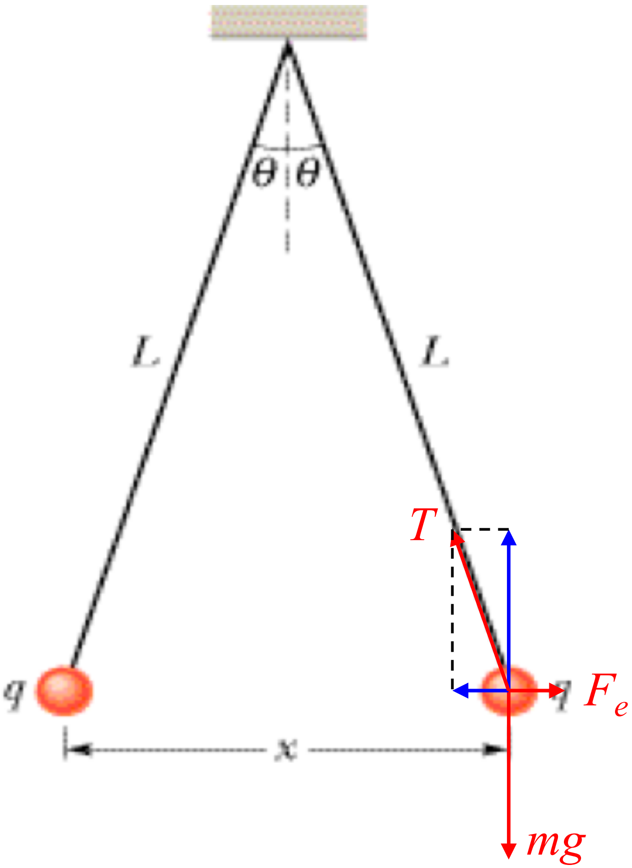
Reviews on Chapters from 21 to 24

Two tiny conducting balls of identical mass m and identical charge q hang from non-conducting threads of length L. Assume that θ is so small that $\tan\theta$ can be replaced by its approximate equal, $\sin\theta$. If L = 180 cm, m = 14 g, and x = 7.3 cm, what is the magnitude of q?



$$T\sin\theta = F_e = k_e\frac{q^2}{x^2}, \quad T\cos\theta = mg$$

$$\rightarrow \tan\theta = \frac{k_e q^2}{mgx^2} \cong \sin\theta$$

$$\frac{x}{2L} = \sin\theta, \quad \frac{k_e q^2}{mgx^2} = \frac{x}{2L}$$

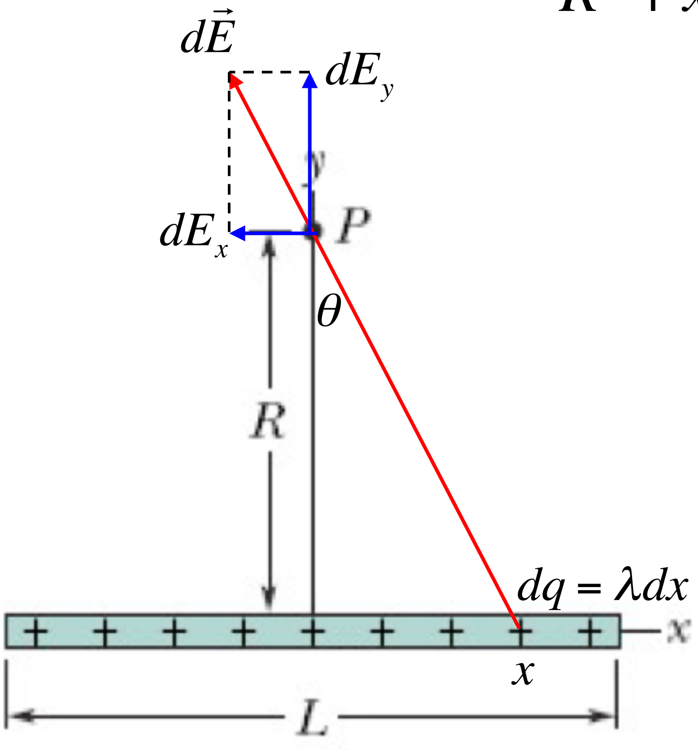
$$\rightarrow q = \pm\sqrt{\frac{mgx^3}{2k_e L}}$$

Reviews on Chapters from 21 to 24

A positive charge q = 7.60 pC is spread uniformly along a thin nonconducting rod of length L = 17.0 cm. What are the (a) x- and (b) y- components of the electric field produced at point P, at distance R = 6.00 cm from the rod along its perpendicular bisector?

