Electric Field

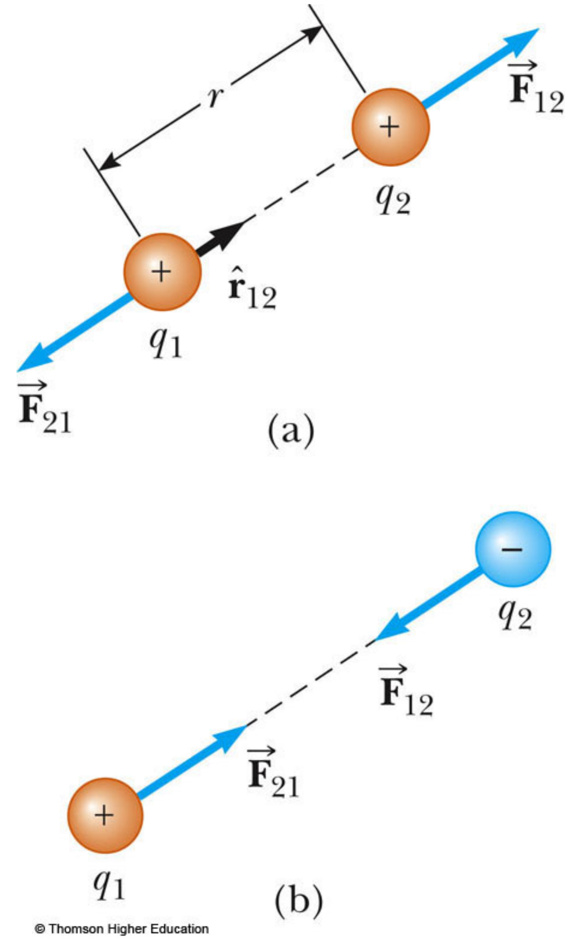
1. A Review on Coulomb's Law
2. Define Electric Field
3. Define Electric Field Line
4. Examples: Calculate Electric Fields
5. Charge Particles Experience Force in an Electric Field

Coulomb's Law, Review

- The formula:

$$F_e = K_e \frac{|q_1||q_2|}{r^2} \quad \text{or} \quad \vec{F}_e = K_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

- The SI unit of charge is the **coulomb** (C)
- k_e is called the **Coulomb constant**
  - k_e = 8.9876 x 10^9 N·m^2/C^2 = $1/(4\pi\varepsilon_o)$
  - ε_o is the **permittivity of free space**
  - ε_o = 8.8542 x 10^{-12} C^2 / N·m^2



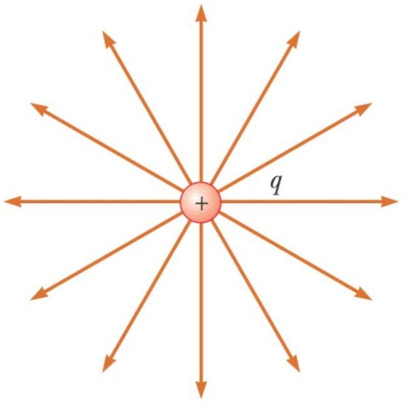
Electric Field, the definition

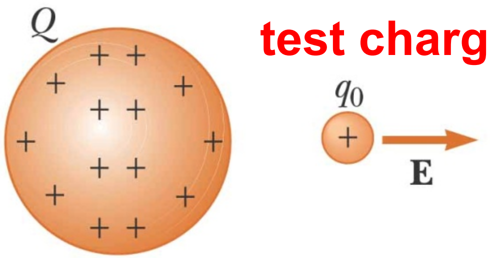
- The electric force acts through space, i.e., with no physical contact between the charged objects.
- One way to offer an explanation, as Faraday initiated, is the concept of a field. We met this situation before, what is that?
- An **electric field** is postulated to exist in the region of space around a charge (called the **source charge**).
- The **strength** and **direction** of that electric field at a point in space is then measured by the force of the electric field exerts on another charge (often called the **test charge**) at that point.
- Mathematically:

$$\vec{E} \equiv \frac{\vec{F}}{q_0}$$

- The electric field, $\vec{E}$, is a vector. The test charge, q_o, is usually a very small charge compared with the source charge, so that its existence does not distort the electrical field generated by the source charge.
- Unit: Newton/Coulomb or N/C.
- Q-1: does the field exist without the test charge?
- Q-2: does an electric field require a source charge to be generated?

source charge



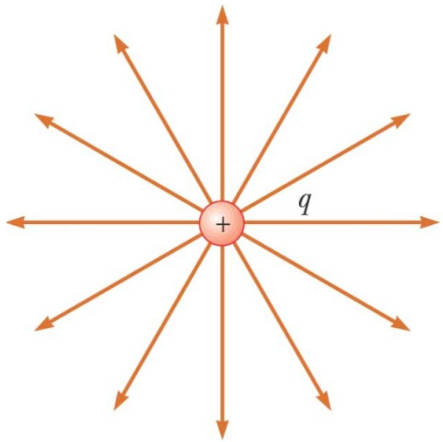


test charge

source charge

Electric Field Lines

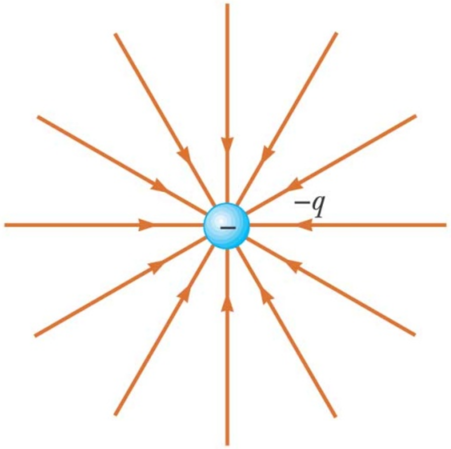
- Electric field is introduced to explain the fact that electric forces act through space.
- We use a set of specially defined lines to illustrate the field. These lines do not exit in space, but they should do so in your mind, and you must be able to “see” them with your mind’s eye.
- Now let’s define these electric field lines:
  - They start from a positive charge, end at a negative charge.
  - Their density in space (number of lines in unit volume) indicates the field strength.
  - The tangent of an electric field line at a given location points to the direction of the field at that location. Hence no lines can cross.



