Charges

positive (+)
negative (-)
conservation

Force (field)

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

→ Force between point charges

$$\vec{F} = q\vec{E}$$

→ Force on charge in the field

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

→ Connect field with its source: charge

Potential (energy)

$$-\Delta V = \int_A^B \vec{E} \cdot d\vec{s}$$

→ Connect field with energy

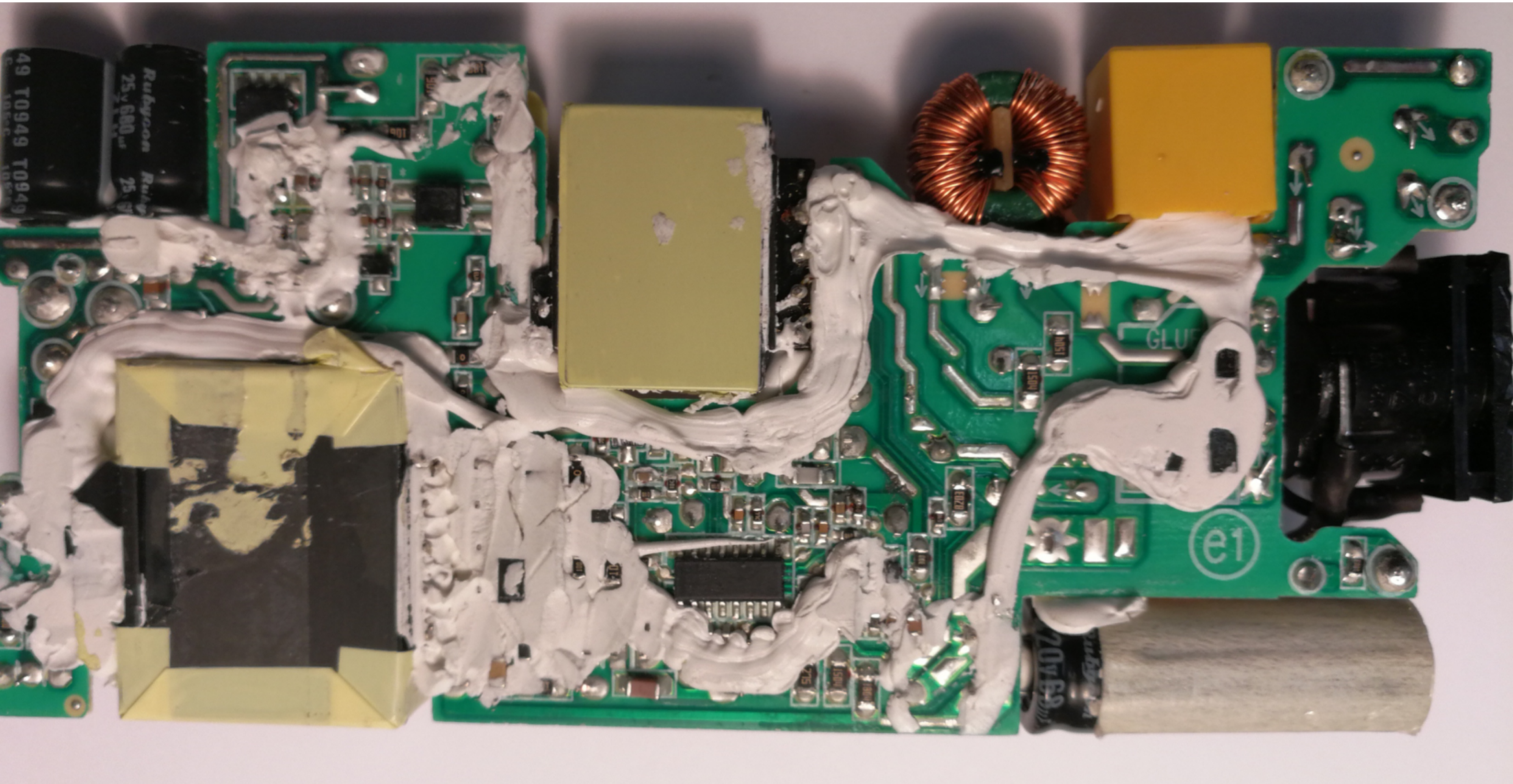
$$U = qV$$

What for?

← To understand the world

Or to move on to capacitor, one of the three passive components in circuits


R050
REV:A
GLUE
N0929
Rubycon
25v 680μF
CARLI ATF
474K 450V
CARLI MPX40/100/21
.33K275V-X2
250V



GLUE
e1

Capacitance and Capacitor

1. Capacitance and the parallel plate capacitor.
2. What's stored in a capacitor? Charge or energy or both?
3. (Real) capacitor with dielectrics.
4. Basic connections of capacitors.

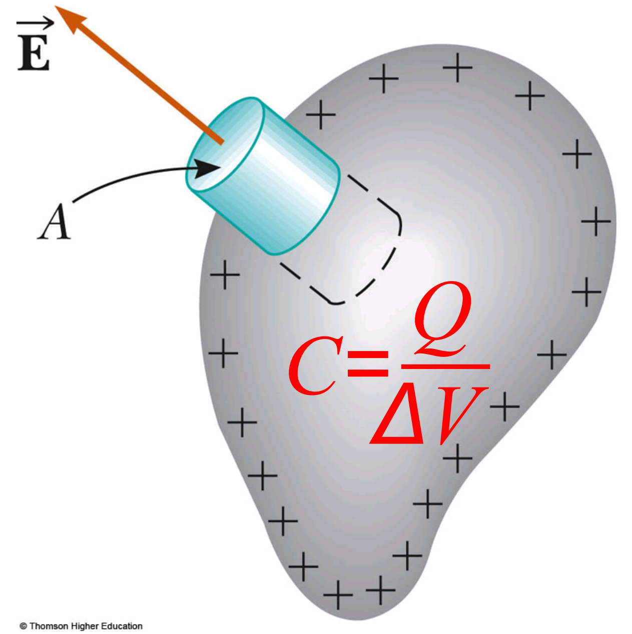
Capacitors; that have capacitance to hold; that a beautiful invention we behold; containers they are, to charges and energy they hold.

