Probability

PHYS 1301 F98
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The naked hulk alongside came,
And the twain were casting dice;
"The game is done! I've won! I've won!"
Quoth she, and whistles thrice.
Samuel Taylor Coleridge, *Rime of the Ancient Mariner*

Most of us have had the experience of listening to the weather report and hearing at one time or another the announcer say "the chance of rain tomorrow is 70%." What does this statement mean? Intuitively, you might say that it is more likely than not that it will rain tomorrow. If we somehow managed to experience many days like today, then we would expect that more often than not it would rain the next day. This weather forecast, like all statements about chance, is a kind of guess. Our ignorance prevents us from making a firm statement about whether or not it definitely would rain the next day. The theory of probability permits us to make sensible and quantitative guesses about matters that have a consistent average behavior.

To be clear about what the word "probability" really means and how you actually calculate it, consider the case where you throw two dice, one green and one red. Each die face can show any integer in the range from 1 to 6 and you are interested in the sum of the two die faces. You want to know what the "probability" of rolling a 5 is because if you can reliably determine this probability you will win much money and the love of your dreams. To begin with, what happens when you roll the pair of dice? Well, the outcome – the sum of the die face - can be any integer between 2 and 12. In our example, there are a number of different ways that a 5 can be rolled. Call this set of different ways A and we have

$$A = \{(1,4), (2,3), (3,2), (4,1)\},$$

where the first number in each ordered pair is the number the red die shows and the second number is the number the green die shows. Each ordered pair of numbers is called an outcome and each roll of the die pair is called an "experiment." You should be able to convince yourself that there are a total of 36 possible outcomes when rolling the dice. The red die can show any integer from 1 to 6 and since for each of these numbers the green die can show any number from 1 to 6, 6 X 6 makes 36. Useful jargon is that the set of all possible outcomes in an experiment is called the "sample space." For our experiment of rolling two dice at a time, the sample space is the set of 36 possible outcomes or ordered pairs of numbers.

What does this have to do with calculating the probability of rolling a 5? Well, by "probability" of a particular outcome of an experiment, we mean our estimate of the most likely fraction of a number of repeated observations that will yield that particular outcome. And do how do you calculate this probability? If you think that each outcome is equally likely, you simply sum up the number of outcomes that will yield a particular event and then divide by the size of the sample space. In our example, there are 4 possible outcomes that produce the "event" of rolling a 5 and there are 36 total possible outcomes in our sample space, so the probability of rolling a 5 is 4/36 = 1/9. Symbolically, we can write that the probability of event A, $P(A) = N_A / N$. Here, N_A is the number of outcomes that produce the event A and N is the size of the sample space.

There are subtleties you should be aware of. To assign a probability to some outcome, it is necessary that the experiment be capable of being repeated**.** For example, it is far from clear that the statement, "the probability that David Graham murdered Adrianne Jones is 65%" (a local murder trial in progress), has any meaning at all. How do we arrange to run many "experiments" with the participation of the defendants and the deceased? Is the deceased supposed to be repeatedly resurrected after her murder so that the experiments can continue? Secondly, as more information becomes available to us, our probability estimate for a particular outcome in the experiment can change. Suppose the experiment is that your sister draws a card from a standard deck and then asks you the probability that it is a queen. If you find out somehow that your sister nervously twitches her ears when she draws either aces or queens, then your answer will certainly depend on the motion of her ears. Having the extra information doesn't change the experiment in any way (your sister twitches her ears whether you know it or not), it does however change your knowledge of the experiment.

When we are playing with our dice, we do not necessarily expect that if we roll the dice 45 times we will observe that <u>exactly</u> 1/9 of the time the sum of the die faces will be 5, even if the dice are honest. This does not mean that our notions of probability are useless. It does mean that to make a probabilistic statement implies that we have a certain amount of ignorance of the experimental situation. If we somehow knew more, we could say exactly what the dice were going to do. However, we can say that if we keep rolling the dice, we do expect that the fraction of times the die face sum to 5 will indeed approach 1/9.