$E_x = 0$, because of the symmetry about y. $\quad dE_y = dE \cdot \cos\theta$

$$dE = k_e \frac{\lambda dx}{R^2 + x^2}, \quad \cos\theta = \frac{R}{\sqrt{R^2 + x^2}}, \quad \rightarrow \quad dE_y = \frac{R k_e \lambda dx}{\left(R^2 + x^2\right)^{3/2}}$$



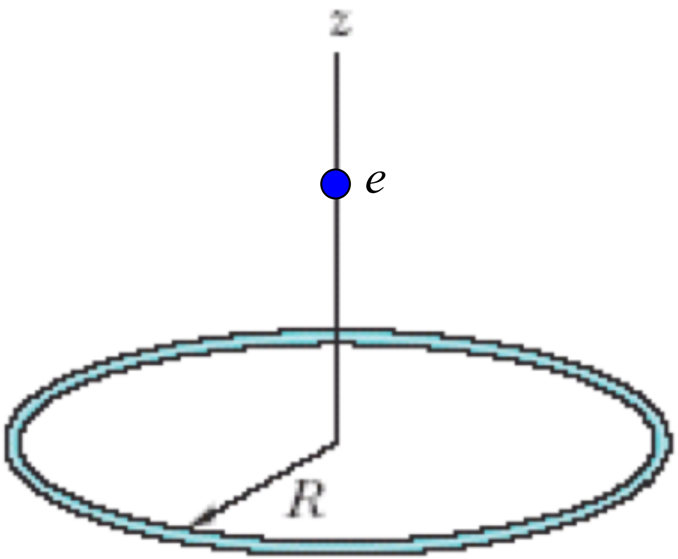
$$E_y = 2\int_0^{L/2} dE_y = 2\int_0^{L/2} \frac{R k_e \lambda dx}{\left(R^2 + x^2\right)^{3/2}}$$

$$= 2 R k_e \lambda \left. \frac{x}{R^2 \sqrt{R^2 + x^2}} \right|_0^{L/2}$$

$$= \frac{k_e \lambda L}{R\sqrt{R^2 + (L/2)^2}} = \frac{k_e q}{R\sqrt{R^2 + (L/2)^2}}$$

Reviews on Chapters from 21 to 24

An electron e is constrained to the central perpendicular axis of a ring of charge of radius R =2.0 m and charge Q = 0.1 mC. Suppose the electron is released from rest a distance z_0 = 0.04 m from the ring center. It then oscillates through the ring center. Calculate its period under the condition that $z_0 << R$.



$$E_z = \oint_{fullcircle} k_e \frac{z\,dq}{\left(R^2+z^2\right)^{\frac{3}{2}}} = \frac{k_e z}{\left(R^2+z^2\right)^{\frac{3}{2}}} \oint_{fullcircle} dq$$

$$= \frac{k_e z Q}{\left(R^2+z^2\right)^{\frac{3}{2}}} \cong \frac{k_e Q}{R^3} z, \quad \text{when } z_0 << R$$

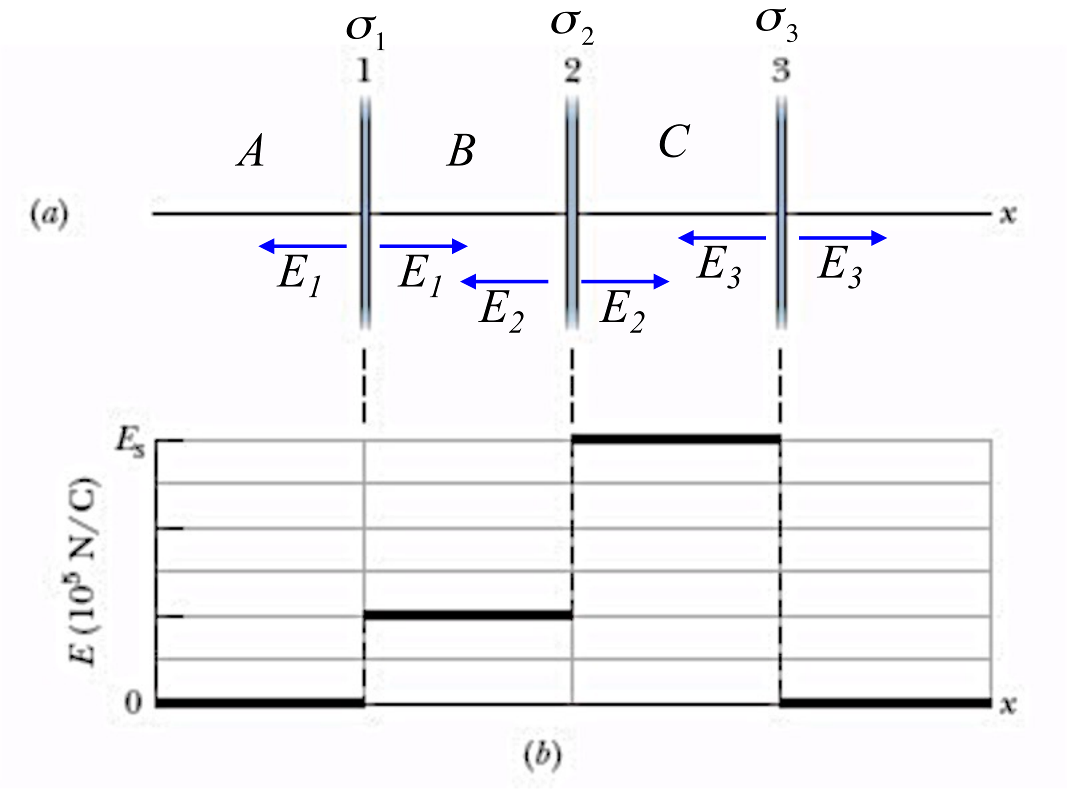
$$F_e = -eE_z = -\frac{k_e e Q}{R^3} z \equiv -kz$$