(a)

Field lines of one point positive source charge in space

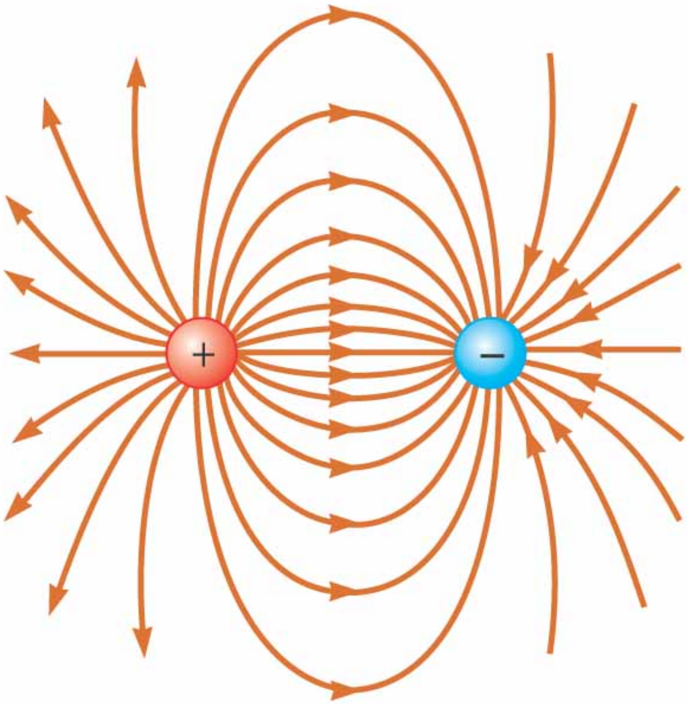
Field lines of one point negative source charge in space



(b)

Draw electric field lines

- Electric dipole: the charges are equal and opposite.
- The charges are equal and positive.
- Can you draw for the case charges are equal but negative?
- A slightly more general case: the charges are not equal, not the same polarity.



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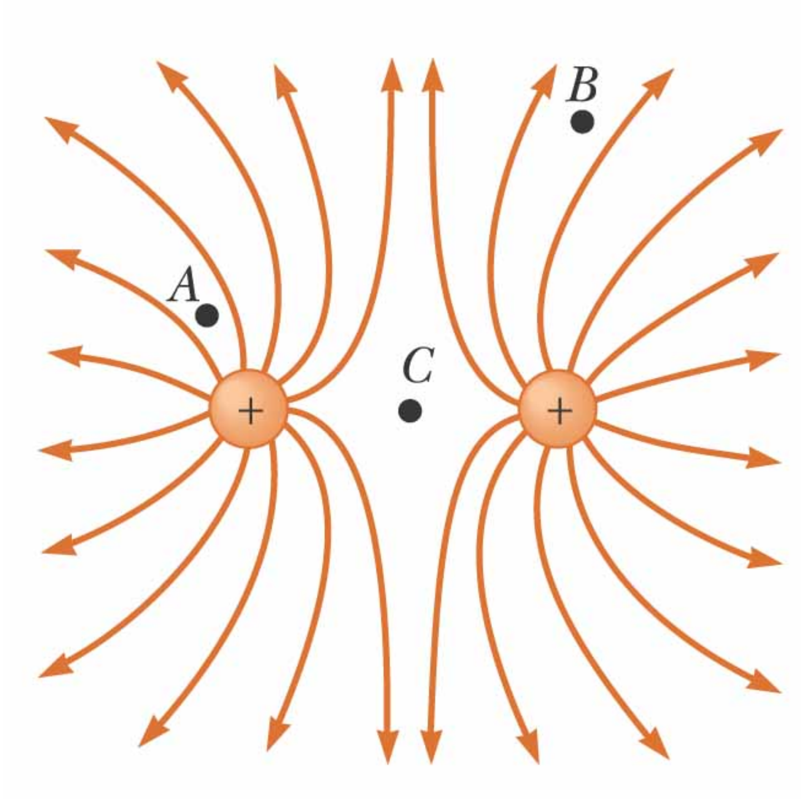

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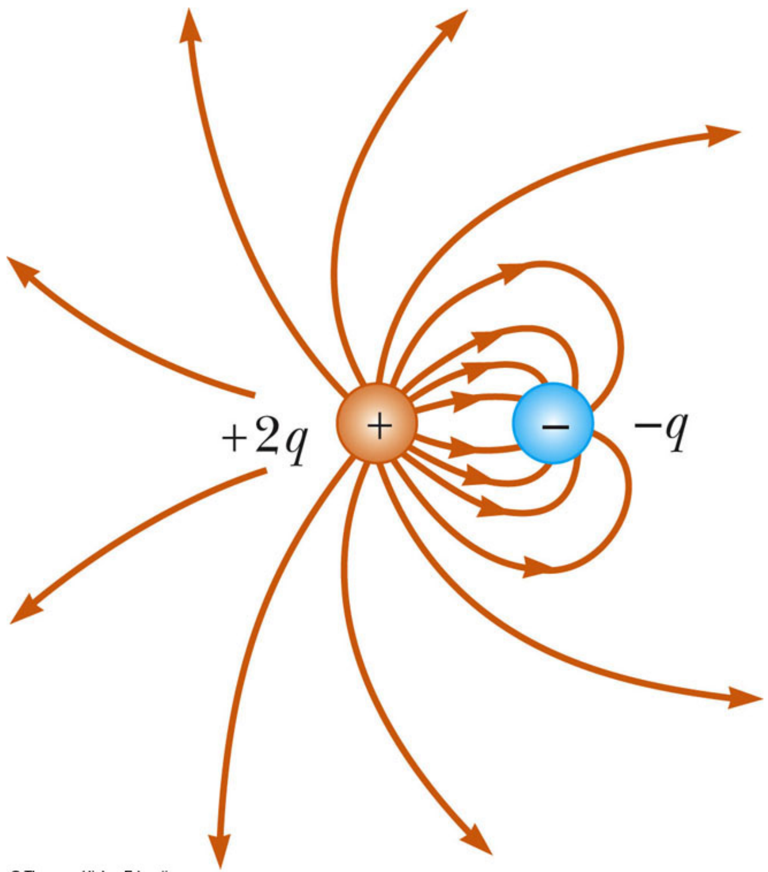
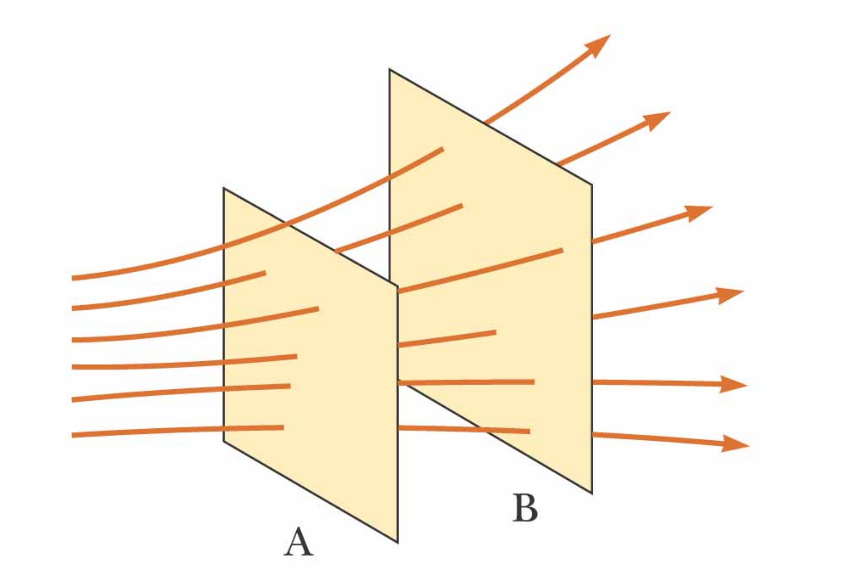