- Capacitors are devices that store electric charges
  - Any conductors can store electric charges, but
  - Capacitors are specially designed devices to story a lot of charges
- Examples of where capacitors are used include:
  - radio receivers
  - filters in power supplies
  - to eliminate sparking in automobile ignition systems
  - energy-storing devices in electronic flashes

Capacitance

- The **capacitance**, C, is defined as the ratio of the amount of the charge Q on the conductor to the potential increase ΔV of the conductor because of the charge:

$$C \equiv \frac{Q}{\Delta V}$$



- This ratio is an indicator of the capability that the object can hold charges. It is a constant once the object is given, regardless there is charge on the object or not. This is like the capacitance of a mug which does not depend on there is water in it or not.
- The SI unit of capacitance is the **farad** (F)

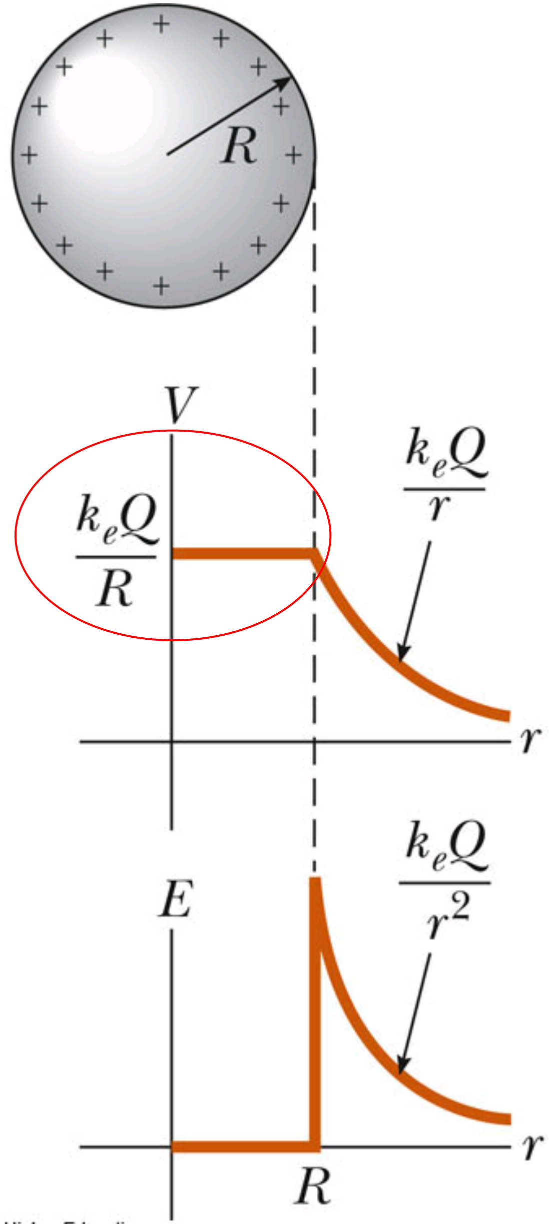
$$1\text{F} = \frac{1\text{C}}{1\text{V}}$$

Capacitance of a one-conductor-system is small: for example, an isolated sphere

- Assume a spherical charged conductor with radius R
- Assume $V = 0$ for at the infinite distance

$$C \equiv \frac{Q}{\Delta V} = \frac{Q}{k_e \dfrac{Q}{R}} = \frac{R}{k_e}$$

- Even for $R = 1$ m, $C = 0.1$ nF
- Note, this is independent of the charge and the potential difference

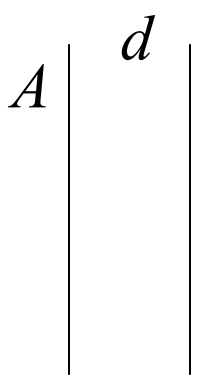


So a sphere is not really a practical container for charges

How to make a capacitor?

- Requirements:
  - Hold charges
  - The potential increase does not appear outside of the device, hence no influence on other devices.
- Is there such a good thing?

Recall the two parallel plates example we talked in Gauss Law chapter. The parallel-plate capacitor:



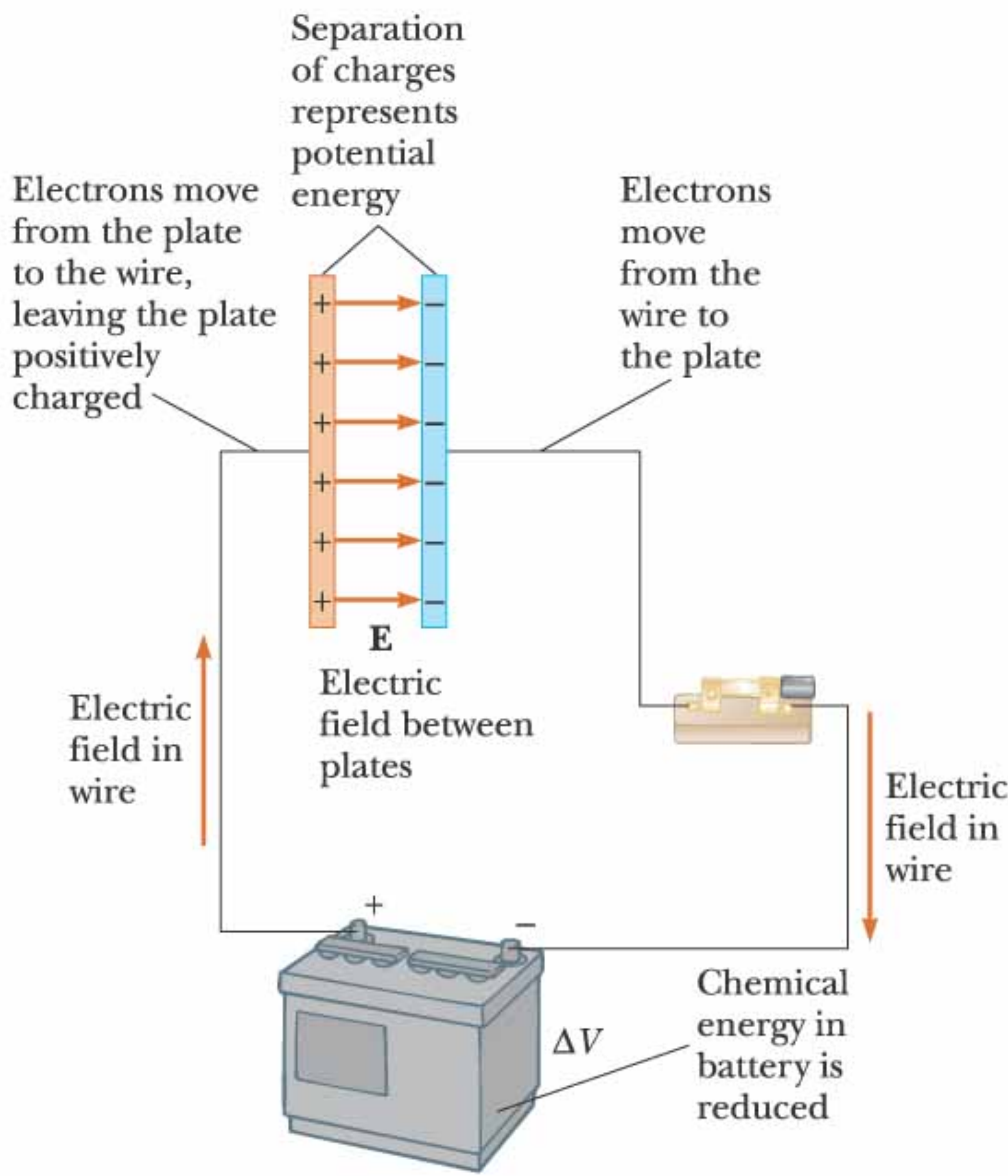
$$\Delta V = Ed$$

$$E = \frac{\sigma}{\varepsilon_0}, \qquad Q = \sigma A$$

$$C \equiv \frac{Q}{\Delta V} = \varepsilon_0 \frac{A}{d}$$

Energy stored in a capacitor

- Consider the circuit to be a system
- When the switch is open, the energy is stored as chemical energy in the battery
- When the switch is closed, the energy is transformed from chemical to electric potential energy
- The electric potential energy is related to the separation of the positive and negative charges on the plates
- So a capacitor can be described as a device that stores energy as well as charge



Energy stored in a capacitor

To study this problem, recall that the work the field force does equals to the electric potential energy loss:

$$W_E = -\Delta U = -Q\Delta V$$

This also means that when the battery moves a charge *dq* to charge the capacitor, the work the battery does equals to the buildup of the electric potential energy:

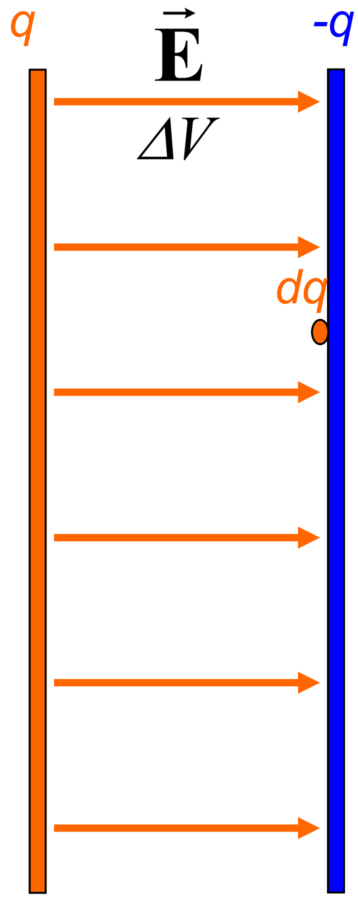
$$W_B = \Delta U$$

When the charge buildup is *q*, move a *dq*, the work is

$$dW_B = \Delta V dq = \frac{q}{C} dq$$

We now have the answer to the final charge *Q*:

$$W_B = \int_0^Q dW_B = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \Delta U$$



Energy in a capacitor, the formula

- When a capacitor has charge stored in it, it also stores electric potential energy that is

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V)^2$$

- This applies to capacitors of any shape and geometry
- The energy stored increases as the charge increases, and as the potential difference increases
- In practice, there is a maximum voltage before the dielectric material breaks down between the plates and discharge occurs

Where does a capacitor store energy?

- The energy can be considered to be stored in the electric field.
- For a parallel-plate capacitor, the energy can be expressed in terms of the field as

$$U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}\left(Ed\right)^2 = \frac{1}{2}\varepsilon_0 AdE^2$$

- It can also be expressed in terms of the energy density (energy per unit volume)

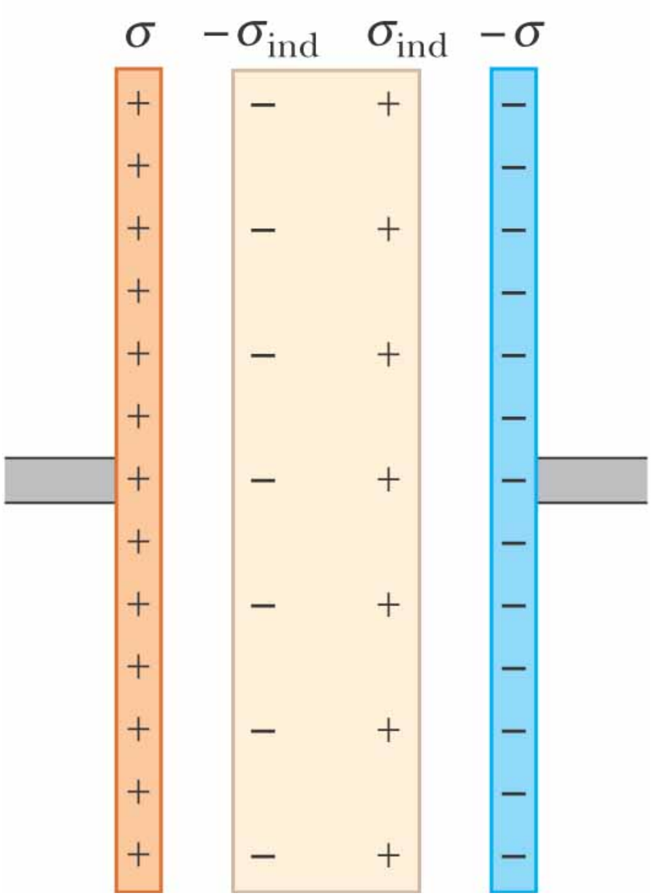
$$u_E = \frac{U_E}{Cap_{volume}} = \frac{\varepsilon_0 AdE^2}{2Ad} = \frac{1}{2}\varepsilon_0 E^2$$

Capacitors with dielectrics

- A *dielectric* is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance
  - Dielectrics include rubber, glass, and waxed paper, etc
- With a dielectric, the capacitance becomes $C = \kappa C_0$
  - The capacitance increases by the factor κ when the dielectric completely fills the region between the plates
  - κ is the **dielectric constant** of the material

Why inserting dielectric increase capacitance

- The electric field due to the plates is directed to the right and it polarizes the dielectric
- The net effect on the dielectric is an induced surface charge that results in an induced electric field
- If the dielectric were replaced with a conductor, the net field between the plates would be zero



Dielectrics strength, the breakdown voltage

- For a parallel-plate capacitor, $C = \kappa\varepsilon_0(A/d)$
- In theory, d could be made very small to create a very large capacitance
- In practice, there is a limit to d
  - d is limited by the electric discharge that could occur though the dielectric medium separating the plates
- For a given d, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material

TABLE 26.1

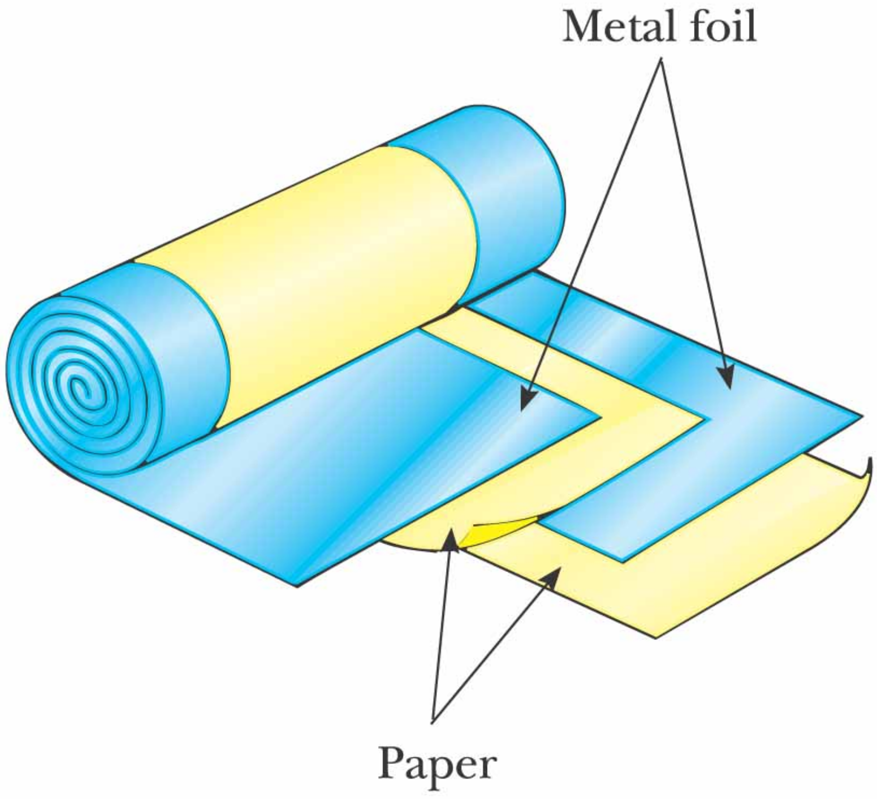
Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

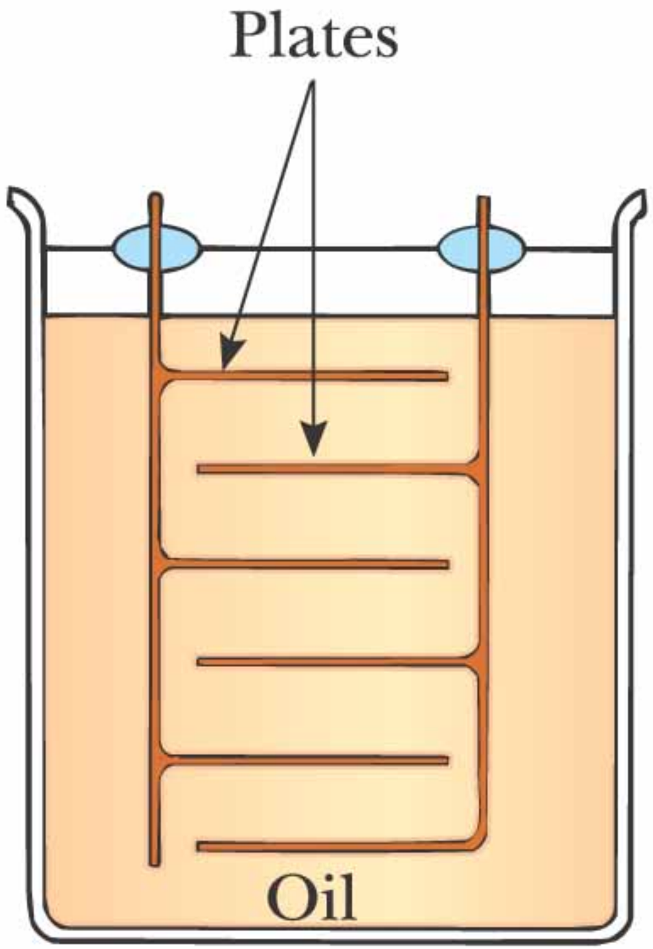
Types of Capacitors – Tubular

- Metallic foil may be interlaced with thin sheets of paraffin-impregnated paper or Mylar
- The layers are rolled into a cylinder to form small package for the capacitor