$$\omega_0 = \sqrt{\frac{k}{m_e}} = \sqrt{\frac{k_e e Q}{m_e R^3}}$$

$$T = \frac{2\pi}{\omega_0} = 2\pi R \sqrt{\frac{m_e R}{k_e e Q}}$$

Reviews on Chapters from 21 to 24

Figure (a) shows three plastic sheets that are large, parallel, and uniformly charged. Figure (b) gives the component of the net electric field along an x axis through the sheets. The scale of the vertical axis is set by E_s = 3.6 × 10^5 N/C. What is the ratio of the charge density on sheet 3 to that on sheet 2?



$$A: -\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_3}{2\varepsilon_0} = 0$$

$$B: -\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_3}{2\varepsilon_0} = 2E_S$$

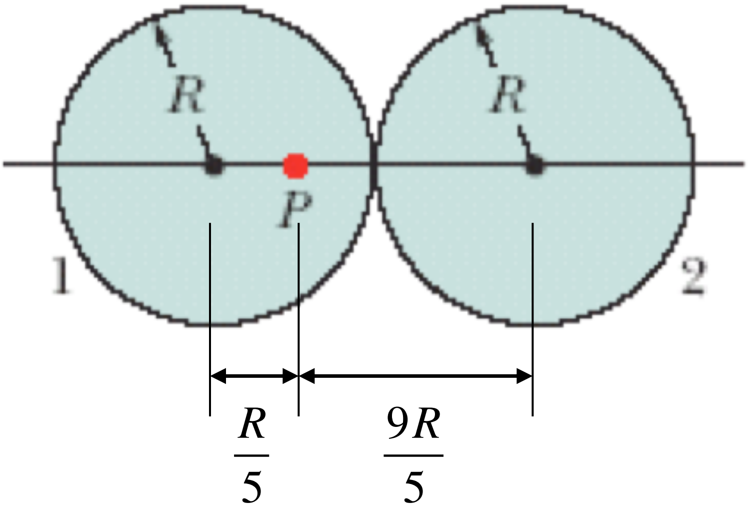
$$C: -\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_3}{2\varepsilon_0} = 6E_S$$

$$\sigma_1 = 2\varepsilon_0 E_S,\ \sigma_2 = 4\varepsilon_0 E_S,\ \sigma_3 = -6\varepsilon_0 E_S$$

$$\frac{\sigma_3}{\sigma_2} = -\frac{3}{2}$$

Reviews on Chapters from 21 to 24

In cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius R. Point P lies on a line connecting the centers of the spheres, at radial distance $R/5$ from the center of sphere 1. If the net electric field at point P is zero, what is the ratio q_2/q_1 of the total charge q_2 in sphere 2 to the total charge q_1 in sphere 1?

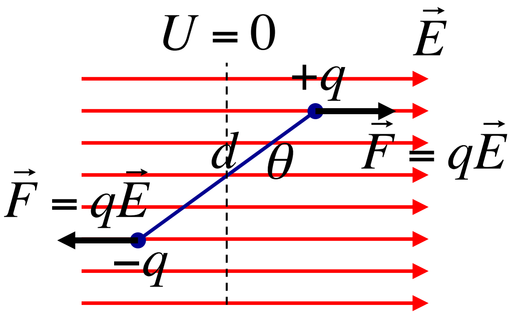


$$E_1 + E_2 = \frac{k_e q_1}{R^3}\frac{R}{5} + \frac{k_e q_2}{(9R/5)^2} = 0$$

$$\frac{q_1}{5} + \frac{25q_2}{81} = 0 \rightarrow \frac{q_2}{q_1} = -\frac{81}{125}$$

Reviews on Chapters from 21 to 24

What is the torque on an electric dipole in a uniform electric field? What is the potential energy it has assuming $U = 0$ when $\vec{p}$ is perpendicular to $\vec{E}$?

