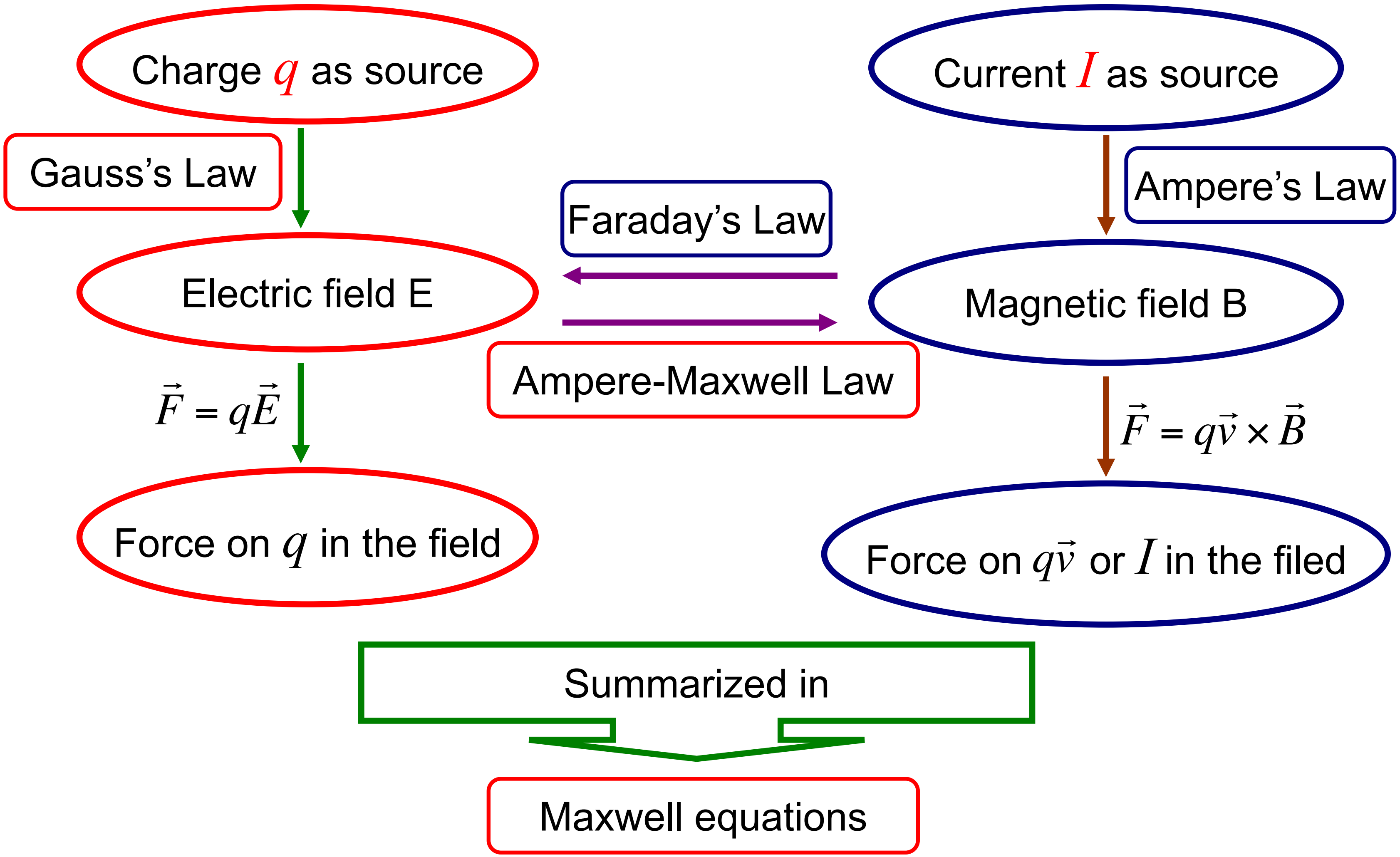
Electricity and Magnetism



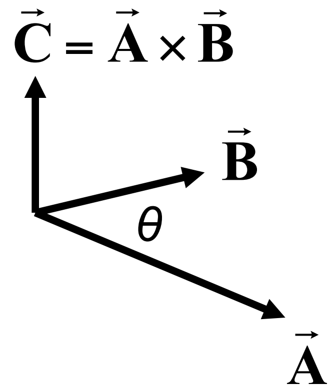
Math:

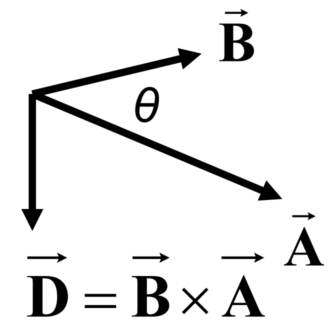
Vector cross product: $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$

Magnitude of the vector $\vec{\mathbf{C}}$:

$$C \equiv \left|\vec{C}\right| = AB\sin\theta$$

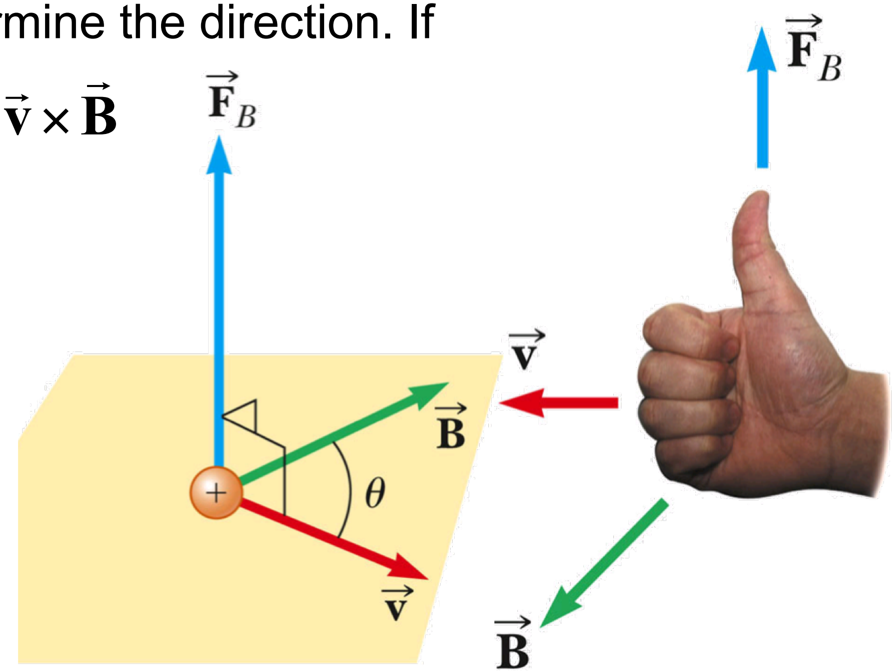
Vector cross product: $\vec{\mathbf{D}} = \vec{\mathbf{B}} \times \vec{\mathbf{A}} = -\vec{\mathbf{C}}$





Determine the direction. If

$\vec{\mathbf{F}}_B = \vec{\mathbf{v}} \times \vec{\mathbf{B}}$



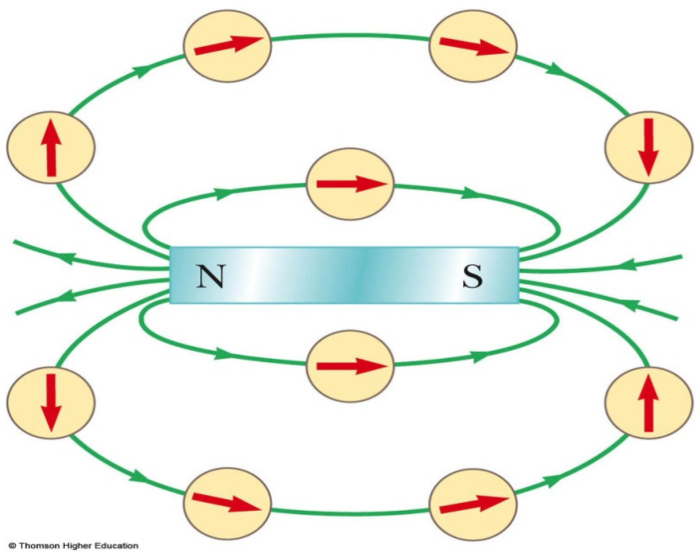
The right-hand rule:

1. Four fingers follow the first vector.
2. Bend towards the second vector.
3. Thumb points to the resultant vector.

The magnetic field

- The field surrounds a magnet is called the magnetic field. The field is a vector, and is symbolized by $\vec{B}$.
- Magnet exists in nature.
- Magnets have two poles, called the north pole and the south pole. Like poles (from different magnets) repel, unlike poles attract.