Let's have some fun, watch this
https://www.youtube.com/watch?v=Y6YdC2UoDYY

Electric flux

- Electric field may not come from static source charges. So there is the need to just draw electric field lines to represent the electric field. In the case in the right side figure:
  - The density of lines through surface A is greater than that through surface B. So the magnitude of the electric field is greater on surface A than on B
  - The electric field strength (number of lines) times the surface area (A or B) is called the electric flux.



How would you define the flux of water coming out of a garden hose?

Electric field by a point source charge q

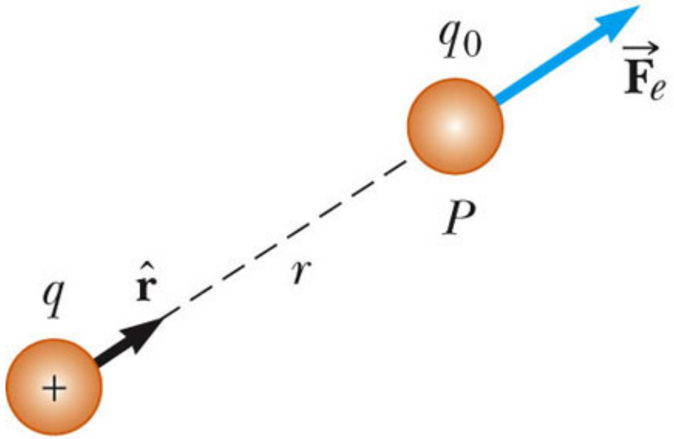
- From the definition: $\vec{E} \equiv \dfrac{\vec{F}}{q_0}$
- Place the test charge q_0 at point P. The force on q_0 is given by Coulomb's law:

$$\vec{F}_e = k_e \frac{q q_0}{r^2} \hat{r}$$

- Then, the electric field will be

$$\vec{E} \equiv \frac{\vec{F}_e}{q_0} = k_e \frac{q}{r^2} \hat{r}$$

- **The electric field only depends on the source charge, not the test charge.**



Electric field by many charges

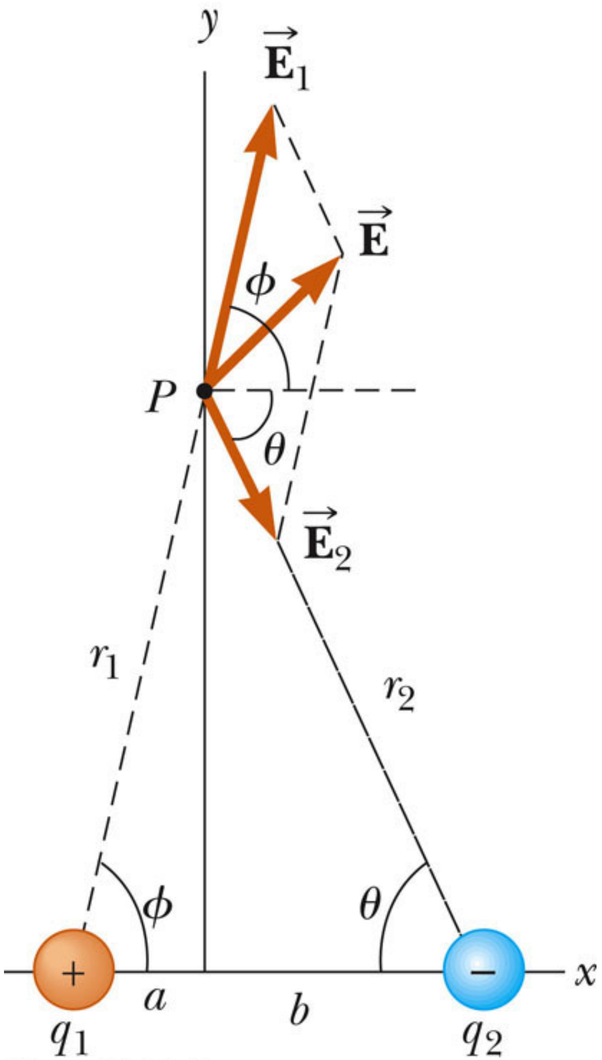
When the charges are still point charges:

from $$\vec{E} \equiv \frac{\vec{F}_e}{q_0} = k_e \frac{q}{r^2}\hat{r}$$

to $$\vec{E} = \sum_i \vec{E}_i = k_e \sum_i \frac{q_i}{r_i^2}\hat{r}_i$$

If you do not feel comfortable about the math here, raise your hand.

Review on math: express a vector in the xyz coordinate system and vector addition.

