

# Quantum Scattering

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## Introduction

This lab will simulate the behavior of a quantum mechanical wave “packet” as it interacts or “scatters” off a potential barrier. The moving display is the value of the “probability density” associated with the wave packet as the wave packet moves along a line in a one-dimensional universe and interacts with a potential barrier. Recall that a potential barrier is a region of space that either attracts or repels wave packets (including wave packets like electrons and baseballs). The barrier (often called a well when it has a negative value) only affects the wave packet when the wave packet is in the same location as the well. You may remember from lecture that the probability density associated with the wave packet is a number proportional to the probability of finding a particle at a particular point in space. You may also recall that this probability density is the square of the wave function and that the wave function is calculated by solving the Schrödinger equation. A wave packet is just a set of wave functions. You can use the words interchangeably for now

It is useful to recall that the larger the probability density is in some region of space, the greater are the chances that you will find the particle there. That the probability density is distributed in space and not just confined in one location means that there is some chance for finding the particle in more than one location, although not at the same time! This behavior is very different from the way you are used to seeing the world behave. When you bounce a tennis ball off a wall or bounce a basketball. You have no problem identifying where the ball is and what its trajectory is. You do not expect the ball to sometimes pass through the wall or the floor and sometimes not to. In both cases the balls always bounce back towards you. This simple behavior is not realized by nature in the sub-atomic world. The sub-atomic equivalent of tennis balls and basketballs, electrons, have the behavior simulated by today's software.

After a wave packet travels to and interacts with the potential well, we will see that part of the packet continues to travel in the original direction while part of it travels exactly backward, retracing its incoming path. The part of the wave packet that travels in the original direction is called the *transmitted* wave packet and the part that travels backward is referred to as the *reflected* wave packet.

By varying the relative energies of the wave packet and the potential barrier, as well as the width of the barrier, you will simulate some uniquely quantum mechanical behavior that contradicts the predictions of classical physics. For example, you will notice that even when the height of the potential barrier is greater than the energy of the incoming wave packet, there is still some probability that some that the wave packet will “tunnel” through the well, emerge on the other side and continue on its

merry way. Classical physics would tell us that this is simply physically impossible, the incoming particle (represented by the wave packet) does not have the energy to do this. It would be like coasting in a wagon down a 10 meter tall hill starting from rest and then being able to coast up *and* over a 20 meter tall hill.

## Procedure

1. Locate the folder *Quantum Scattering*. The instructor will help you if you can't find it. Double click (left mouse button) on the icon *Quantum Scattering* Click *OK* when queried.
2. You first need to understand the display. Click *GO* and watch the display. The display will repeat itself until you click *STOP*. When the simulation is running, you will see the probability density move from left to right and interact with the potential barrier. The potential barrier is the object which looks like a rectangle. During the time the probability density overlaps in space with the potential barrier, the shape of the probability density will become complicated. Afterwards, you will see some the probability density continue to the right while some of it will be reflected and move to the left.

You may notice other details as well. The small moving dot represents the motion of a classical particle with the same energy as the wave packet. The classical particle does not know anything about quantum mechanics and moves as if quantum mechanics did not exist. You will also notice a decimal number on the lower right hand side of the display. This is the *transmission probability* associated with the interaction of the wave packet and the barrier. It is the probability that a particle (represented by the probability density) traverses the barrier well and continues along the original direction of the packet. This number is automatically calculated by the simulation program and depends on the energy of the particle, the strength of the potential barrier and the width of the barrier.

A horizontal line labeled Energy represents the energy of the particle. The potential barrier is represented by the other line you see and is labeled  $V(x)$ . The potential is zero everywhere except where it sticks up (or down). Notice a number representing the ratio of the strength of the potential barrier to the particle energy, **Barrier Height ( $V/E$ )**. You can change this ratio by clicking on the corresponding number. The **barrier width** is also displayed. You also change its value by clicking on the corresponding number.

3. Spend 5-10 minutes familiarizing yourself with the simulation program. Change the ( $V/E$ ) to  $V/E=1.5$ . Click *GO* and watch what happens. Click the *Reset* and *Stop* buttons to learn what they do. What does the *Continue* button do? Change the barrier width to 0.02 and observe what happens. Notice that the transmission probability changes when either the barrier width or the  $V/E$  ratio changes. *Stop* and *Step* the display.
4. Now we are ready to start. **Question 1:** How does a free wave packet (potential barrier equals zero everywhere) change with time? Sketch the probability density at the beginning of its motion and at the end of its motion. **Question 2:** What is this telling you about how an object (a baseball, say) would behave if it moved long enough in a region of space where it interacted very little with its surroundings?

**Question 3:** What is the relationship between the location of the classical dot and the *maximum* value of the probability density?

5. Set  $V/E=2.00$  and the barrier width to 0.01. *Step* the simulation just as the classically moving dot rebounds from the left hand side of the potential barrier.

**Question 4:** Sketch the probability density. **Question 5:** What is the significance of the peaks in the probability density just as the classical dot rebounds?

6. **Question 6:** For the following two situations, compare the quantum and classical transmission probabilities:

a:  $V/E=2.0$ ; width=0.01; rectangular barrier.

b:  $V/E=-2.0$ ; width=0.05; rectangular barrier.

7. Now set  $V/E=1.5$ . Run the simulation for the widths 0.001, 0.002, 0.004, 0.006, 0.008 0.010, 0.015 and 0.018. **Question 7:** Tabulate the transmission probabilities and then graph the natural logarithm (that's the key marked "*ln*" on your calculator) of the transmission probability versus the barrier width. What do you observe? Does this behavior make sense to you?

8. This time, set  $V/E=-2.5$  (note the minus sign). Run the simulation for barrier widths 0.005 and then from 0.01 - 0.9 by steps of 0.01. (Notice all the zeroes!)

**Question 8.** Tabulate the transmission probabilities. What do you observe? Does this make sense to you?

# Quantum Scattering

Name\_\_\_\_\_ Section\_\_\_\_\_

**Abstract:**

**Questions:**

**Question 1:**

**Question 2:**

**Question 3:**

**Question 4:**

**Question 5:**

### Question 6:

### Question 7:

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