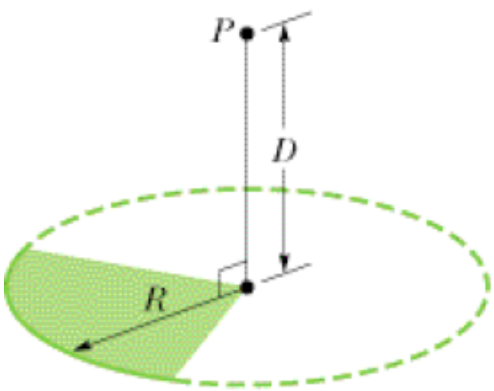


$$U = -W = -2F \cdot \frac{1}{2} d \cos\theta = -qEd\cos\theta = -PE\cos\theta = -\vec{P} \cdot \vec{E}$$

$$\tau \equiv \vec{r} \times \vec{F} = 2 \cdot \frac{1}{2} \vec{d} \times q\vec{E} = \vec{P} \times \vec{E}$$

Reviews on Chapters from 21 to 24

A plastic disk of radius R = 80 cm is charged on one side with a uniform surface charge density 8.0 fC/m^2, and then three quadrants of the disk are removed. The remaining quadrant is shown in the figure. With V = 0 at infinity, what is the potential in volts due to the remaining quadrant at point P, which is on the central axis of the original disk at distance D = 0.8 cm from the original center?

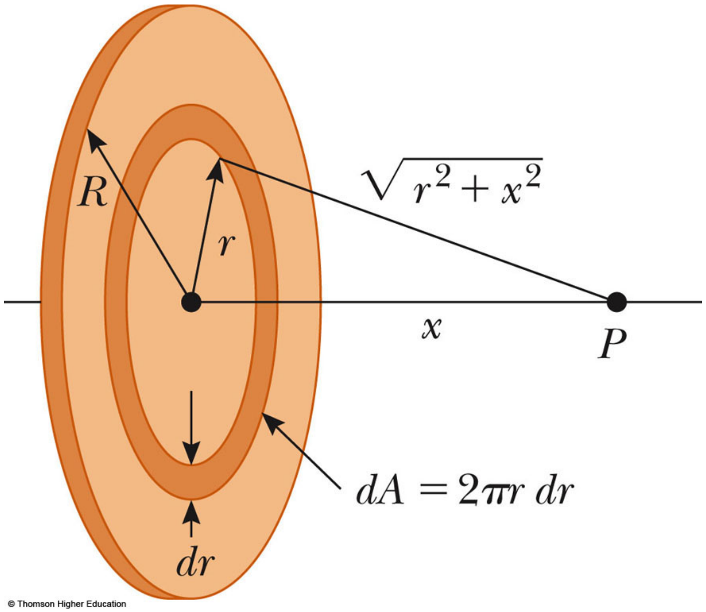


For full disk $V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{\left(R^2 + D^2\right)} - D\right]$

For 1/4 disk $V = \frac{\sigma}{8\varepsilon_0}\left[\sqrt{\left(R^2 + D^2\right)} - D\right]$

V for a Uniformly Charged Disk

- The ring has a radius R and surface charge density of σ
- P is along the perpendicular central axis of the disk



$$dV = k_e \frac{dq}{\sqrt{r^2 + x^2}} = \frac{1}{4\pi\varepsilon_0} \frac{\sigma(2\pi r)dr}{\sqrt{r^2 + x^2}} = \frac{\sigma}{4\varepsilon_0} \frac{d\left(r^2 + x^2\right)}{\sqrt{r^2 + x^2}}$$

$$V = \int_0^R \frac{\sigma}{4\varepsilon_0} \frac{d\left(r^2 + x^2\right)}{\sqrt{r^2 + x^2}} = \frac{\sigma}{4\varepsilon_0} \left. \frac{\left(r^2 + x^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right|_0^R = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{\left(R^2 + x^2\right)} - x\right]$$

Compare this with the same problem in Chapter 22. And when $R \rightarrow \infty$

$$E_x = -\frac{dV}{dx} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

$$E_x \rightarrow \frac{\sigma}{2\varepsilon_0}$$

Compare with Gauss Law type 3