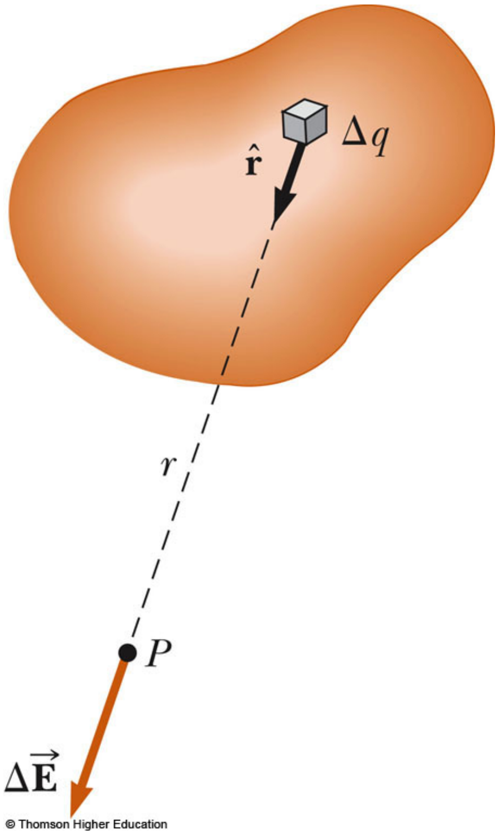


Electric field by continuous charges

When the charges are distributed over volume V:

from $$d\vec{E} \equiv \frac{d\vec{F}_e}{q_0} = k_e \frac{dq}{r^2}\hat{r}$$

to $$\vec{E} \equiv \int d\vec{E} = k_e \int_V \frac{dq}{r^2}\hat{r}$$



Again if you do not feel comfortable about the math here, raise your hand.

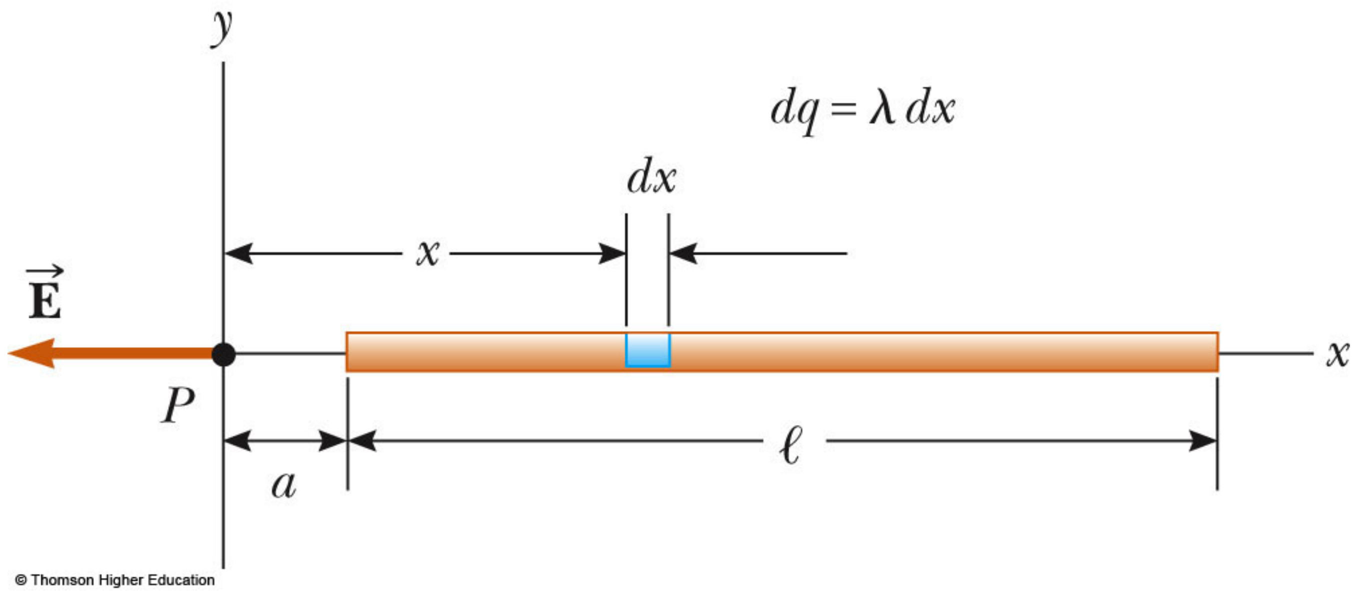
Review on math: make sure that you understand the following examples or you will have to go and study your calculus textbook.

Electric field from a continuous charge distribution

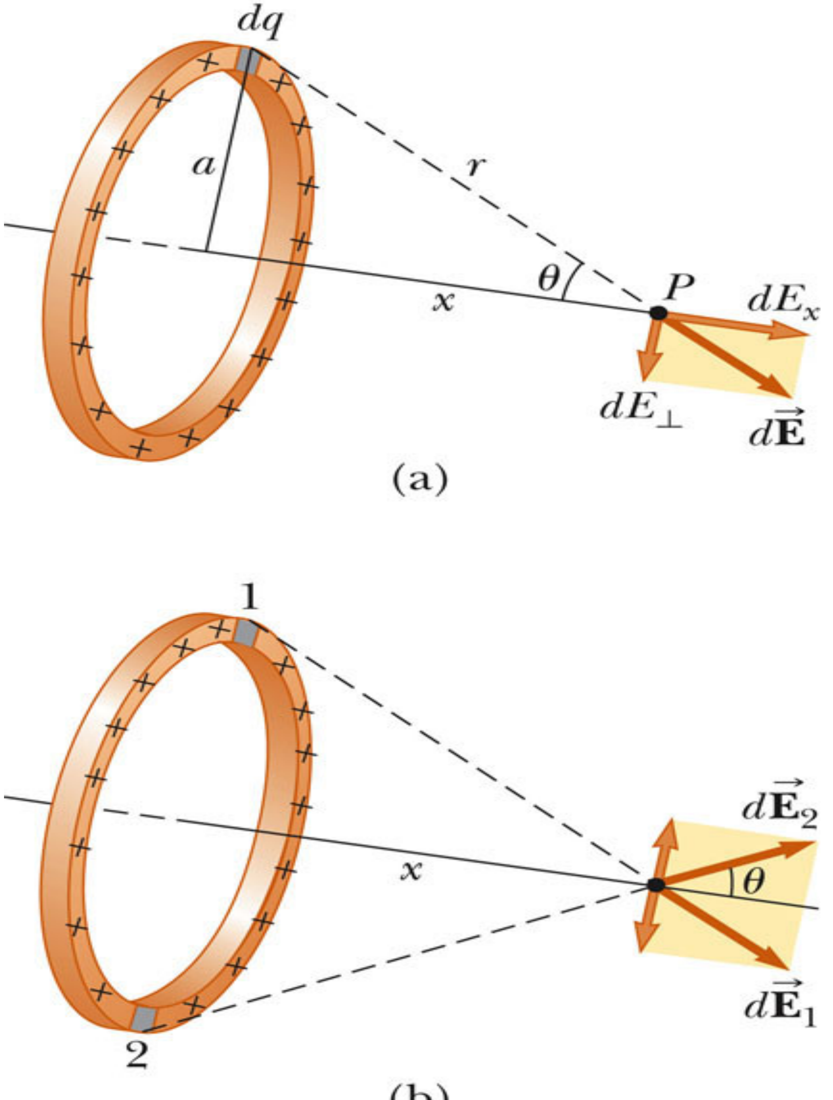
Condition: a straight wire of length ℓ has a charge density of λ.

