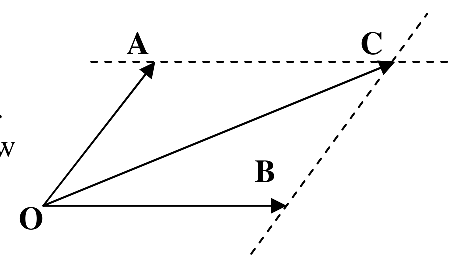
Quiz 0 Name: ____________________

ID: ____________________

Mathematics is the language in sciences. Certain amount of knowledge in mathematics is required to understand the material in this course. To help you check how well you are prepared in math for this course, please complete this quiz in class. The grade of this quiz will not be counted towards your final course grade. However if you find this quiz difficult, you may need to review your math text books.

1. The formula that links R with R_1 and R_2 is this: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
   If $R_1 = 2$, $R_2 = 3$, what is the value of R?
   Analysis: simple application of one formula, one unknown, one equation.
   $\frac{1}{R} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, $R = \frac{6}{5}$. Check: no unit associated. Done.



2. Two vectors **A** and **B** are drawn as in the following figure. Both are originated from point **O**. Vector **C** = **A** + **B**. Show vector **C** in the same figure.
   Analysis: vector addition, graphic method.
   Draw AC//OB, BC//OA, connect OC to be the vector **C**.

3. In a Cartesian coordinate system, vector **A** = 2**i** + 3**j** + 4**k**, vector **B** = 3**i** + 4**j** + 5**k**. Here the vectors **i, j** and **k** are the unit vectors for the x-, y- and z- axes. Write the expression for vector **C** and scalar D if:
   Analysis: simply apply vector algebra rules in Cartesian system.

   (1) **C** = **A** + **B** = (2+3)**i**+(3+4)**j**+(4+5)**k** = 5**i**+7**j**+9**k** check: result a vector.

   (2) D = **A**·**B** = 2·3+3·4+4·5 = 38 check: result a scalar.

   (3) $\mathbf{C} = \mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}\mathbf{k} = -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ check: result a vector.

4. Solve the following equation for variable x, make sure to get all possible solutions.

$$\frac{4}{(9-x)^2} = \frac{1}{x^2}$$

step 1, simlify: $4x^2 = (9-x)^2$

step 2, move to one side: $4x^2 - (9-x)^2 = 0$

step 3, use $a^2 - b^2 = (a+b)(a-b)$: $(2x+9-x)(2x-9+x) = 0$

step 4, from: $(2x+9-x) = 0, \quad x = -9$

from: $(2x-9+x) = 0, \quad x = 3$

step 5, check back in the orignal equation:

$\frac{4}{(9+9)^2} = \frac{1}{(-9)^2}$ and $\frac{4}{(9-3)^2} = \frac{1}{(3)^2}$. both equal signs hold.

5. Solve the following equations for variables *x, y* and *z*.

$$3x+3y+4z=0 \quad (1)$$
$$x-\ y+2z=3 \quad (2)$$
$$3x+\ y-4z=16 \quad (3)$$

Analysis: 1st equations, 3 unknowns, 3 equations, solvable.

Strategy: reduce to 2 unknowns and 2 equations.

Step 1, $(1)+(2)$ to get rid of z: $6x+4y=16$

simplify: $3x+2y=8 \quad (4)$

Step 2, $(1)-2(2)$ to get rid of z:

$x+5y=-6 \quad (5)$

Step 3, solve (4) and (5) for x and y:

$3x+2y=8 \quad (4)$

$x+5y=-6 \quad (5)$

$x=4, \quad y=-2$

from (2), $z=-\frac{3}{2}$.

final answer: $x=4, \quad y=-2, \quad z=-\frac{3}{2}$.

Check: equations (1), (2) and (3) hold with these numbers.

6. If $y=3x^2+\sin(6x)+\ln(x)+\frac{1}{x}$, What is $\frac{dy}{dx}$?

$$\frac{dy}{dx}=6x+6\cos(6x)+\frac{1}{x}-\frac{1}{x^2}$$

7. Solve this integral: $U(R)=\int_{\infty}^{R}\frac{dr}{r^2}$

$$U(R)=-\frac{1}{r}\bigg|_{\infty}^{R}=-\frac{1}{R}$$

8. Solve the equation for *q*. The variable *q is* a function of *t* . Here *R* and *C* are constants. When $t=0,\ q=Q_0$.

$$\frac{q}{C}+R\frac{dq}{dt}=0$$

Step 1, re-arrange to seperate q and t: $\frac{q}{C}=-R\frac{dq}{dt}$, then: $\frac{dq}{q}=-\frac{1}{RC}dt$

Step 2, take integral on both sides: $\int\frac{dq}{q}=-\frac{1}{RC}\int dt$

we have: $ln(q)=-\frac{t}{RC}+C_1$, or: $q(t)=C_2e^{-\frac{t}{RC}}$,

here $C_2=e^{C_1}$, both C_1 and C_2 are constants.

Step 3, from the initial conditions that $q=Q_0$ when $t=0$,

we have: $Q_0=C_2e^{-\frac{0}{RC}}=C_2$,

So the final answer: $q(t)=Q_0e^{-\frac{t}{RC}}$