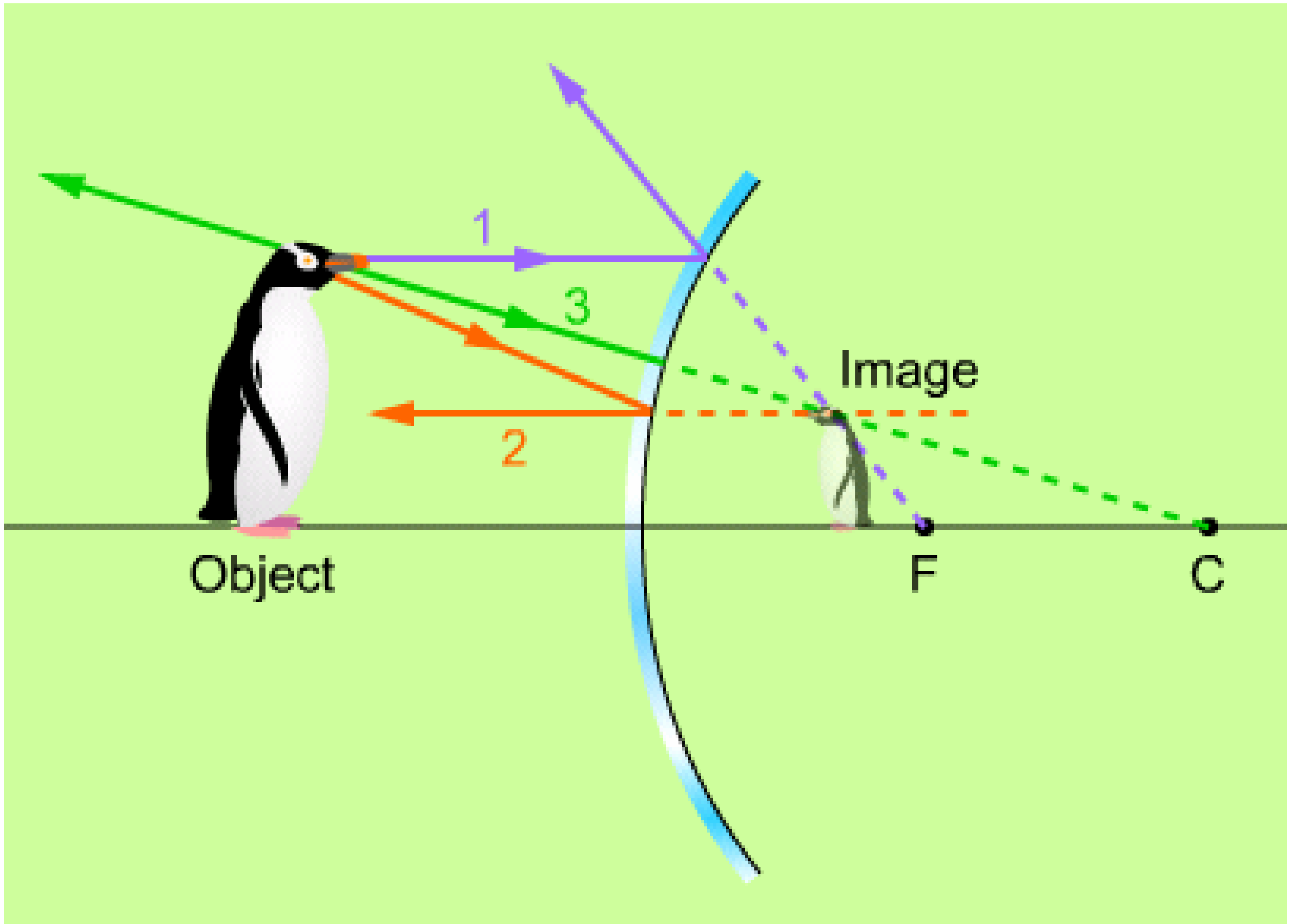


Image always upright, smaller, virtual

Optics -- refraction

1. Index of refraction
   - Light travels slower in medium than in vacuum.

$$n \equiv \frac{c}{v}$$

2. refraction
   - Snell's law
   - Total internal reflection.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 \sin\theta_c = n_2 \quad \text{so } n_1 > n_2$$

3. Lens
   - Focal length and the radius, lens maker's equation.
   - lens equation and the sign conventions.
   - The magnification.
   - The 3-ray diagram.
   - Multi-lens problems.

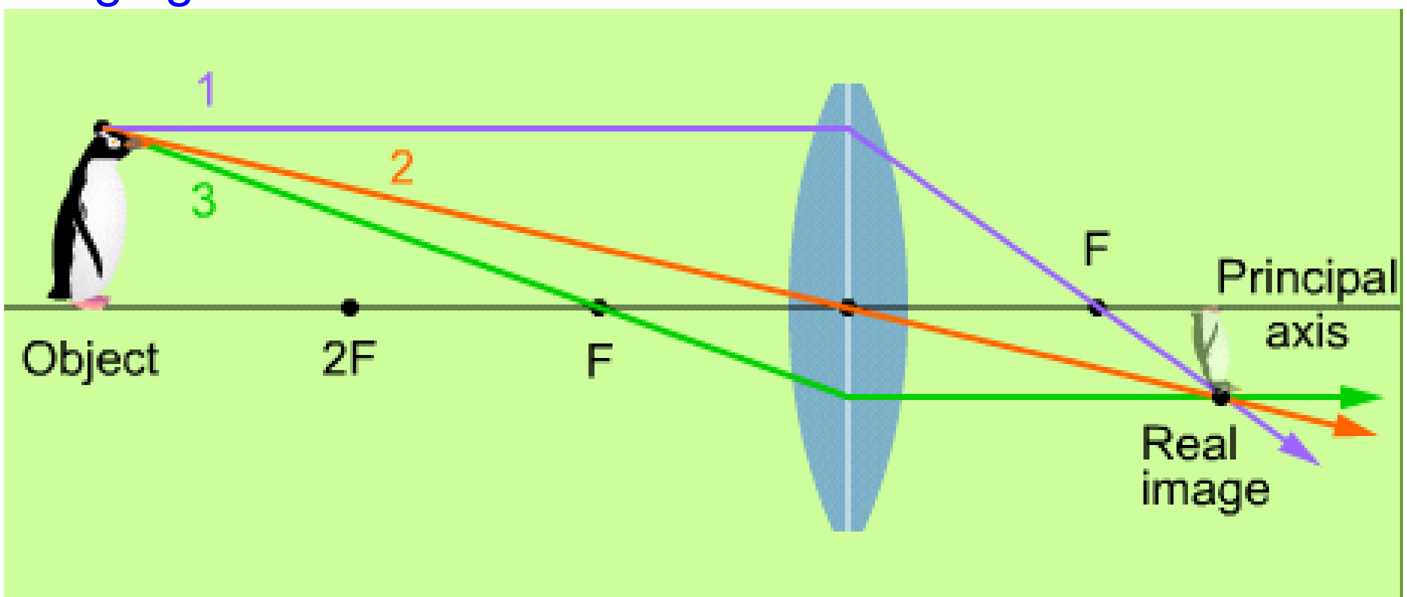
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

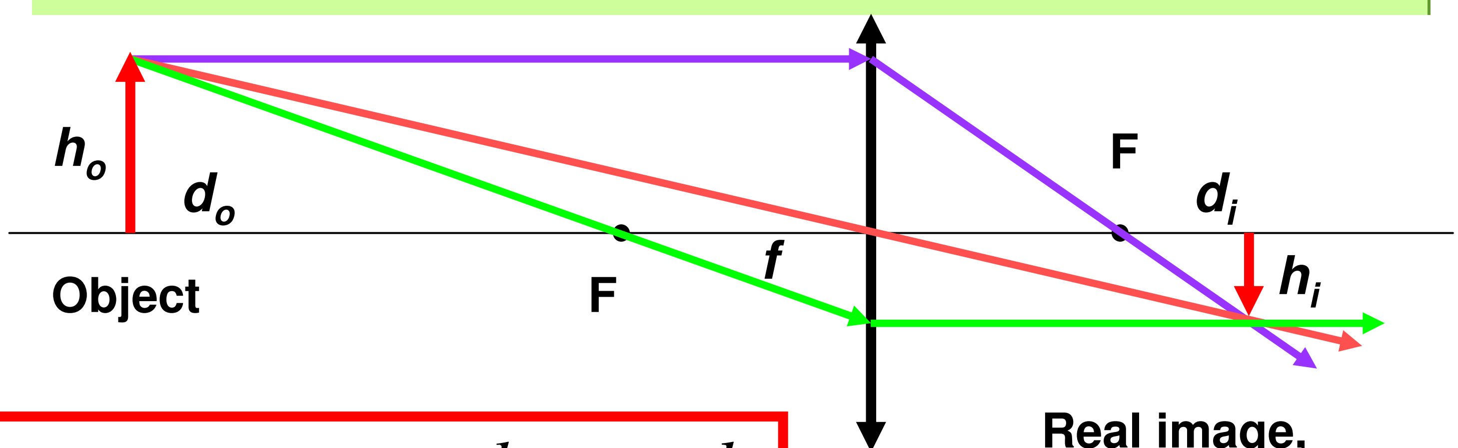
$$\frac{1}{f} = \left(\frac{n_{lens}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$m \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