Question: what is the electrical field strength at point P which is on the line of the wire but a distance a away from it?

Based on what we just discussed, can you find the answer?



Electric field from a continuous charge distribution



Condition: a ring of radius a with evenly distributed positive charge Q.

Question: what is the electrical field strength at point P which is at a distance x away from the center of the ring?

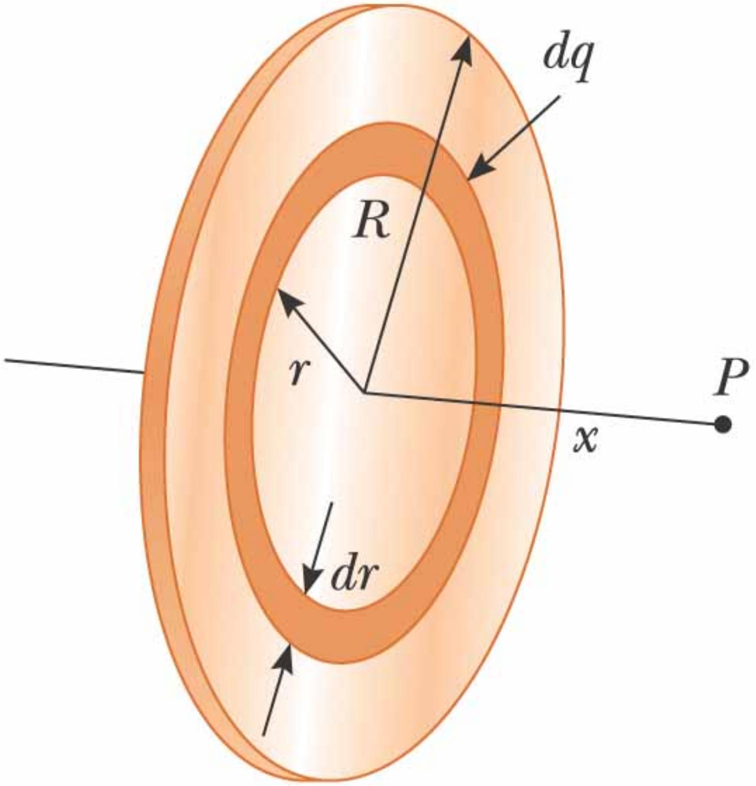
With the first example, this time it must be easier for you to start out and seek for a solution.

Electric field from a continuous charge distribution

Condition: a disk of radius R with evenly distributed charge Q.

Question: what is the electrical field strength at point P which is at a distance x away from the center of the disk?

This time I really need you to find a solution to this problem.



Easy:　　　　difficult:　　　　too difficult:

Charge Particles Experience Force in an Electric Field

- From the definition of the electric field:

$$\vec{E} \equiv \frac{\vec{F}}{q_0}$$ ← Force the test charge experiences

We know that charge particles experience force in an electric field:

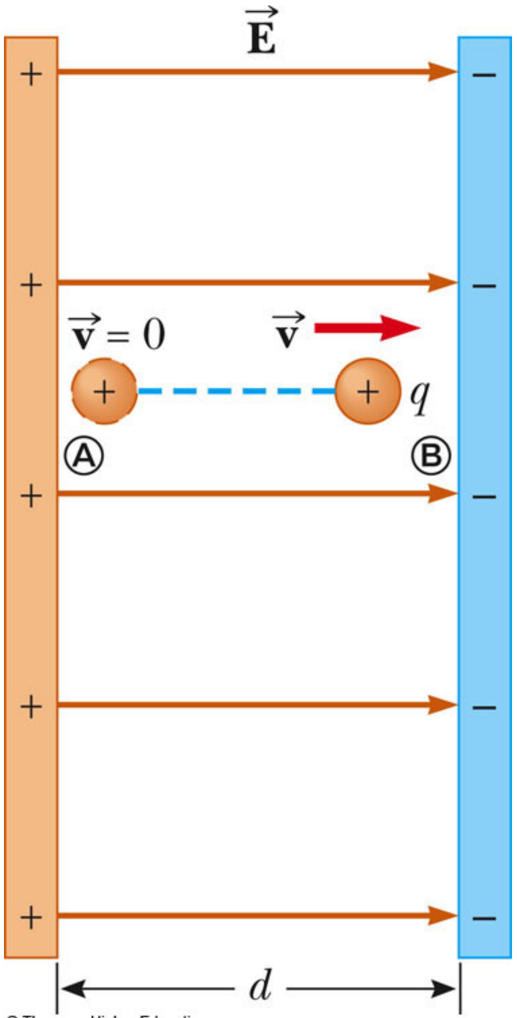
$$\vec{F} = q_0\vec{E}$$

- This formula, although a simple transformation from the definition, is a lot more useful.

Two more examples, 1

Condition: electric field $\vec{E}$ fills between the two plates separated with d. A positive charge q with mass m is released from the positively charged plate.