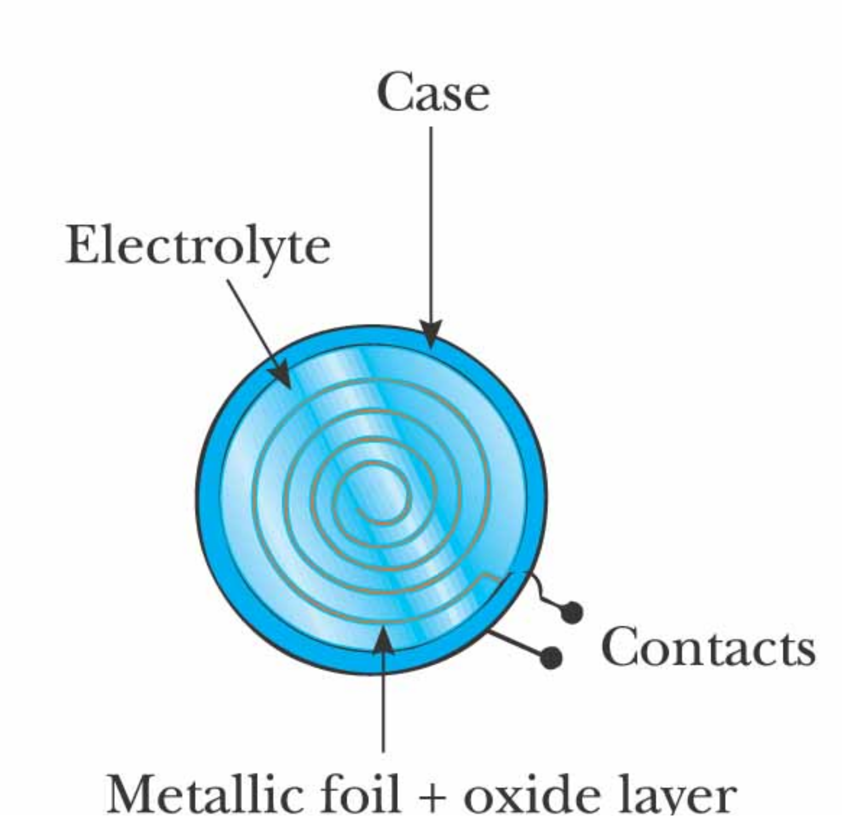
Types of Capacitors – Oil Filled

- Common for high-voltage capacitors
- A number of interwoven metallic plates are immersed in silicon oil



Types of Capacitors – Electrolytic

- Used to store large amounts of charge at relatively low voltages
- The electrolyte is a solution that conducts electricity by virtue of motion of ions contained in the solution



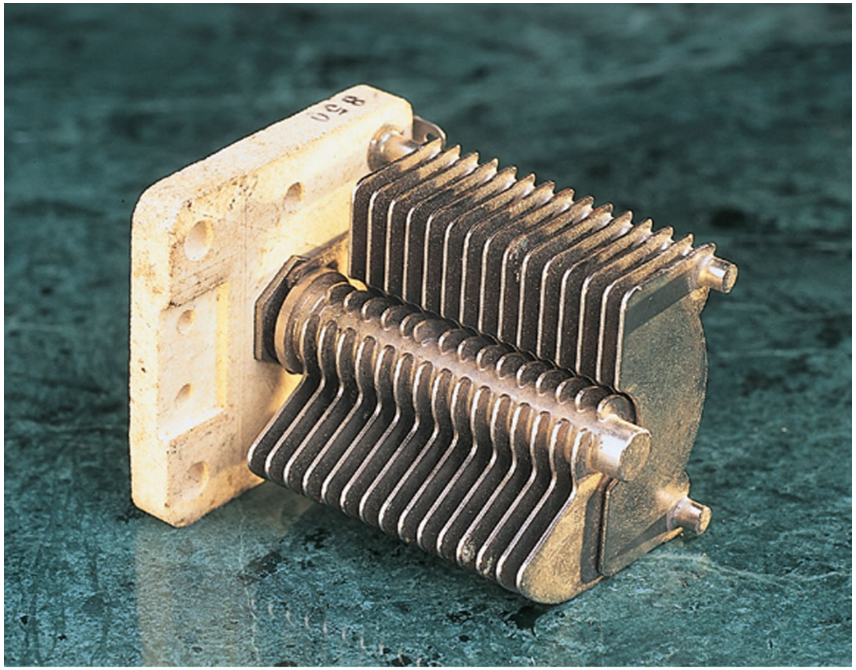
Use digikey.com to check on capacitor types and specs

Types of Capacitors – Variable

- Variable capacitors consist of two interwoven sets of metallic plates
- One plate is fixed and the other is movable
- These capacitors generally vary between 10 and 500 pF
- Used in radio tuning circuits

- MOSFET or diode Varicap

http://en.wikipedia.org/wiki/Varicap



Recap

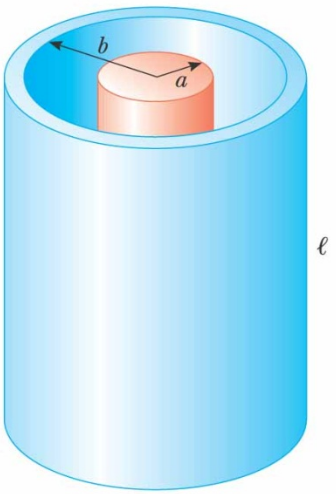
The **capacitance C** is defined as

$$C \equiv \frac{Q}{\Delta V}$$

To calculate the capacitance, one starts by introduce *Q* to the object, and use the Laws we have so far to calculate for the *ΔV*.

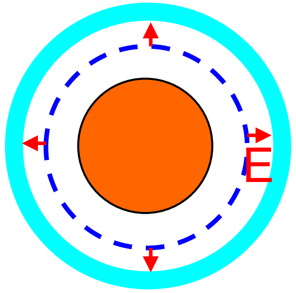
For example: capacitance of a cylindrical capacitor

Step 1: introduce Q to the rod (radius a) and $-Q$ to the shell (inner radius b):



Step 2: Use Gauss's Law to calculate the electric field between the rod and the shell to be (see the example in Gauss's Law chapter):

$$\vec{E} = \frac{Q/l}{2\pi\varepsilon_0 r}\hat{r}$$



Step 3: Calculate for ΔV:

$$\Delta V = \int_a^b \vec{E}\cdot dr = \int_a^b \frac{Q/l}{2\pi\varepsilon_0 r}dr = \frac{Q}{2\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right)$$

Step 4: Capacitance is $C \equiv \dfrac{Q}{\Delta V} = {2\pi\varepsilon_0 l}\big/{\ln(b/a)}$

Recap

Capacitance of a parallel plate capacitor $C = \varepsilon_0 \dfrac{A}{d}$

The electric energy stored in a capacitor $U_E = \dfrac{Q^2}{2C} = \dfrac{1}{2}C(\Delta V)^2$

A question: why C here is not a function of ΔV while U_E is?

The dielectrics in a capacitor: $C = \kappa \cdot C_{vacuum}$

Example

A parallel plate capacitor, made of two very smooth plates, is charged with ΔV. Maintain this potential difference over the two place, and insert a glass plate in between the two parallel plates.

(a) will the capacitance of this capacitor increase?

(b) will the energy stored in this capacitor increase?

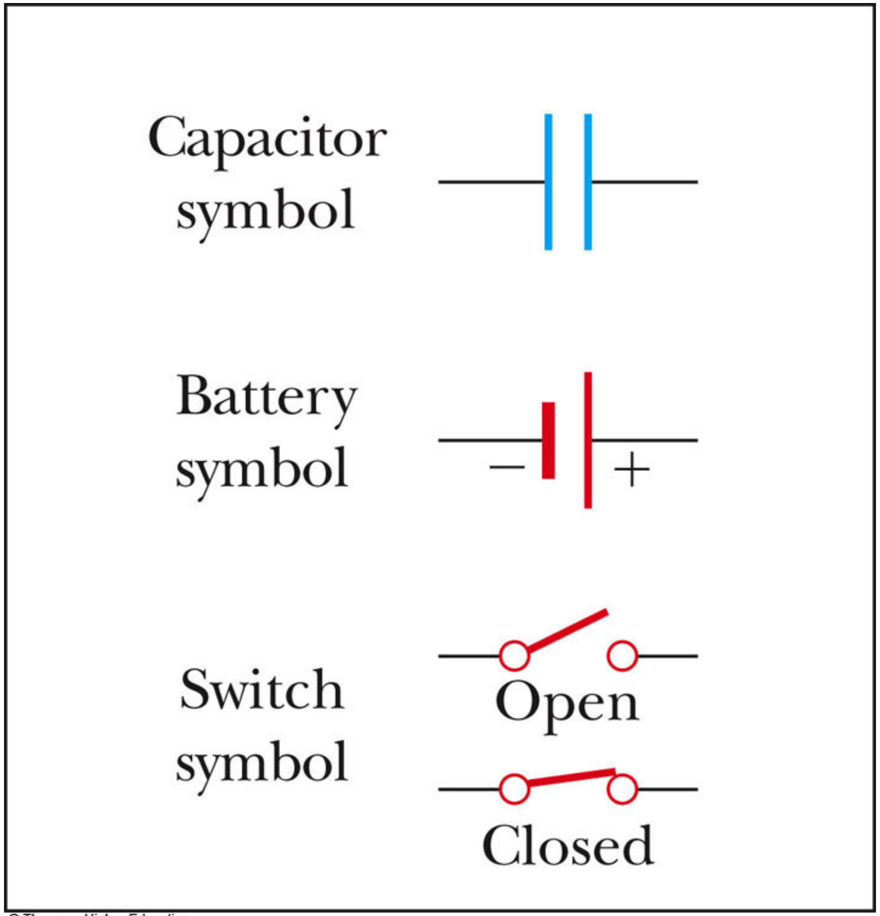
(c) will the charge stored in either plate change? If yes, increase or decrease (only consider absolute value)?

(d) will you need to push the glass plate in or it will be sucked in by the electric force (ignore friction)?

Now maintain the charge stored in the capacitor instead of the potential difference, (e) will you need to push the glass plate in or it will be sucked in by the electric force (ignore friction)?

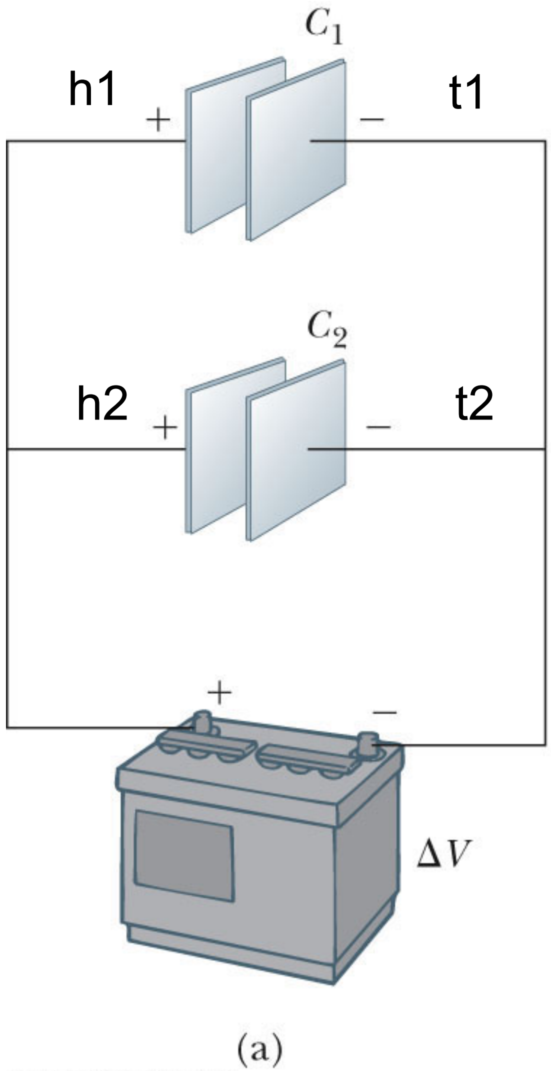
Circuit and Its Symbols

- A circuit diagram is a simplified representation of an actual circuit
- Circuit symbols are used to represent the various elements
- Lines are used to represent wires with zero resistance
- The battery's positive terminal is indicated by the longer line. The potential difference ΔV is measured over the battery from + to -.



Connect capacitors

- Connection in parallel: head to head and tail to tail. As oppose to connection in series: head - tail (of No.1) to head – tail (of No. 2).
- When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged
- This process takes place independently on C_1 and C_2.



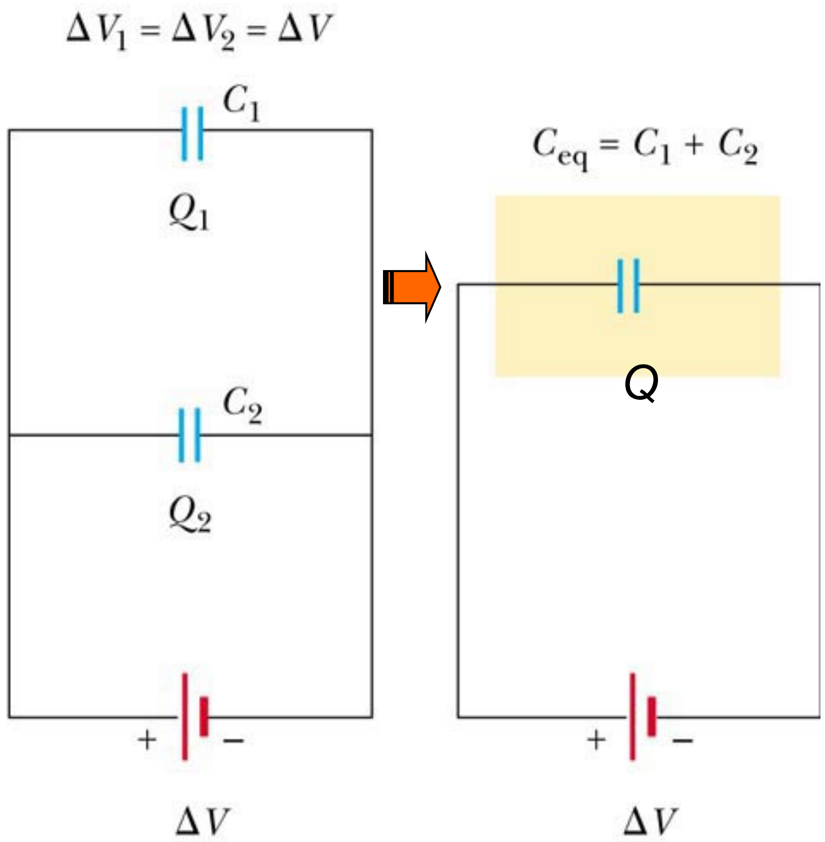
Capacitors in Parallel, equal potential difference (true for anything connected in parallel)

- The flow of charges ceases when the voltage across the capacitors equals that of the battery
- The potential difference across the capacitors is the same
  - And each is equal to the voltage of the battery
  - $\Delta V_1 = \Delta V_2 = \Delta V$
    - ΔV is the battery terminal voltage
- The capacitors reach their maximum charge when the flow of charge ceases
- The total charge is equal to the sum of the charges on the capacitors
  - $Q_{\text{total}} = Q_1 + Q_2$

The two conditions we use to derive the formula for capacitors connected in parallel.

the equivalent capacitor C_{eq}

- The capacitors can be replaced with one capacitor with a capacitance of C_{eq}
  - The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors



To derive the formula:

From

$$\Delta V = \Delta V_1 = \Delta V_2, \quad Q = Q_1 + Q_2$$

And from: $C \equiv \dfrac{Q}{\Delta V}$

One has:

$$C_{eq} \equiv \frac{Q}{\Delta V} = \frac{Q_1 + Q_2}{\Delta V} = \frac{Q_1}{\Delta V_1} + \frac{Q_2}{\Delta V_2} = C_1 + C_2$$

Capacitors in Parallel

- *One can expand the formula into:*

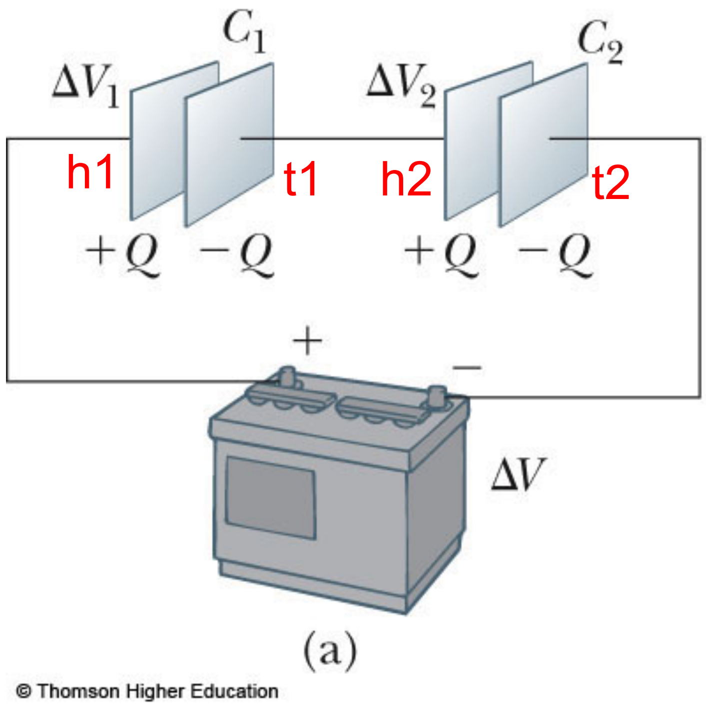
$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$

When more than two are connected in parallel.

- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors
  - Essentially, the areas are combined

Capacitors connected in series

- Connection in series: head - tail (of No.1) to head – tail (of No. 2).
- When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery
- As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge
- All of the right plates gain charges of $-Q$ and all the left plates have charges of $+Q$



The equivalent capacitor C_{eq}

The conditions we use:

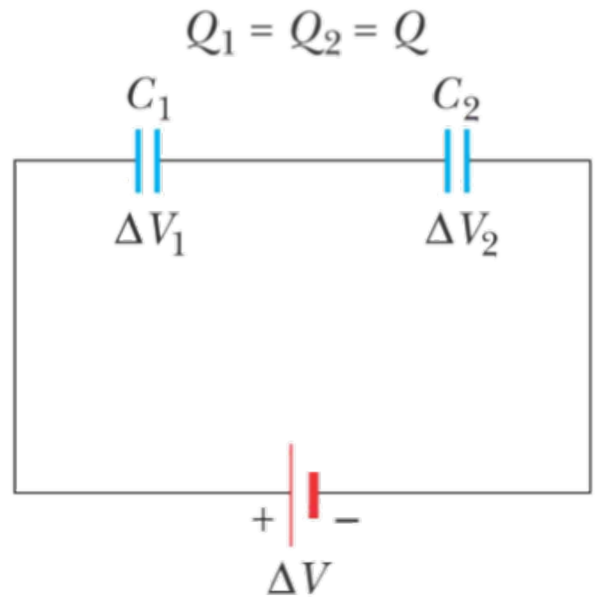
$$\Delta V = \Delta V_1 + \Delta V_2, \quad Q = Q_1 = Q_2$$