Like field lines in an electric field, magnetic field lines are used to illustrate the field. Outside a magnet, field lines start from the north pole, end at the south pole. Field lines can be traced out by a small compass.



Units of magnetic field

- The SI unit of magnetic field is the tesla (T)

$$T = \frac{Wb}{m^2} = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m}$$

From flux

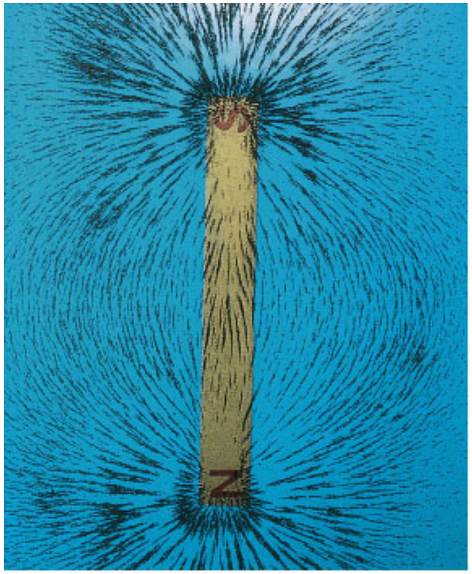
From force

Wb (Weber) is the unit for magnetic field flux.

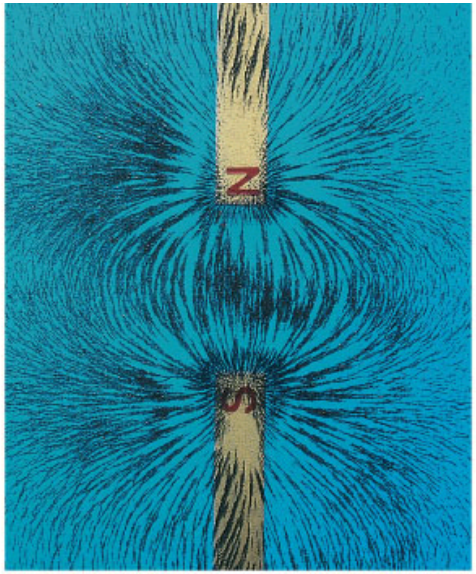
- A non-SI but commonly used unit is gauss (G)

1 T = 10^4 G

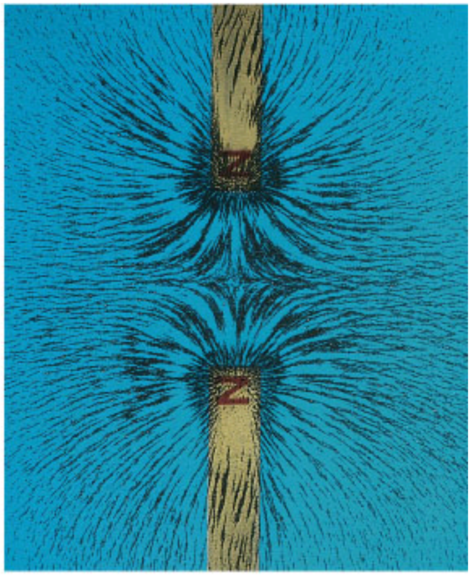
Magnetic field lines of bar magnets



Field lines of one magnet



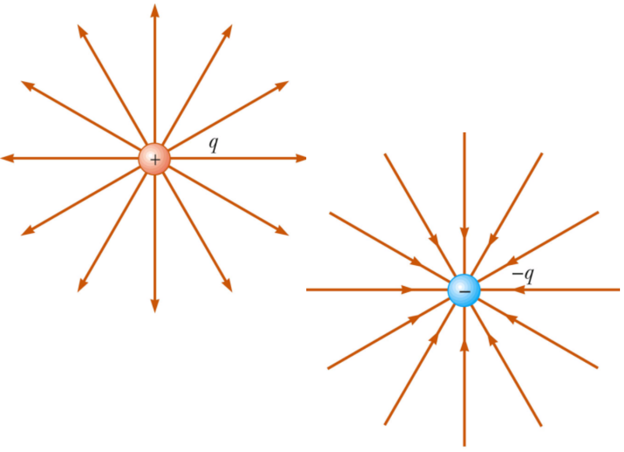
of N and S poles

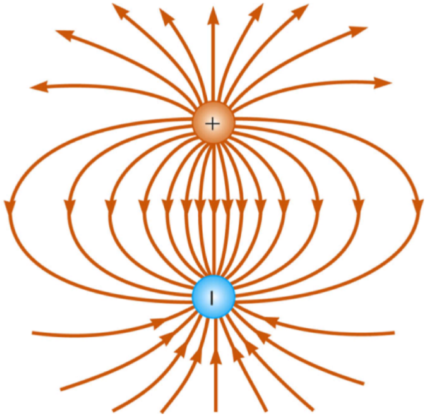


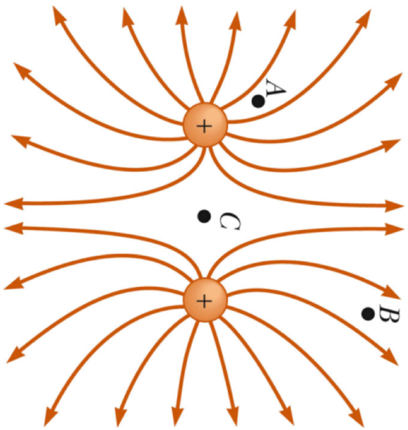
of N and N poles

Magnetic field

Comparison: there exist electric monopoles, the point charges. Magnet monopoles do not exist (have not been found). No matter how small a magnet is, it has two poles, N and S.



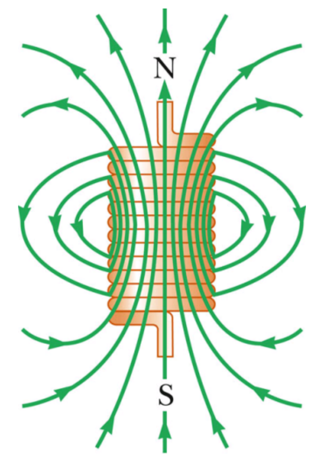


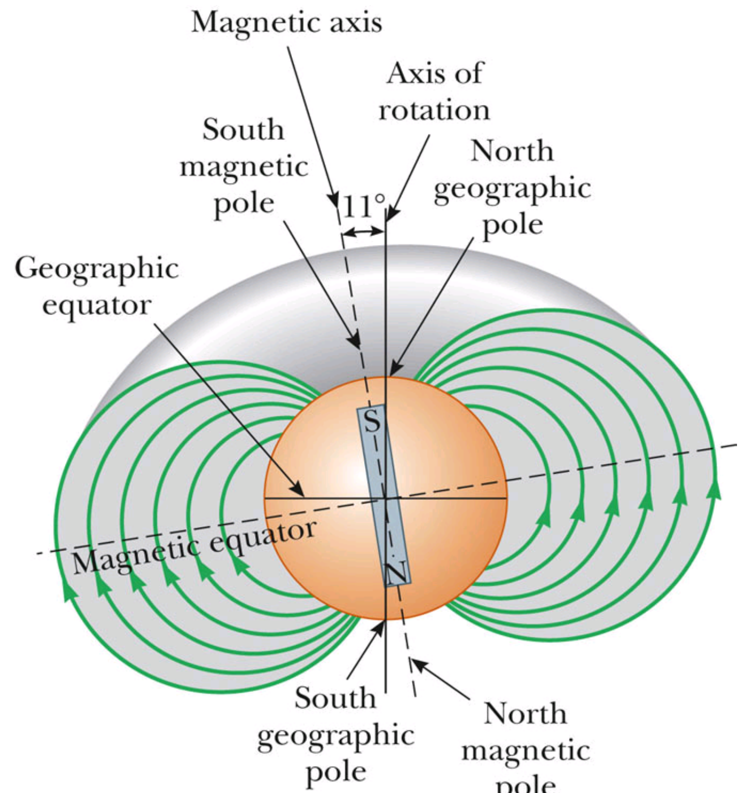


Electric field

What generates the magnetic field?