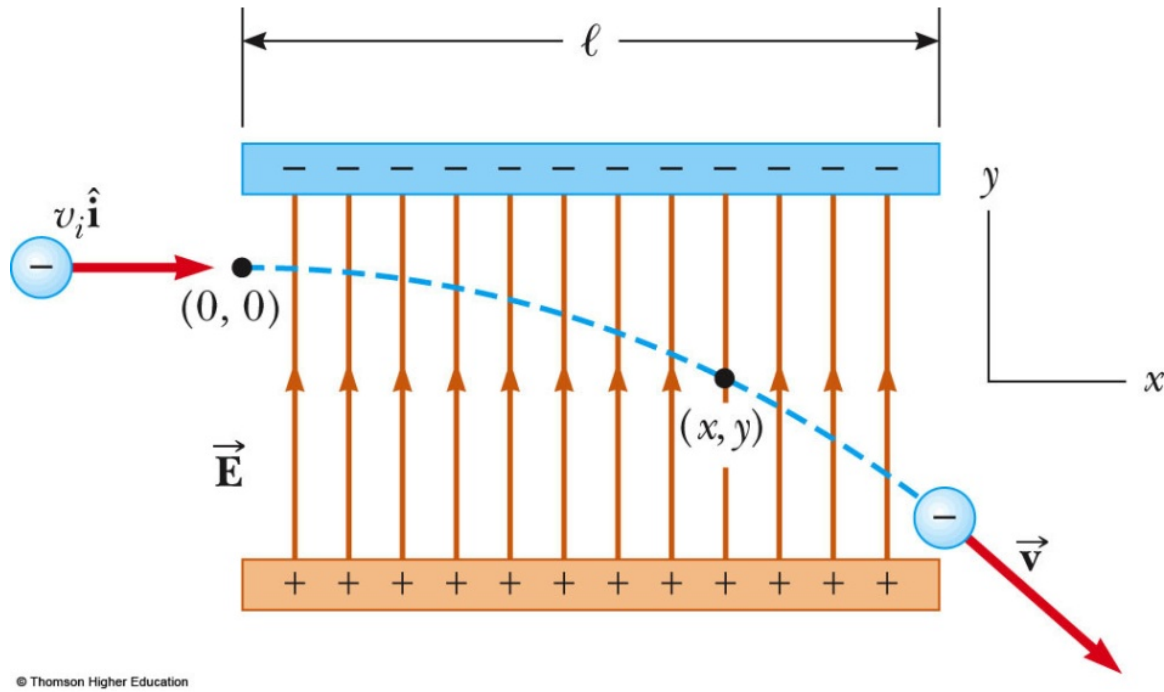
Question: what is the velocity of this charge q when it reaches the negatively charged plate?



Two more examples, 2

Condition: electric field $\vec{E}$ fills between the two plates with a length ℓ. A negative charge -q with mass m enters the field at $(0,0)$ and $v_i\hat{x}$ in the xy coordinate system as indicated. Assume the field is wide enough in y direction.

Question: where this charge leaves the field?



Reading material and Homework assignment

Please watch this video (about 50 minutes):
http://videolectures.net/mit802s02_lewin_lec02/

Please check wileyplus webpage for homework assignment.

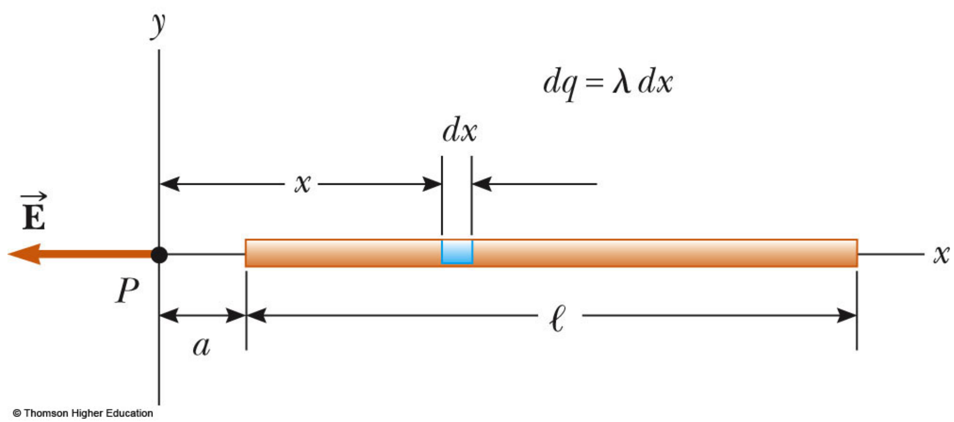
Electric field from a continuous charge distribution

Condition: a straight wire of length ℓ has a charge density of λ.

Question: what is the electrical field strength at point P which is on the line of the wire but a distance a away from it?

Analyze:

One dimensional problem: set up x-axis;



Continuous charge distribution: $d\vec{E} = k_e \dfrac{dq}{x^2}(-\hat{x})$, and $dE = -k_e \dfrac{dq}{x^2}$

What is dq? $dq = \lambda dx$

Now the final answer: $$E = \int_a^{a+l} dE = \int_a^{a+l}\left(-k_e\lambda\frac{dx}{x^2}\right) = \frac{k_e\lambda l}{a(a+l)}$$

$$\int_a^{a+l}\left(k_e\lambda\frac{dx}{x^2}\right) = k_e\lambda\left(-\frac{1}{x}\right)\Bigg|_a^{a+l} = \frac{k_e\lambda l}{a(a+l)}$$

If you cannot follow, you'll have a lot of math to catch up.

Electric field from a continuous charge distribution

Condition: a ring of radius a with evenly distributed positive charge Q.

Question: what is the electrical field strength at point P which is at a distance x away from the center of the ring?