- Converging lens

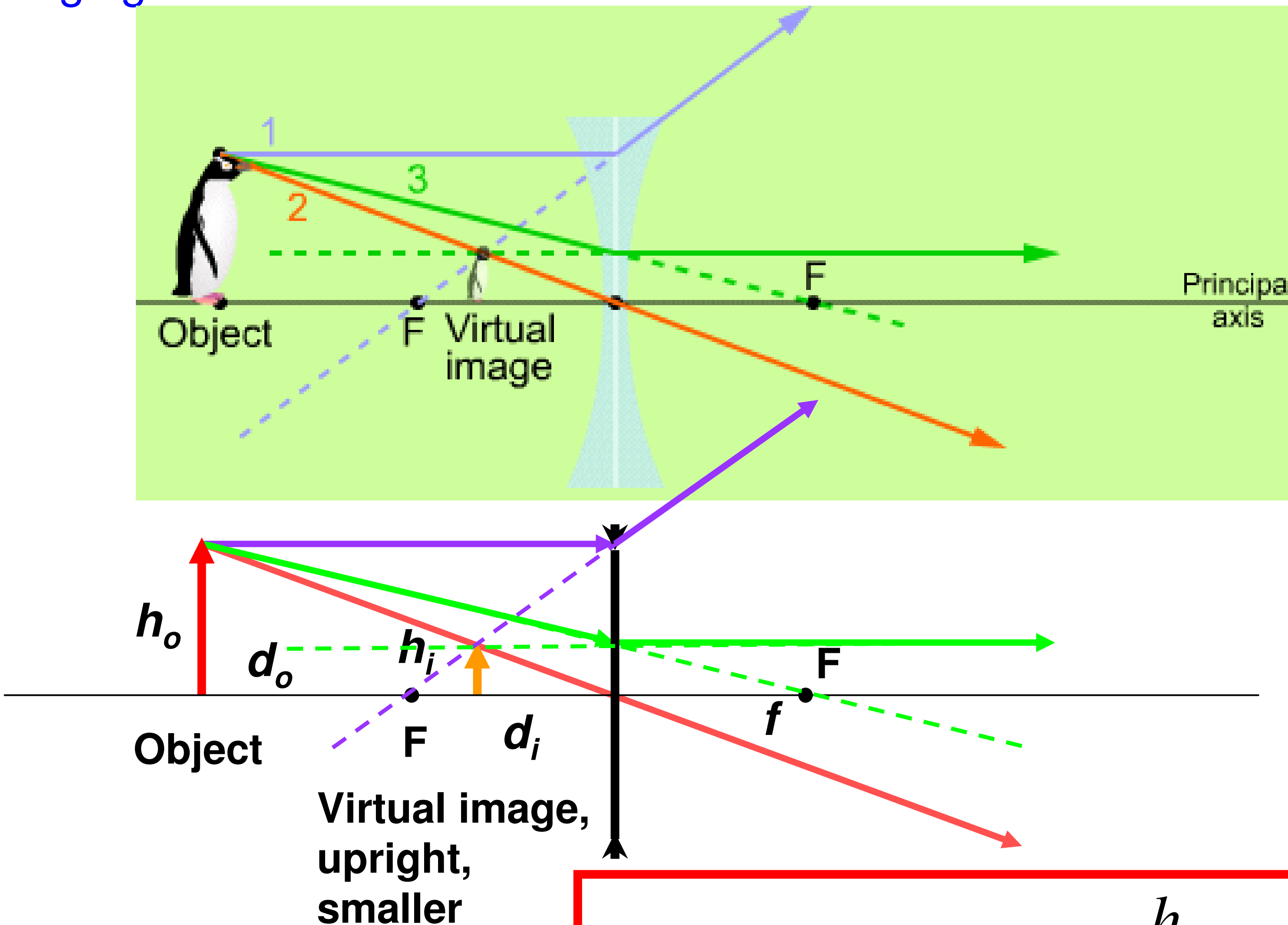




Real image, inverted, smaller

$$\text{magnification}: \quad m \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

- Diverging lens



$$\text{magnification}: \quad m \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$