- Current (or moving charges, or changing electric field) generates magnetic field. We will get back to this topic in the following chapter.
- Magnet can take the form of a permanent magnet (ex. the bar magnet) or a solenoid.
- The Earth itself is also a big magnet.

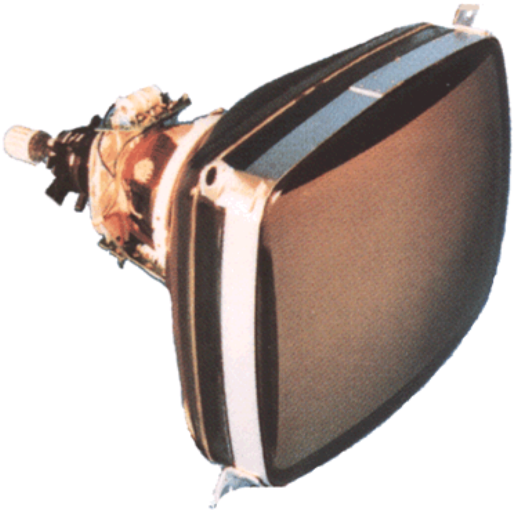




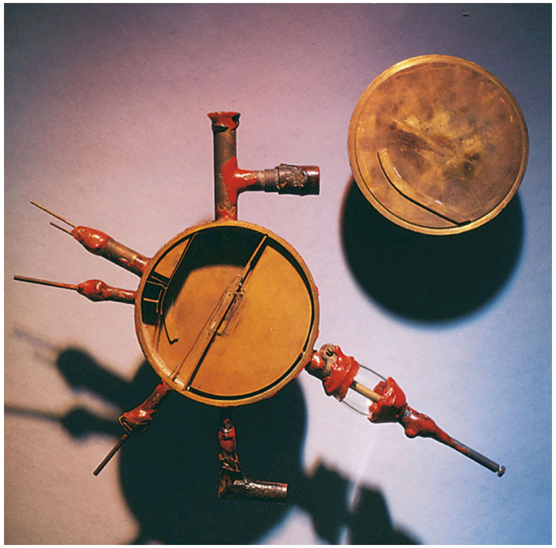
Magnetic force on

- A moving charge
  - CRTs (old TV tube)
  - Particle accelerator
  - Particle mass spectrometer
  - Particle detection
    and homeland security

- A current carrying conductor.
  - Electric motor
  - Hall effect



A CRT (Cathode ray tube)

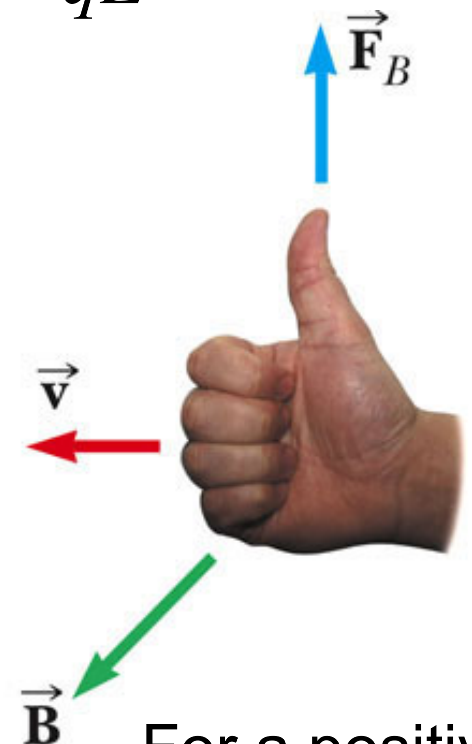


A cyclotron accelerator

Magnetic force on a moving charge

The formula: $\vec{F}_B = q\vec{v} \times \vec{B}$ compare $\vec{F}_E = q\vec{E}$

Here $\vec{F}_B$ is the magnetic force
q is the charge
$\vec{v}$ is the velocity of the charge
$\vec{B}$ is the magnetic field



For a positive charge

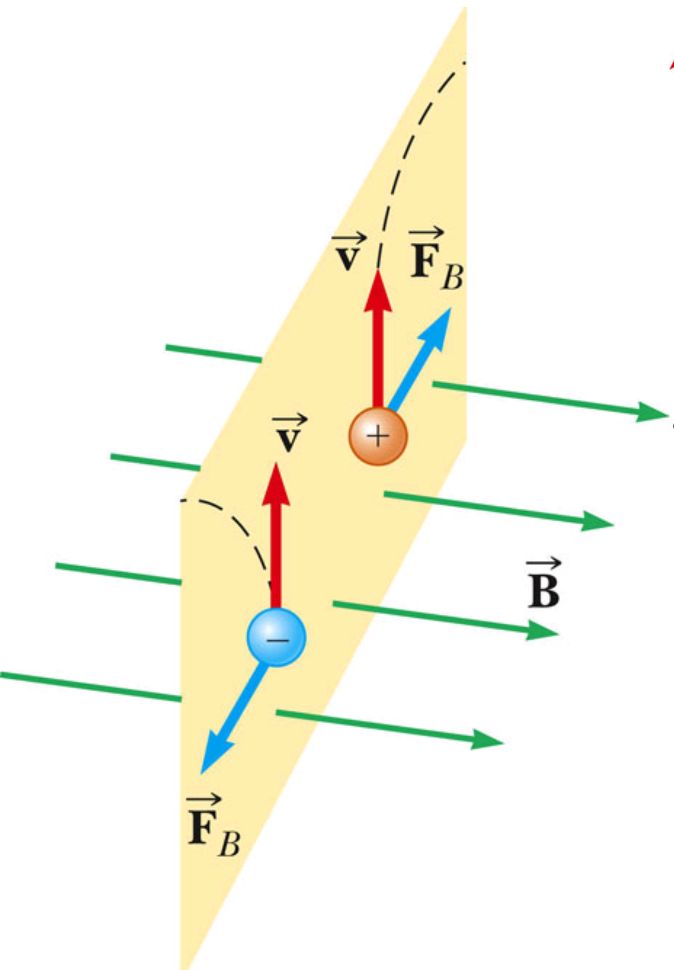
The direction of the force is determined by the charge, the vector product of the velocity and the magnetic field.