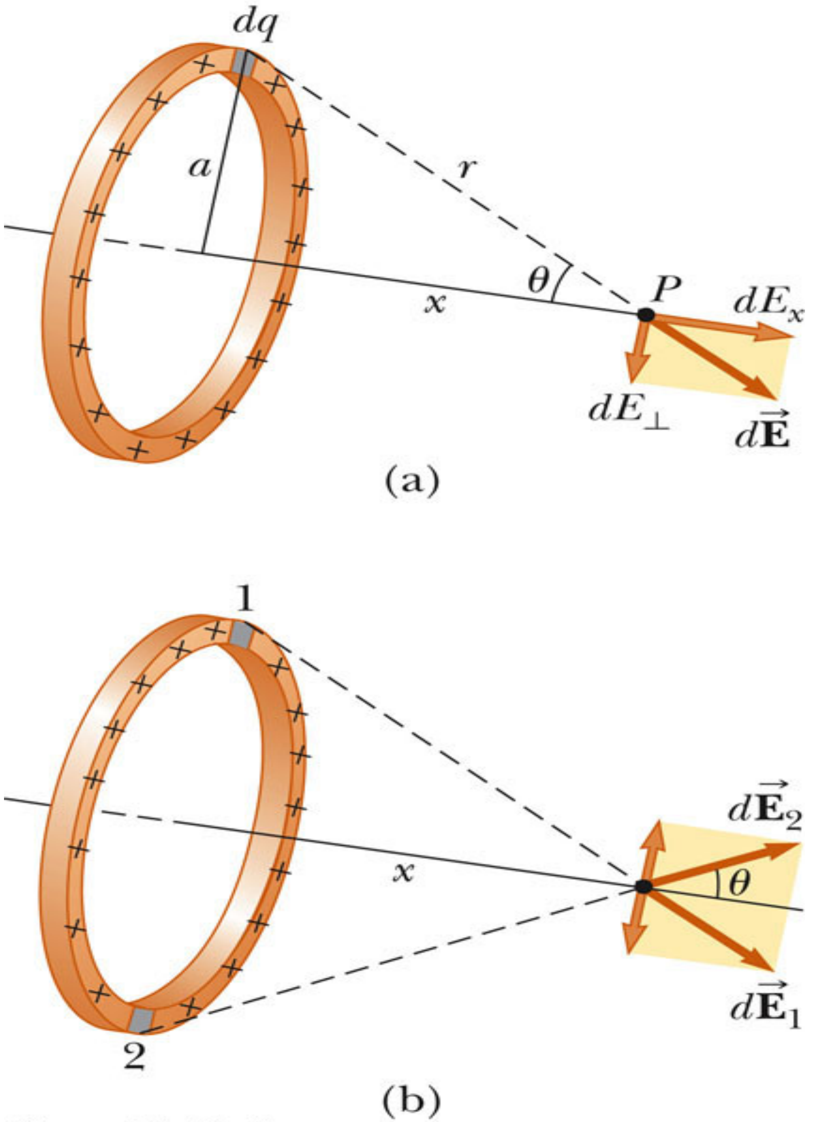
Analyze: Three dimensional problem: set up xyz-axes; Continuous charge distribution but based on the ring symmetry:

$$d\vec{E}_x = k_e \frac{dq}{a^2 + x^2} \cos\theta \hat{x}$$

$$dE_x = k_e \frac{dq}{a^2 + x^2} \frac{x}{\sqrt{a^2 + x^2}} = k_e \frac{x dq}{\left(a^2 + x^2\right)^{\frac{3}{2}}}$$

What is dq? $dq = \left(\frac{Q}{2\pi a}\right) dl$

Where dl is a section on the ring.



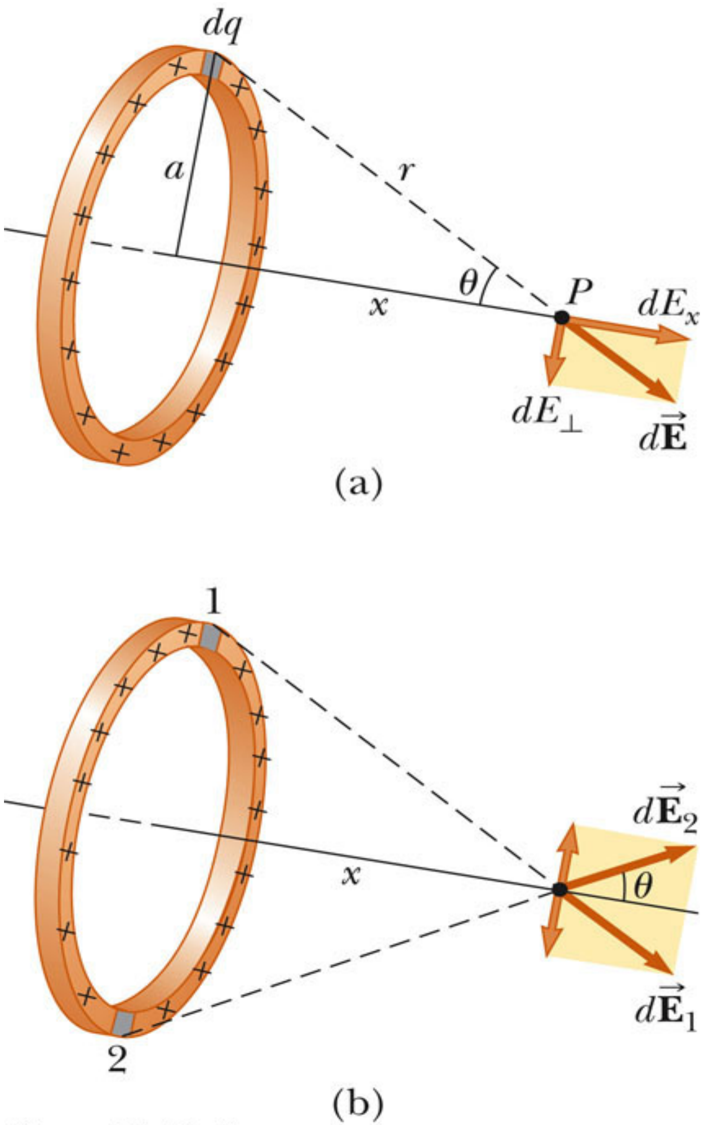
Electric field from a continuous charge distribution

The rest is simple math:

$$E_x = \oint_{fullcircle} k_e \frac{x\,dq}{\left(a^2+x^2\right)^{\frac{3}{2}}}$$

$$= \frac{k_e x}{\left(a^2+x^2\right)^{\frac{3}{2}}} \oint_{fullcircle} dq$$

$$= \frac{k_e x Q}{\left(a^2+x^2\right)^{\frac{3}{2}}}$$



With two weeks and 3 lectures we introduced several concepts: charge, electric field, conductor, insulator, ... There is one physics law (Coulomb's) and one definition (electric field). The rest is about problem solving skills.

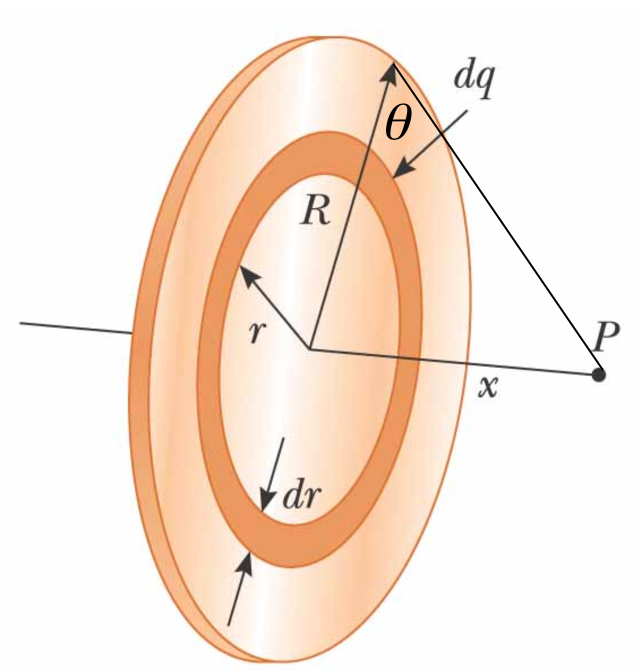
After these two weeks, we'll need to speed up.