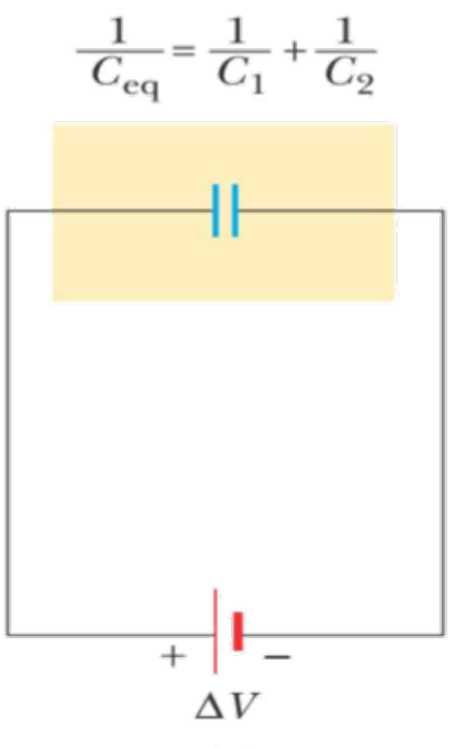
And from: $C \equiv \dfrac{Q}{\Delta V}$

One has:

$$C_{eq} \equiv \frac{Q}{\Delta V}, \text{ or } \frac{1}{C_{eq}} = \frac{\Delta V}{Q}$$

$$\text{so } \frac{1}{C_{eq}} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q_1} + \frac{\Delta V_2}{Q_2} = \frac{1}{C_1} + \frac{1}{C_2}$$





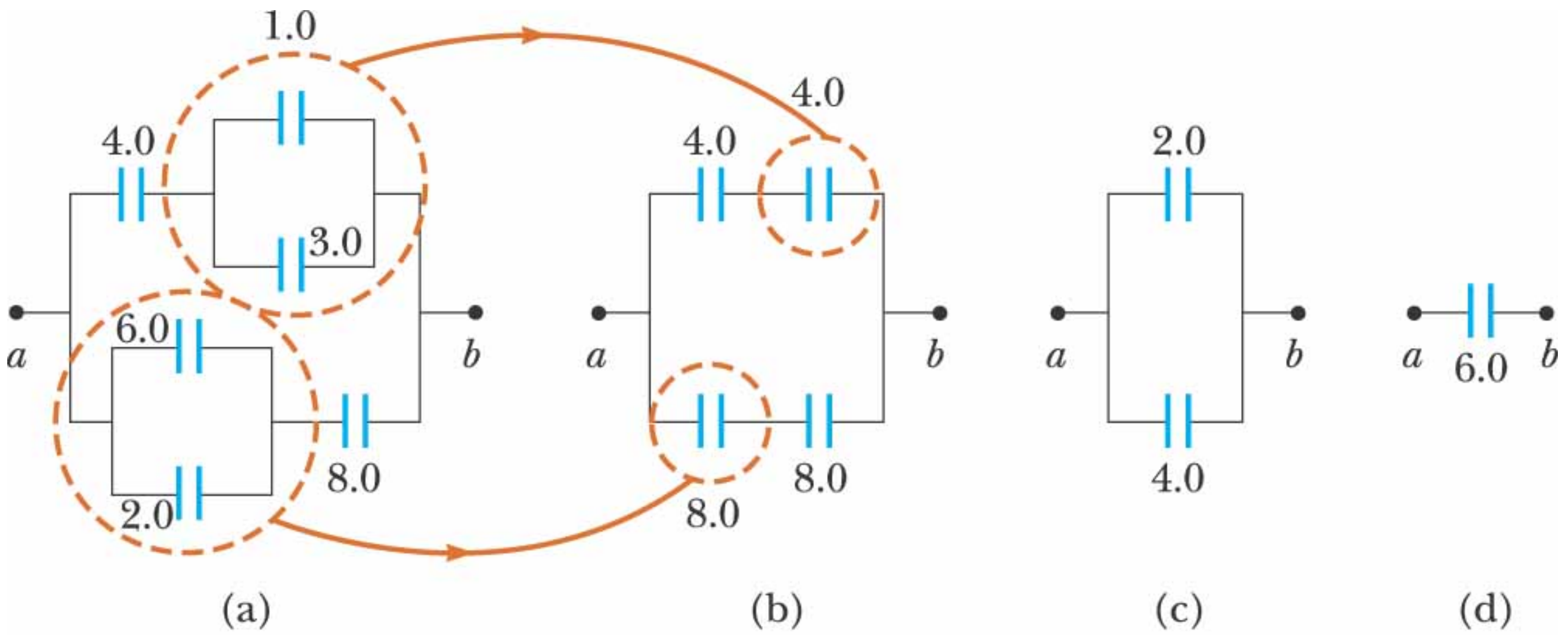
Capacitors in Series

- The equivalent capacitance of more than two in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

- The equivalent capacitance of a series combination is always less than any individual capacitor in the combination

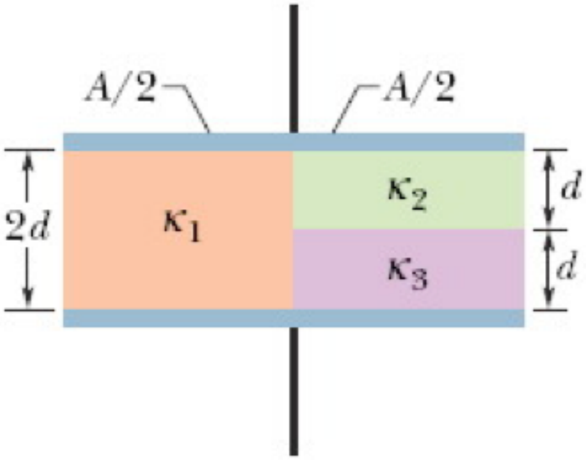
Example



- The 1.0-μF and 3.0-μF capacitors are in parallel as are the 6.0-μF and 2.0-μF capacitors
- These parallel combinations are in series with the capacitors next to them
- The series combinations are in parallel and the final equivalent capacitance can be found

Example

A parallel-plate capacitor of plate area A = 100 cm^2 and plate separation $2d$= 10.0 mm. The left half of the gap is filled with material of dielectric constant κ_1 = 4.0; the top of the right half is filled with material of dielectric constant κ_2 = 3.0; the bottom of the right half is filled with material of dielectric constant κ_3 = 6.0. What is the capacitance?



Reading material and Homework assignment

Please watch this video (about 50 minutes each):
http://videolectures.net/mit802s02_lewin_lec07/ and
http://videolectures.net/mit802s02_lewin_lec08/

Please check wileyplus webpage for homework assignment.