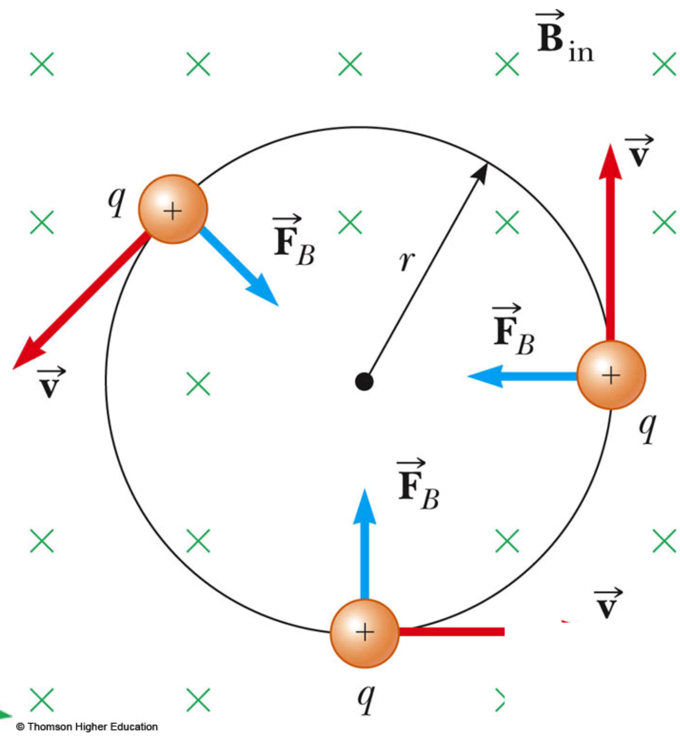
The magnitude:

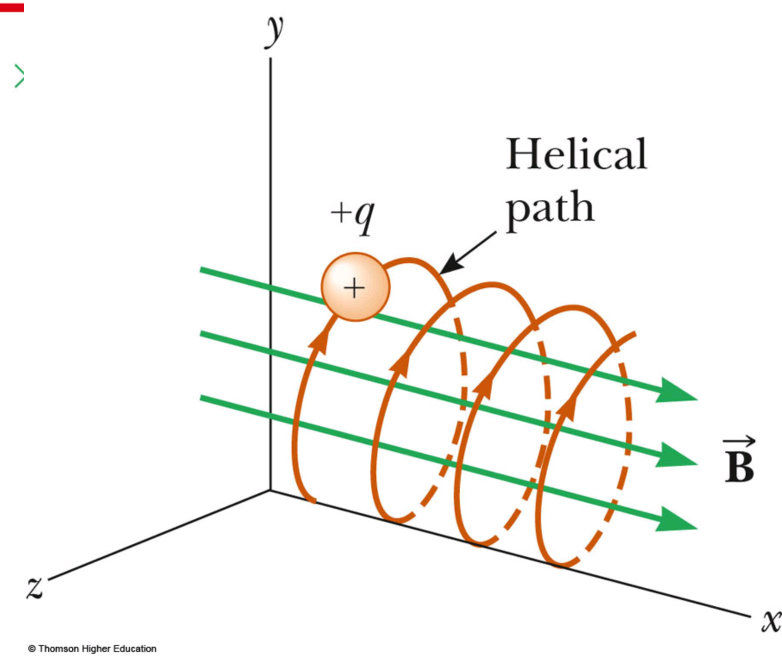
$$F_B = qvB\sin\theta$$

or $F_B = qvB$ When the velocity and the field are perpendicular to each other.

A few examples





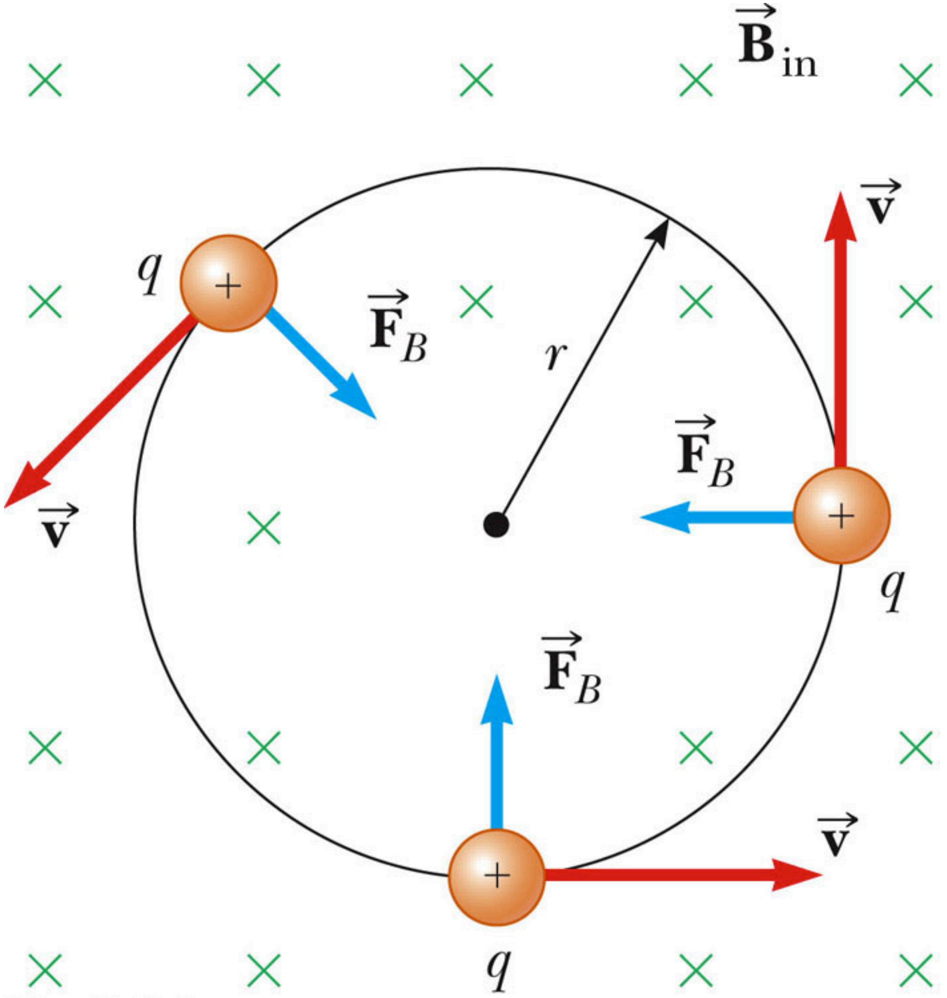


A charged particle in a magnetic field

- Consider a particle moving in an external magnetic field with its velocity perpendicular to the field

- The force is always directed toward the center of the circular path

- The magnetic force causes a centripetal acceleration, changing the direction of the velocity of the particle



Force on a charged particle

- Equating the magnetic and centripetal forces:

$$F_B = qvB = \frac{mv^2}{r}$$

- Solving for r : $r = \dfrac{mv}{qB}$

  r is proportional to the linear momentum ($p{=}mv$) of the particle and inversely proportional to the magnetic field B. This is how particle's momentum is measured in a particle physics experiment.

Motion of a charged particle

- The angular speed of the particle is

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- The period of the motion is

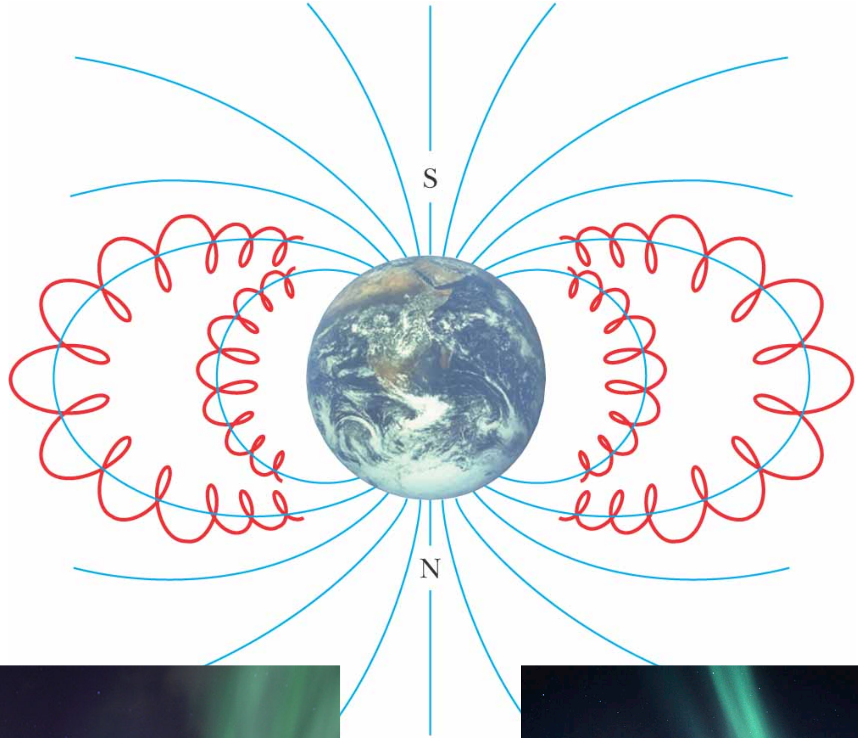
$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

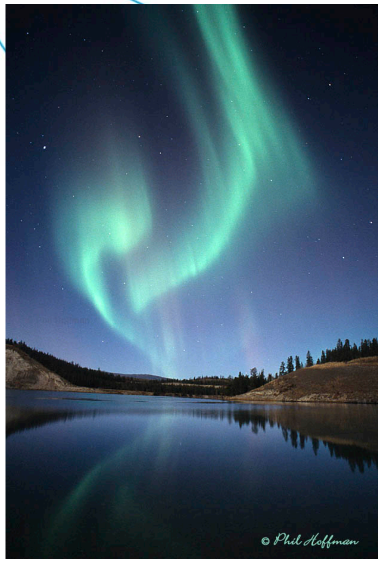
  and this is not a function of the velocity, making cyclotron simple and passible.

The Van Allen Radiation Belts

- The Van Allen radiation belts consist of charged particles surrounding the Earth in doughnut-shaped regions
- The particles are trapped by the Earth's magnetic field
- The particles spiral from pole to pole

May result in Auroras





Electric and magnetic fields

- Direction of force
  - The electric force acts along the direction of the electric field: $\vec{F}_E = q\vec{E}$
  - The magnetic force acts perpendicular to the magnetic field (vector cross product): $\vec{F}_B = q\vec{v} \times \vec{B}$
- Motion
  - The electric force acts on a charged particle regardless the particle is moving or not.
  - The magnetic force acts on a charged particle only when the particle is in motion.

Electric and magnetic fields

- Work
  - The electric force does work in displacing a charged particle
  - The magnetic force associated with a steady magnetic field does no work when a charged particle is displaced
    - This is because the force is perpendicular to the displacement
    - Prove: $dW = \vec{F}_B \cdot d\vec{s} = \left(q\vec{v} \times \vec{B} \right) \cdot \vec{v}dt = 0$

Charged particles moving in electric and magnetic fields

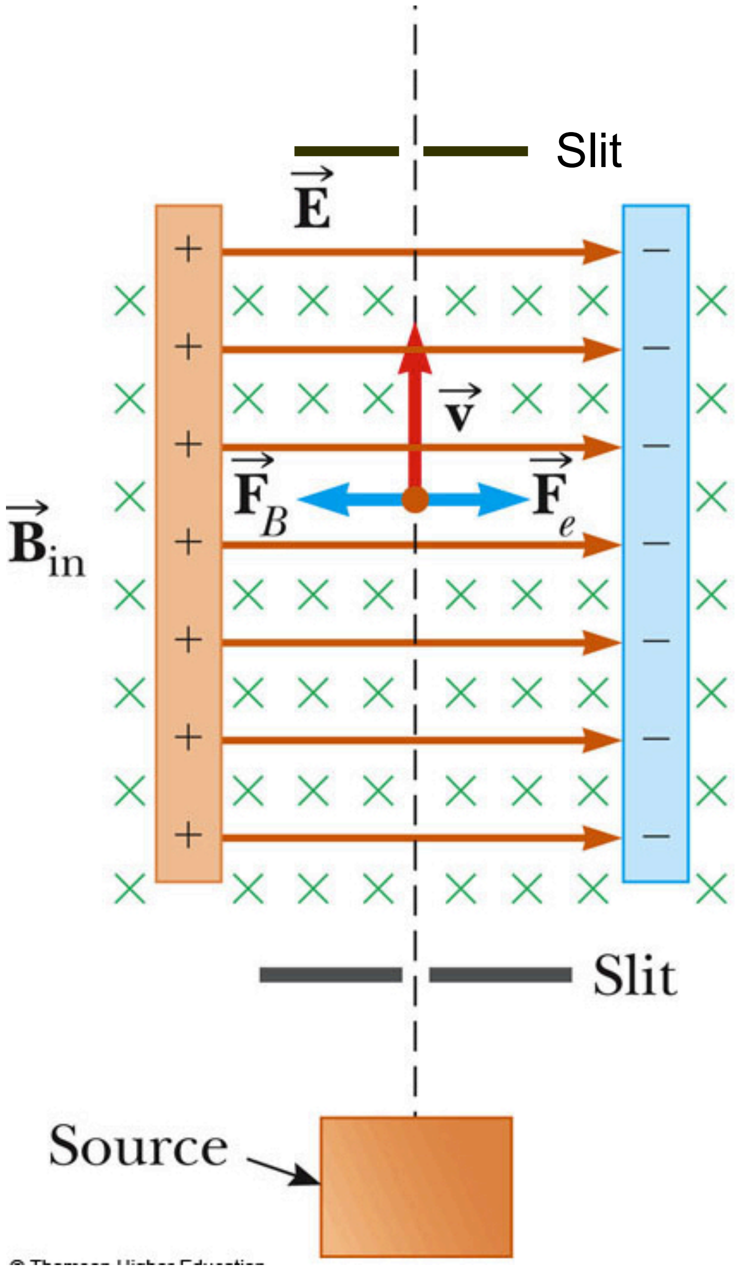
- In many applications, charged particles will move in the presence of both magnetic and electric fields
- In that case, the total force is the sum of the forces due to the individual fields
- In general (The Lorentz force):

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Velocity selector

- A uniform electric field is perpendicular to a uniform magnetic field
- When the force due to the electric field is equal but opposite to the force due to the magnetic field, the particle moves in a straight line
- This selects particles with velocities of the value

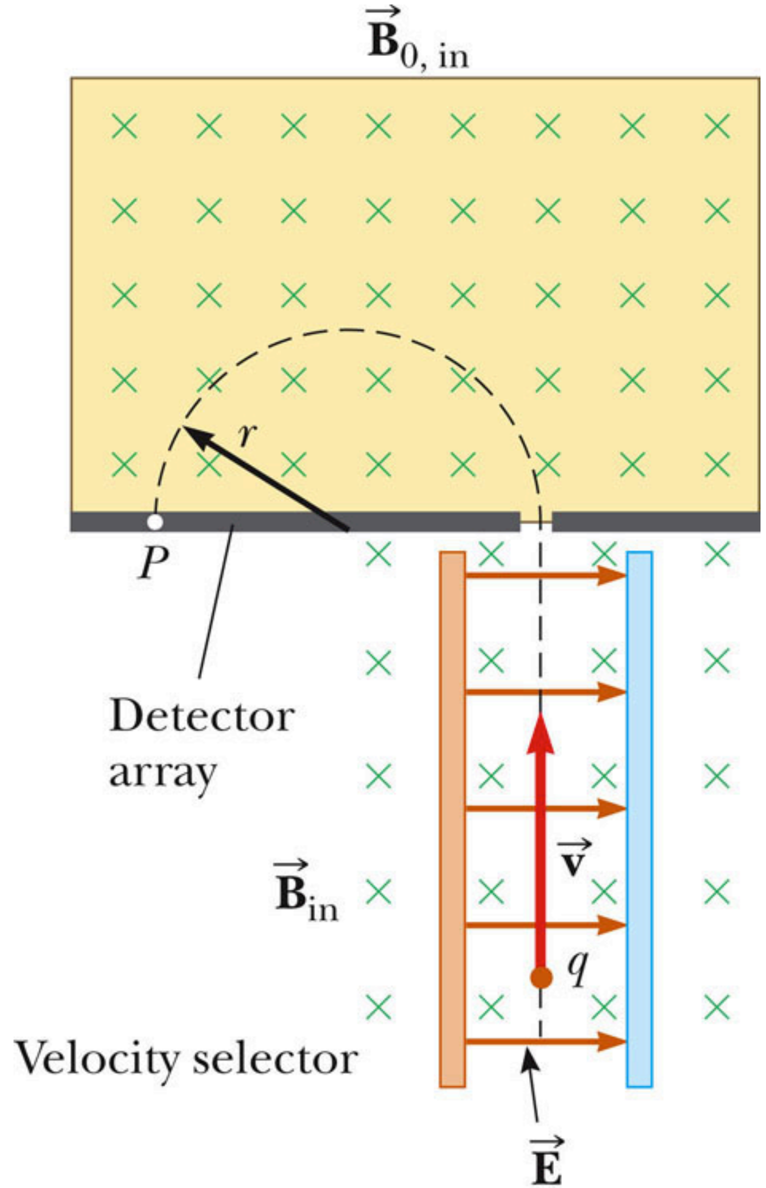
$v = E / B$



Mass spectrometer

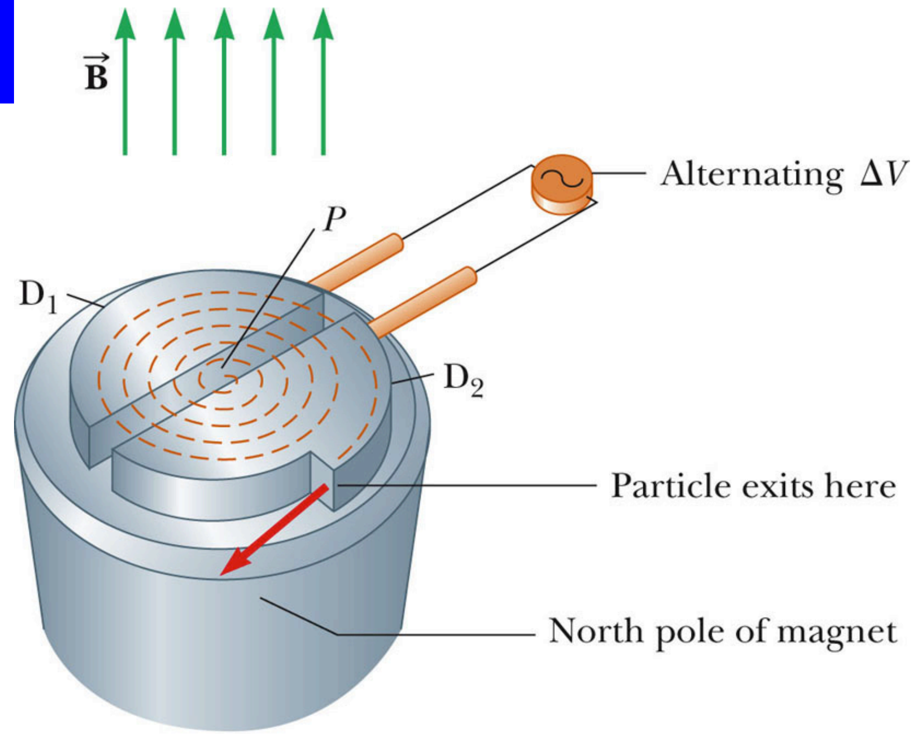
- A mass spectrometer separates ions according to their mass-to-charge ratio
- A beam of ions passes through a velocity selector and enters a second magnetic field where the ions move in a semicircle of radius r before striking a detector at P.

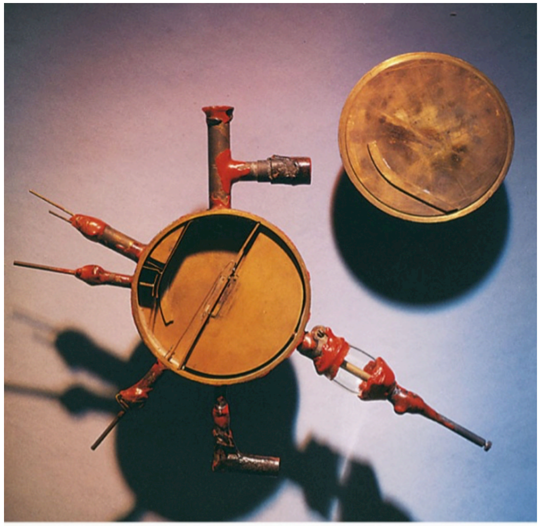
  From $r = \dfrac{mv}{qB}$ and v from the velocity selector, the mass m of the particle is measured.
- If the ions are positively charged, they deflect to the left; If the ions are negatively charged, they deflect to the right



Cyclotron

- A **cyclotron** is a device that accelerates charged particles to very high speeds (a few words about particle accelerators and their applications).
- D_1 and D_2 are called *dees* because of their shape
- A high frequency alternating potential is applied to the dees
- A uniform magnetic field is perpendicular to them





(b)

Cyclotron, the calculation

- The cyclotron's operation is based on the fact that the particle's revolution period T is independent of the speed of the particle and of the radius R of the path

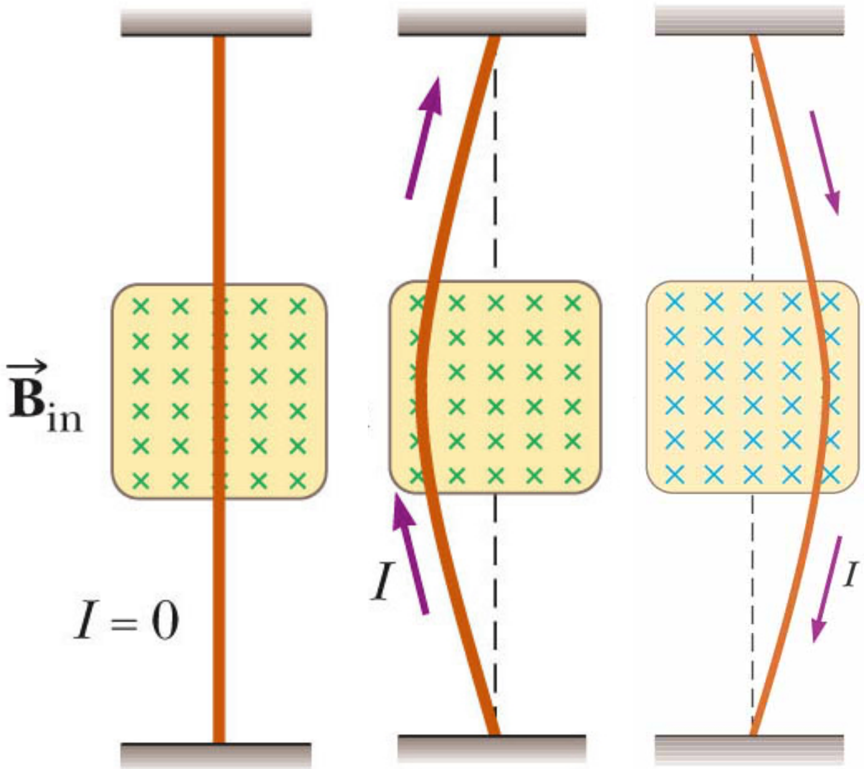
$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

$$K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m} \longleftarrow \left\{ \begin{aligned} &\vec{F}_B = qvB = m\frac{v^2}{R} \\ &v = \frac{qBR}{m} \end{aligned} \right.$$

- When the energy of the ions in a cyclotron exceeds about 20 MeV, relativistic effects come into play

Magnetic force on a current carrying conductor, a wire

- A force is exerted on a current-carrying wire placed in a magnetic field The current is a collection of many charged particles in motion
- The direction of the force is given by the right-hand rule



Force on a wire

- The magnetic force is exerted on each moving charge in the wire

  $$\vec{F}_B = q\vec{v}_d \times \vec{B}$$

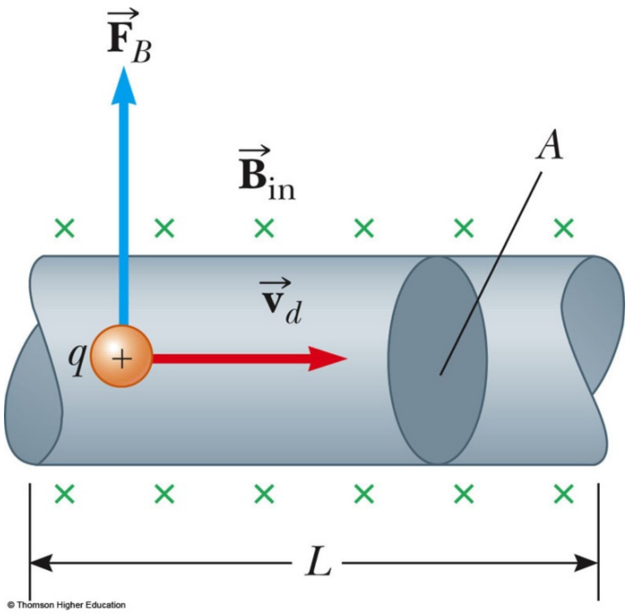
- The total force is the product of the force on one charge and the number of charges

  $$\vec{F}_B = \left(q\vec{v}_d \times \vec{B}\right) nAL = qnA\vec{v}_d L \times \vec{B}$$

- In terms of the current, this becomes

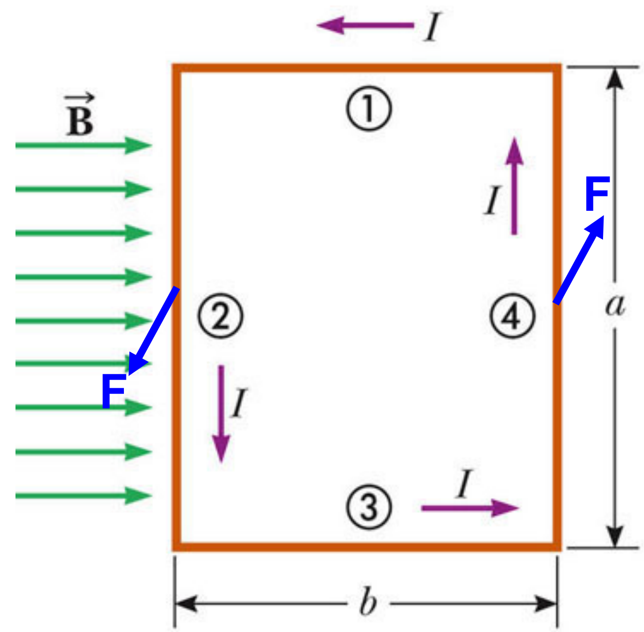
  $$\vec{F}_B = I\vec{L} \times \vec{B} \qquad \because I = nq\vec{\mathbf{v}}_d \cdot \vec{\mathbf{A}}$$

  - I is the current
  - $\vec{L}$ is a vector that points in the direction of the current
    - Its magnitude is the length L of the segment

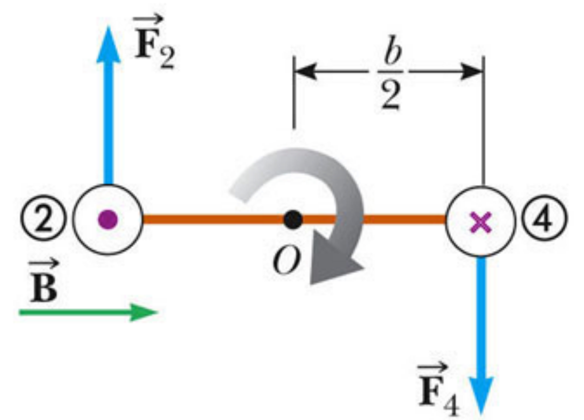


Torque on a current loop

- The rectangular loop carries a current I in a uniform magnetic field
- No magnetic force acts on sides 1 & 3.
- The magnitude of the magnetic force on sides 2 & 4 is
  - $F_2 = F_4 = I\,a\,B$
- The direction of F_2 is out of the page; The direction of F_4 is into the page
- The forces are equal and in opposite directions, but not along the same line of action → rotation.
- The forces produce a torque around point O (bottom view)



Front view



bottom view

Torque on a current loop

- The maximum torque is found by:

$$\tau_{\max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = \left(F_2 + F_4\right)\frac{b}{2} = 2IaB\frac{b}{2} = IabB$$

- The area enclosed by the loop is $A = ab$, so

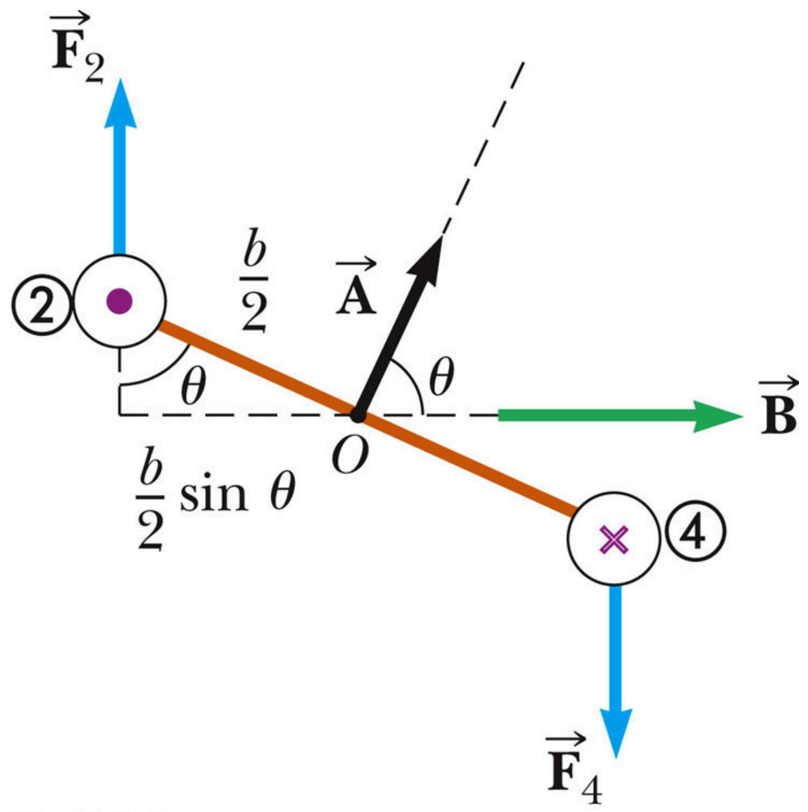
$$\tau_{\max} = IAB$$

This maximum value occurs only when the field is parallel to the plane of the loop

Torque on a current loop

- Assume the magnetic field makes an angle of $\theta < 90^o$ with a line perpendicular to the plane of the loop (the direction of a loop, next slide)
- The net torque about point O will be $\tau = IAB\sin\theta$
- When the direction of the loop area is defined, the torque can be expressed in its vector format:

$$\vec{\tau} = I\vec{A} \times \vec{B}$$



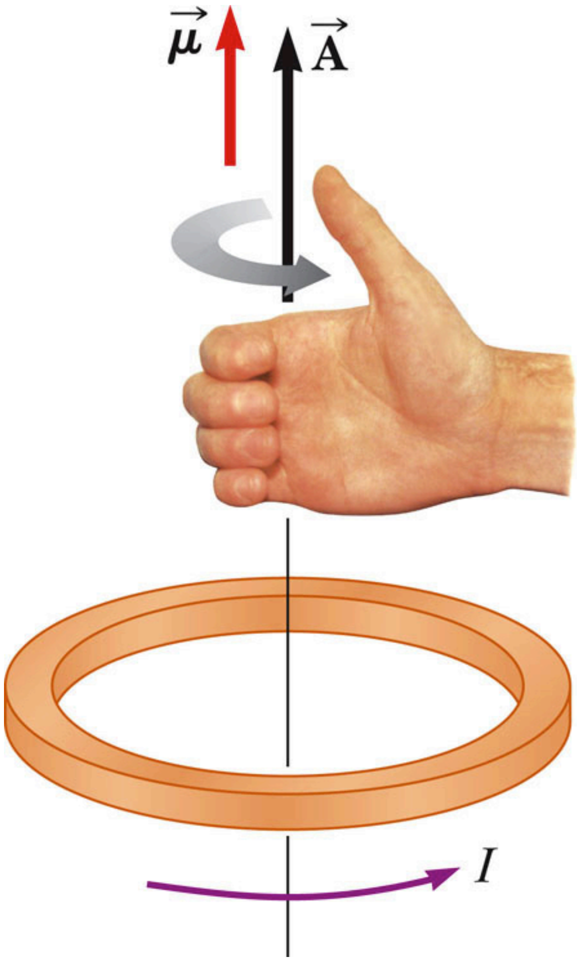
Direction of a current loop and the magnetic dipole moment

- The right-hand rule can be used to determine the direction of $\vec{A}$
- Curl your fingers in the direction of the current in the loop
- Your thumb points in the direction of $\vec{A}$
- The product $I\vec{A}$ is defined as the **magnetic dipole moment,** $\vec{\mu}$, of the loop

  Often called the magnetic moment
- SI units: A · m^2
- Torque in terms of magnetic moment:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Analogous to $\vec{\tau} = \vec{\mu} \times \vec{E}$ for electric dipole



Potential Energy

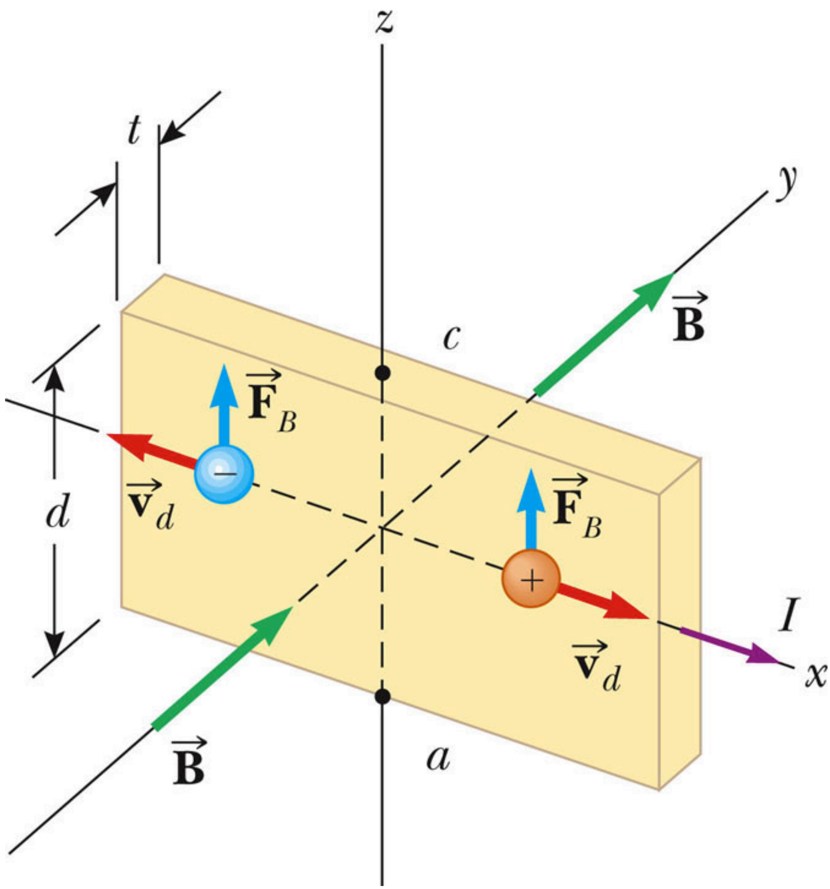
- The potential energy of the system of a magnetic dipole in a magnetic field depends on the orientation of the dipole in the magnetic field:

$$U = -\vec{\mu} \cdot \vec{B}$$

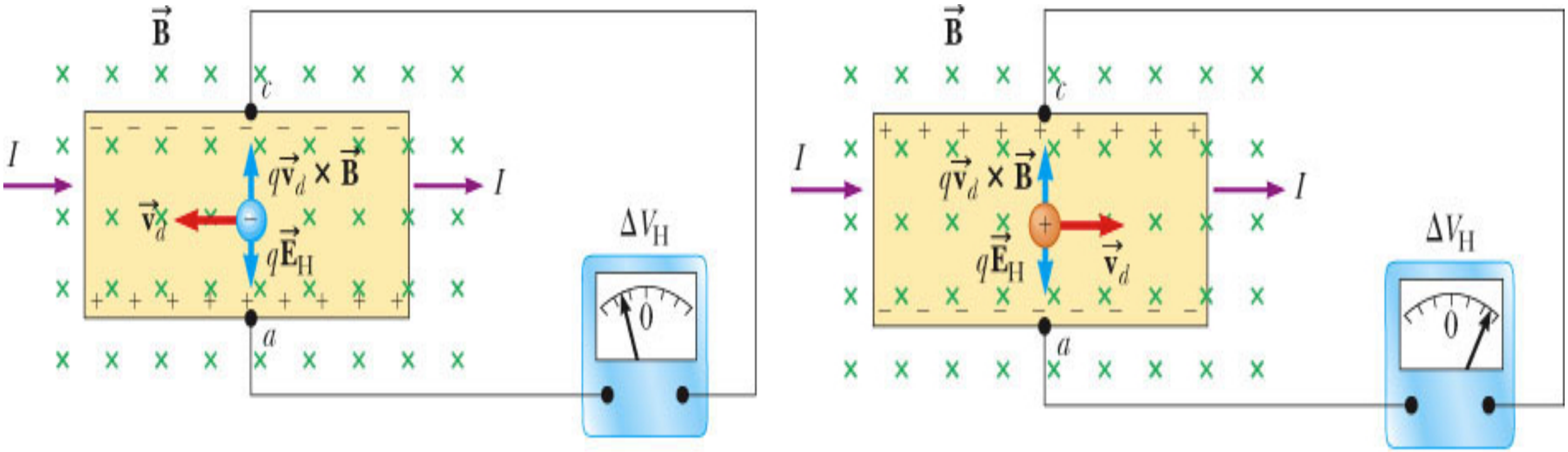
  - $U_{\min} = -\mu B$ and occurs when the dipole moment is in the same direction as the field
  - $U_{\max} = \mu B$ and occurs when the dipole moment is in the direction opposite the field

Hall Effect, a way to measure magnetic field

- When a current carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field
- This phenomena is known as the Hall effect
- It arises from the deflection of charge carriers (either positive, or negative, but not both) to one side of the conductor as a result of the magnetic forces they experience
- In the figure, the Hall voltage is measured between points a and c



Hall voltage, negative or positive carriers



- When the charge carriers are negative, the upper edge of the conductor becomes negatively charged, c is at a lower potential than a
- When the charge carriers are positive, the upper edge becomes positively charged, c is at a higher potential than a

Hall voltage as a function of the magnetic field

$$\Delta V_H = E_H d = v_d B d$$

- d is the width of the conductor
- v_d is the drift velocity
- If B and d are known, v_d can be found

- $\Delta V_H = \dfrac{IB}{nqt} = \dfrac{R_H IB}{t}$
  - $R_H = 1 / nq$ is called the Hall coefficient
  - A properly calibrated conductor can be used to measure the magnitude of an unknown magnetic field

Reading material and Homework assignment

Please watch this video (about 50 minutes each):
http://videolectures.net/mit802s02_lewin_lec11/

Please check wileyplus webpage for homework assignment.