So, how many times do we have to roll the dice before we are confident that our probability calculation is really correct? The answer is there is no specific number of times we need to roll the dice that will definitely tell us one way or the other that our probability calculation is absolutely correct! The reason is that there is some chance, no matter how small, that the dice after many throws happen to come up summing to 5 at a rate different from 1/9. (For example, if you threw the dice 99,000 times, it is certainly possible that the number of times the dice summed to 5 could be different from 11,000 even if there is no cheating.) The important point here is that we expect that the more often we roll the dice, the more likely the summed results approach our probabilistic predictions. Differences between the actual results of our experiments and our probabilistic predictions are called " statistical fluctuations." If our probabilistic predictions are sensible, then we expect the statistical fluctuations to become smaller as the number of times we perform the experiment becomes larger, i.e., the larger the "statistics" we collect.

To summarize, in today's lab we want to check two important ideas about probability. First, we want to check this idea that the probability for an experiment can be estimated by counting outcomes and using the simple expression $P(A) = N_A / N$. We will use rolls of dice as our "experiment."

Secondly, we would like to verify that as the number of experiments (rolls of the dice) increases, the statistical fluctuations decrease. If our ideas of probability are sensible, then we expect that our theoretical estimate of the fraction of time a particular sum shows should more closely match the actual observations as we perform more and more experiments.

Procedure:

You will test our ideas about probability using a jar containing a pair of dice. Each of you should have your own jar. A "roll" of the dice just means you briefly shake the jar and let the dice come to rest. You are interested in the sum of the numbers showing on the dice.

1. Fill out the second row in table 1. Calculate the different outcomes on a piece of scratch paper and just enter the results in the table.

2. Complete table 1. For the 3rd row, keep your answers in fractional form for now.

3. Now you will perform many individual experiments by shaking the glass jar and observing the dice sum after each "roll." Perform at least 50 rolls of the dice and record how often you get the sums 2 – 12 in row 2 of table 2 ("observed number of events"). Using the total number of rolls, fill out row 3 of table 2 ("relative fraction of observed events"). Using a piece of scratch paper to tally the results of the individual experiments will make your life easier.

4. Move on to table 3, "Class Rolls." You will need to chat with all your fascinating neighbors. Fill out row 2 by summing together the results for each event (a particular sum) from everyone in the lab. In this way you will have results from hundreds of rolls. Using the total number of rolls in table 3, compute the entries for row 3.

5. Add up all the probabilities for all possible outcomes in table 1. (This means just sum the bottom row together.) **Question 1.** What answer do you get? What does this number mean?

6. Compare each box in the last row of table 1 with its corresponding box in table 2. **Question 2.** Do you see any similarities? Do you see any differences? **Question 3.** For what outcome is the difference between theory and your private rolls the largest?

7. Now compare each box in the last row of table 1 with its corresponding box in table 2. **Question 4.** Do you see any similarity in the pairs of boxes? What is it? **Question 5.** In general, is the agreement between the box pairs of table 1 and table 2 better or worse than the agreement between box pairs of table 1 and table 3? Give reasons for your answer. Explain why the agreement (or disagreement) makes sense based on the ideas of probability.

8. **Question 6.** Calculate the probability of the dice sum showing a sum between 2 and 5 inclusive. (Express your answer as P(2-5) = …) Use table 1 to help you. **Question 7.** Using the results of table 3, does the observed fraction of times the dice showed a sum of 2-5 agree or disagree with your prediction for its probability? Explain your reasoning.

9. **Question 8.** We said above when explaining our formula for calculating the probability of event *A* to happen that we assumed that all outcomes of an experiment were equally likely. That is, we assumed we were just as likely to roll a (2,1) as a (6,4). Suppose now that this is not true because, say, someone cheated and loaded the dice so that 5's are very likely to show up. Do you think this circumstance would our effect our formula? Explain why or why not?

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Abstract

Table 1. Theory

Sum	2	3	4	5	6	7	8	9	10	11	12
Number of outcomes											
Probability											

Table 2. Private Rolls.

Sum	2	3	4	5	6	7	8	9	10	11	12
Observed number of events											
Observed relative Fraction of events											

Table 3. Class Rolls.

Sum	2	3	4	5	6	7	8	9	10	11	12
Observed number of events											
Observed relative Fraction of events											

Question 1.

Question 2.

Question 3.

Question 4.

Question 5.

Question 6.

Question 7.

Question 8.