Electric field from a continuous charge distribution

Condition: a disk of radius R with evenly distributed charge Q.

Question: what is the electrical field strength at point P which is at a distance x away from the center of the disk?

Analyze: build up the second example problem,



for a ring of radius r to dr: $dE_r = k_e \frac{x\,dq}{\left(r^2+x^2\right)^{\frac{3}{2}}}$

What is dq: $dq = \frac{Q}{\pi R^2} 2\pi r dr = \frac{2Q}{R^2} r dr$

Final answer:

$$E_x = \int_0^R dE_r = \int_0^R \frac{2k_e x Q}{\left(r^2+x^2\right)^{\frac{3}{2}} R^2} r\,dr = \frac{2k_e x Q}{R^2}\int_0^R \frac{r dr}{\left(r^2+x^2\right)^{\frac{3}{2}}}$$

$$= \frac{2k_e x Q}{R^2}\left(-\frac{1}{\sqrt{x^2+r^2}}\right)\Bigg|_0^R = \frac{2k_e Q}{R^2} - \frac{2k_e x Q}{R^2\sqrt{x^2+R^2}} = \frac{Q}{2\pi\varepsilon_0 R^2}\left(1-\sin\theta\right) = \frac{\sigma}{2\varepsilon_0}\left(1-\sin\theta\right)$$

See here for the integral: integrals.wolfram.com

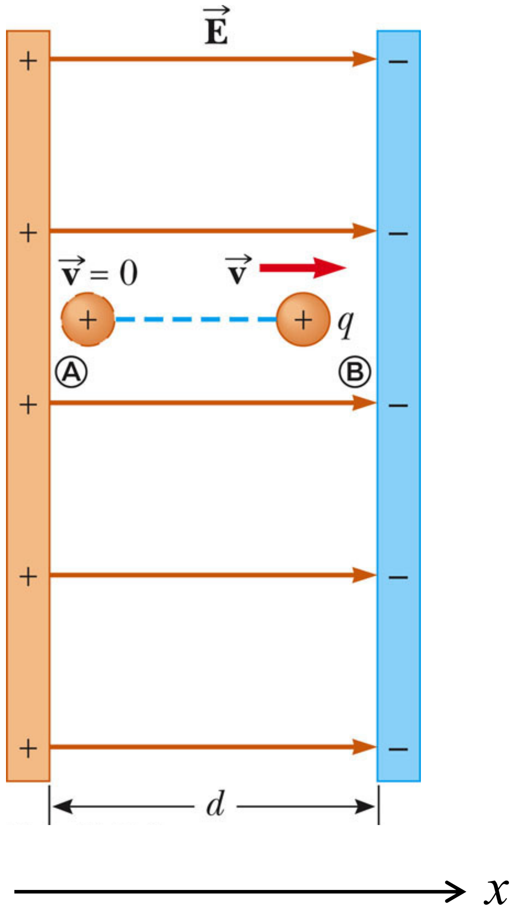
Two more examples, 1

Analyze: set up x-axis along the electric field.

The force on the charge is $\vec{F} = qE\hat{x}$

The kinetic energy the charge gained by this force is $\frac{1}{2}mv^2 = W = \vec{F} \cdot d\hat{x} = qEd$

Final answer: $\vec{v} = \sqrt{\frac{2qEd}{m}}\hat{x}$



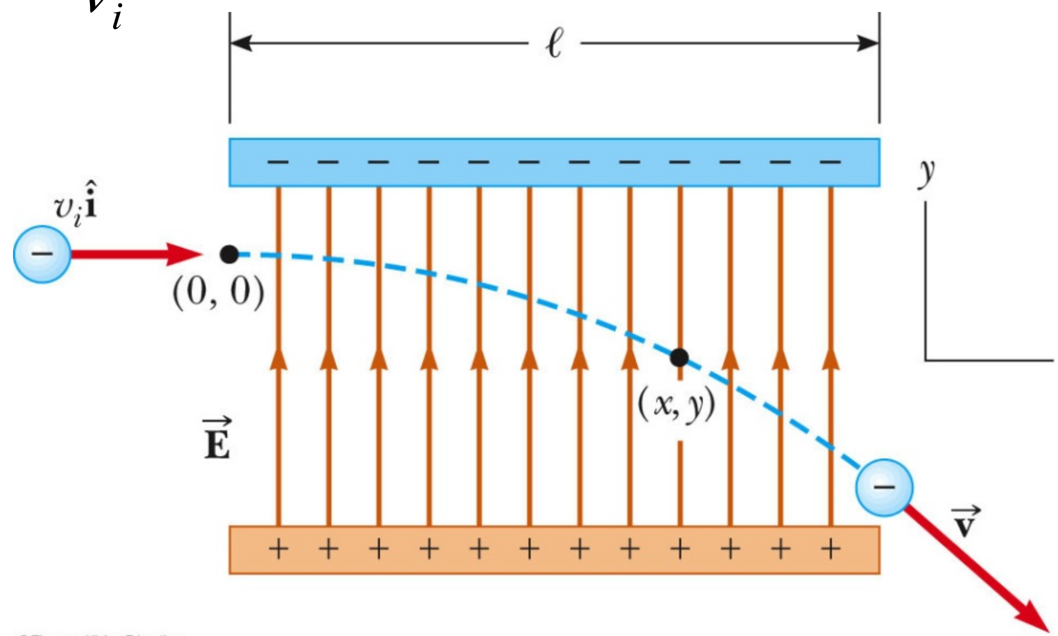
This time the math needed is middle school level. But you'll need the knowledge in Mechanics to work on this problem. If you did not have a clue about how to start on this problem, you'll need to immediately review Mechanics (PHYS 1303).

Two more examples, 2

Analyze: the charge leaves at $(0,y)$.

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{-qE}{m}t^2, \qquad t = \frac{l}{v_i}$$

$$y = -\frac{qEl^2}{2mv_i^2}$$



You find this easy:

Difficult:

Too difficult: