## Chapter 34 Images



## Lens



## Review: Spherical mirror formulas

Focal length and sphere radius:

$$
f=\frac{|r|}{2} \quad \begin{aligned}
& \text { Converging } \\
& \text { mirror }
\end{aligned}
$$



Object distance, image distance and focal length:

$$
\frac{1}{p}+\frac{1}{i}=\frac{1}{f}
$$

Object distance, image distance and magnification:

$$
m \equiv \frac{H_{I}}{H_{O}}=-\frac{i}{p}
$$

## Thin lenses -- again

- Lenses are commonly used to form images by refraction.
- We discuss about spherical lenses only. They are part of two spheres.
- There are two types of lenses
- Converging
- Diverging
- When the thickness of the lens is negligible, the lens is called thin lens.
- Principal axis: the line that goes through the two centers of the two spheres.
- Paraxial rays are those close to the principal axis.
- Lenses are used in optical instruments
- Cameras
- Telescopes, binoculars
- Microscopes


Principal axis

diverging

## Converging and diverging Lens Shapes

- Converging:
- positive focal lengths
- thickest in the middle

- Diverging:
- negative focal lengths
- thickest at the edges



## 34.7: Thin Spherical Lenses:

A spherical lens is a transparent object with two refracting surfaces whose central axes coincide. The common central axis is the central axis of the lens.

A lens that causes light rays initially parallel to the central axis to converge is called a converging lens. If, instead, it causes such rays to diverge, the lens is a diverging lens.

A thin lens is a lens in which the thickest part is thin relative to the object distance $p$, the image distance $i$, and the radii of curvature $r_{1}$ and $r_{2}$ of the two surfaces of the lens. If one considers only light rays that make small angles with the central axis, and if $f$ is the focal length, then

Also, $\quad \frac{1}{f}=\frac{1}{p}+\frac{1}{i} \quad$ (thin lens),
The lens maker's equation in air. More about it later.

$$
\frac{1}{f}=(n-1)\left(\frac{1}{r_{1}}-\frac{1}{r_{2}}\right) \quad(\text { thin lens in air })
$$

## 34.7: Thin Lenses:

To find the focus, send in rays parallel to the central axis.


The bending occurs only at the surfaces.

## Focal Length of a Converging Lens



- The parallel rays pass through the lens and converge at the focal point
- The parallel rays can come from the left or right of the lens


## Focal Length of a Diverging Lens



- The parallel rays diverge after passing through the diverging lens
- The focal point is the point where the rays appear to have originated


## The lens maker's formula (lens in vacuum or air)


$\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$
$n$ : index of refraction of the lens material.
$R_{1}$ : radius of near surface.
$R_{2}$ : radius of far surface.

The sign conventions for the radii of the two surfaces:

1. The near or far surface of the lens is with respect to the focal point $F$. Near side is surface 1 and $R_{1}$, far side is surface 2 and $R_{2}$.
2. The sign of the radius is then defined as:
"十" if the center is on the far side; "-" if the center is on the near side
3. In this convention, positive $f$ means converging lens, negative $f$ means diverging lens.


More examples


## Steps to determine the signs

- Pick up the focal point, hence the focal length in question.
- Mark the near surface $S_{1}$, and far surface $S_{2}$.
- Based on the curve of these two surfaces, mark the center $C_{1}$ for $S_{1}$ and $C_{2}$ for $S_{2}$.
- Mark $R_{1}$ for $S_{1}$ and $R_{2}$ for $S_{2}$.
- Now decide on the sign of $R_{1}$ and $R_{2}$, based on its center on far side ( + ) or near side (-).


## The lens maker's formula (lens in a medium)



$$
\frac{1}{f}=\left(\frac{n_{\text {lens }}}{n_{\text {medium }}}-1\right)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)
$$

## Example: prove for a thin lens, the focal length

 on both side of the lens is the same.

## More examples

A thin lens has a focal lens of $f_{\mathrm{a}}=5 \mathrm{~mm}$ in air. The index of refraction of the lens material is 1.53. If this lens is placed in water ( $n=1.33$ ), what will the lens' focal length in water?

## One more example

A thin lens has a near surface with a radius of curvature of -5.00 cm and a far surface with a radius of curvature of +7.00 cm . (a) Is the lens converging or diverging? (b) What is the focal length of the lens if the index of refraction of the material is 1.74?

# Notes on Focal Length and Focal Point of a Thin Lens 

- Because light can travel in either direction through a lens, each lens has two focal points
- One focal point is for light passing in one direction through the lens and the other is for light traveling in the opposite direction
- However, there is only one focal length
- Each focal point is located the same distance from the lens


## Ray Diagrams for Thin Lenses - converging

- Ray diagrams are convenient for locating the images formed by thin lenses or systems of lenses
- For a converging lens, the following three rays are drawn:
- Ray 1 is drawn parallel to the principal axis and then passes through the focal point on the back side of the lens
- Ray 2 is drawn through the center of the lens and continues in a straight line
- Ray 3 is drawn through the focal point on the front of the lens (or as if coming from the focal point if $p<f$ ) and emerges from the lens parallel to the principal axis

- The image is real
- The image is inverted
- The image is on the back side of the lens


## The 3-ray diagram again



Ray 1 :
Parallel to axis, then passes through far focal point

Ray 2:
Passes unchanged through center of lens

Ray 3:
Passes through near focal point, then parallel to axis

magnification : $m \equiv \frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}}$ inverted, smaller

## Object distance, 5 cases: 1 -- 2



## Object distance, 5 cases: 3 -- 4



## Object distance, 5 cases: 5



Like in the converging mirror case, there are 5 possible object locations that produce different images.

While in the diverging lens case, like in the diverging mirror case, no matter where the object is placed, you always get a virtual, upright and smaller image.

## Ray Diagrams for Thin Lenses - Diverging

- For a diverging lens, the following three rays are drawn:
- Ray 1 is drawn parallel to the principal axis and emerges directed away from the focal point on the front side of the lens
- Ray 2 is drawn through the center of the lens and continues in a straight line
- Ray 3 is drawn in the direction toward the focal point on the back side of the lens and emerges from the lens parallel to the principal axis


1. The image is virtual
2. The image is upright
3. The image is smaller
4. The image is on the front side of the lens

## The 3-ray diagram again



Ray 1
Parallel to axis, virtual ray passes through near focal point


Ray 2
Straight through center of lens

Ray 3
Virtual ray through far focal point, virtual ray parallel to axis


## 34.7: Thin Lenses, Two Lens System, where the calculation helps

## Step 1

Neglecting lens 2, locate the image $I_{l}$ produced by lens 1 .
Determine whether the image is on the left or right side of the lens, whether it is real or virtual, and whether it has the same orientation as the object. Find the lateral magnification $m_{l}$.

## Step 2

Neglecting lens 1 , treat $I_{l}$ as though it is the object for lens 2. Locate the image $I_{2}$ produced by the second lens.


If $I_{l}$ lies to the right of lens 2 (past lens 2 ), treat it as the object for lens 2 , but take the object distance $p_{2}$ as a negative number when locating the final image position, $I_{2}$.
Find the lateral magnification $\mathrm{m}_{2}$.
Total magnification is: $\quad M=m_{1} m_{2}$.
If $M$ is positive, the final image has same the orientation as the object.

Example, Image produced by a system of two thin lenses:

Figure 34-18a shows a jalapeño seed $O_{1}$ that is placed in front of two thin symmetrical coaxial lenses 1 and 2, with focal lengths $f_{1}=+24 \mathrm{~cm}$ and $f_{2}=+9.0 \mathrm{~cm}$, respectively, and with lens separation $L=10 \mathrm{~cm}$. The seed is 6.0 cm from lens 1. Where does the system of two lenses produce an image of the seed?

Fig. 34-18


(c)

Fig. 34-18

## The Eye

- The normal eye focuses light and produces a sharp image.
- The image distance is unknown but fixed. The focal length changes in a range.
- The near point corresponds to the shortest focal length, the far point, which is infinity for an healthy eye, a finite distance for near-sighted, corresponds to the longest focal length.



## Simple Magnifying Lens



When object is between the near point and the eye, image is no longer on the retina, the eye sees a blurred image.

When object is too small (to have an opening angle of $\theta$ at the near point $P_{\mathrm{n}}$ ), a magnifying lens is placed before the eye so that the image of the object by this lens has an opening angle of $\theta^{\prime}>\theta$ : magnified.

## Simple Magnifying Lens



The angular magnification (not the lateral magnification) of a simple magnifying lens is

$$
M_{\theta}=\frac{\theta^{\prime}}{\theta} \cong \frac{25 \mathrm{~cm}}{f}
$$

where $f$ is the focal length of the lens and 25 cm is a reference value for the near point value.

## Compound Microscope:

## Eyepiece



$$
\left|\leftarrow-f_{\mathrm{ob}} \rightarrow\right| \leftarrow f_{\mathrm{ob}} \rightarrow|\leftarrow s \longrightarrow|<f_{\mathrm{ey}} \rightarrow 1
$$

A compound microscope is made of two focusing lenses (called the objective and the eyepiece). The focal lengths $f_{\text {ob }}$ and $f_{\text {ey }}$ are small compared with the distance $s$ between the two lenses. The two lenses are configured in such a way that the object, placed just outside of the focal point $F_{1}$ of the objective produces an magified real image just inside the focal point $F_{1}$ ' of the eyepiece. When looking through the eyepiece, that real image forms a much magnified virtual image to the eye.

## Compound Microscope:


$1 \leftarrow f_{\mathrm{ob}} \rightarrow 1 \leftarrow f_{\mathrm{ob}} \rightarrow|\leftarrow s \longrightarrow| \leftarrow f_{\mathrm{ey}} \rightarrow 1$
If the lateral magnification produced by the objective is $m$, and the total magnification of the microscope is $M$, then

$$
\begin{aligned}
& m=-\frac{i}{p} \cong-\frac{s}{f_{\text {object lens }}} \\
& M=m M_{\theta} \cong-\frac{s}{f_{\text {object lens }}} \frac{25 \mathrm{~cm}}{f_{\text {eyepiece }}}
\end{aligned}
$$

## Refracting Telescope



A refracting telescope is also made of two focusing lenses (called the objective and the eyepiece). The objective focal length is longer than that of the eyepiece: $f_{\mathrm{ob}}>$ $f_{\text {ey }}$. The two lenses are configured in such the distance between them is the sum of the two focal lengths: $s=f_{\mathrm{ob}}+f_{\text {ey }}$. Object far away from the objective forms a real image just inside the focal point if the eyepiece. When looking through the eyepiece, that real image forms a much magnified virtual image to the eye.

## Refracting Telescope




The angular magnification $m_{\theta}$ of the telescope is $\theta_{e y} / \theta_{o b}$. For rays close to the central axis, we can write $\theta_{\mathrm{ob}} \approx h / f_{o b}$ and $\theta_{e y} \approx h / f_{e y}$, which gives

$$
M_{\theta} \cong-\frac{f_{\text {objective }}}{f_{\text {eyepiece }}}
$$

