Q1. STRATEGY: FIND SPEEDS OF μ & π, AND THEN FIND DIFFERENCES OF SPEEDS TO FIND DISTANCE BY WHICH MUON WINS. IGNORE PARTICLE DECAY.

$$E_T = 10 \text{ GeV}$$

$$= \gamma m c^2$$

$$\Rightarrow \gamma = E_T / mc^2$$

$$1 - \beta^2 = \left(mc^2/E_T\right)^2$$

$$\beta = \left[1 - \left(mc^2/E_T\right)^2\right]^{1/2} \qquad (1)$$

$$m_\mu c^2 = 106 \text{ MeV}$$
$$m_\pi c^2 = 140 \text{ MeV}$$

BOTH MASSES $< E_T$.
EXPAND EQ (1) USING BINOMIAL THEOREM:

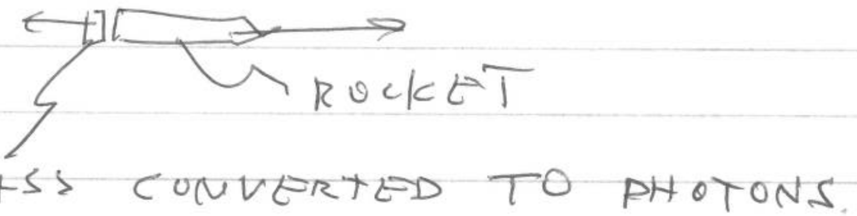
$$\beta \simeq 1 - \frac{1}{2}\left(mc^2/E_T\right)^2$$

$$\Rightarrow \beta_\mu \simeq 1 - 5.6 \times 10^{-5}$$
$$\beta_\pi \simeq 1 - 9.8 \times 10^{-5}$$

Q2.

ROCKETS WORK BY CONSERVATION OF LINEAR MOMENTUM.

ROCKET FRAME:



LET PHOTONS BE FIRED IN NEG. DIR.
CONSIDER SMALL CHUNK OF CONVERTED MASS.

$$dE_\gamma = (-dm)c^2 \quad dm \text{ IS NEG}$$

ROCKET MASS DECREASES.

$$dP_\gamma = -dE/c = dm \cdot c \quad \text{(NEG., OK)}.$$

IN ALICE'S FRAME, WE HAVE FROM CONS OF LIN MOMENTUM

$$dP_\gamma^{\,A} + d(\gamma m v)^A = 0$$

WE GET $dP_\gamma^{\,A}$ FROM ABOVE EXPRESSION FOR P_γ AND USE L-TRANSFORM.

~~CONSOLIDATING RESULTS, WE HAVE~~ $\longrightarrow$

Q2. CONT.

$$\begin{pmatrix} E \\ P_\gamma c \\ 0 \\ 0 \end{pmatrix}^{A} = \begin{pmatrix} \gamma & +\beta\gamma & 0 & 0 \\ +\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ P_\gamma c \\ 0 \\ 0 \end{pmatrix}^{R}$$

NOTE (+) SIGN IN FRONT OF $\beta\gamma$.
EXPANDING,

$$c\, dP_\gamma^{A} = \beta\gamma(-dm)c^2 + \gamma(dm)c^2$$

AGAIN, CONS OF LIN MOMENTUM

$$dP_\gamma^{A} + d(\gamma m v)^{A} = 0$$

$$\beta\gamma(-dm)c + \gamma(dm)c + d(\gamma m v) = 0$$

EXPAND LAST TERM

$$d(\gamma m v) = (d\gamma)mv + \gamma m\, dv + \gamma v\, dm$$

$$d\gamma = d\left(\frac{1}{\sqrt{1-\beta^2}}\right) = -\frac{1}{2}(1-\beta^2)^{-3/2}(-2\beta)d\beta$$

$$= \beta d\beta / (1-\beta^2)^{3/2} = \gamma^3 \beta d\beta$$

Q2. CONT.

CONSOLIDATING RESULTS

$$\beta\gamma(-dm)c + \gamma\, dm\, c + d(\gamma m v) = 0$$

$$-\beta\gamma(-dm)c + \gamma(dm)c + \gamma^3\beta^2 d\beta\, mc$$

$$+ \gamma mc\, d\beta + \gamma\beta c\, dm = 0$$

$$dm + \gamma^2\beta^2 d\beta \cdot m + m\, d\beta = 0$$

$$dm + m\, d\beta\left[\frac{\beta^2}{1-\beta^2} + 1\right] = 0$$

$$dm + m\, d\beta\left[\frac{1}{1-\beta^2}\right] = 0$$

$$\boxed{dm/m + \frac{d\beta}{1-\beta^2} = 0}$$

Q2 b)

$$\int_{m_0}^{m} dm/m + \int_0^{\beta} d\beta/1-\beta^2 = 0$$

$$\int_{m_0}^{m} dm/m + \frac{1}{2}\left[\int_0^{\beta} \frac{d\beta}{1+\beta} + \frac{d\beta}{1-\beta}\right] = 0$$

$$\ln m\Big|_{m_0}^{m} + \frac{1}{2}\ln(1+\beta)\Big|_0^{\beta}$$

$$- \frac{1}{2}\ln(1-\beta)\Big|_0^{\beta} = 0$$

$$\ln m/m_0 + \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right) = 0$$

$$m/m_0 = \left(\frac{1+\beta}{1-\beta}\right)^{-1/2}$$

$$\boxed{m = m_0\left(\frac{1-\beta}{1+\beta}\right)^{1/2}}$$

Q2(c)

$$E = \gamma m c^2$$

$$= \gamma m_0 \left(\frac{1-\beta}{1+\beta}\right)^{1/2} c^2$$

$$= \frac{m_0}{\sqrt{1-\beta^2}} \left(\frac{1-\beta}{1+\beta}\right)^{1/2} c^2$$

$$= \frac{m_0 c^2}{1+\beta}$$

$$\boxed{\lim_{\beta \to 1} E_f = \frac{m_0 c^2}{2}}$$

Q3:

FIRST, CONVINCE YOURSELF THAT IN ALICE'S FRAME THE DISTANCE BTWN CORRESPONDING POINTS ON THE TWO ROCKETS IS ~~[illegible]~~ CONSTANT. E.G., THE NOSE ON EACH ROCKET OR THE TAIL ON EACH ROCKET.

SINCE ROCKETS BEHAVE IDENTICALLY, EACH NOSE OR EACH TAIL TRAVELS THE SAME TOTAL DISTANCE. IF THEY START 1 M APART, THEY ARE ALWAYS 1 M APART, AGAIN AS MEASURED IN ALICE'S FRAME.

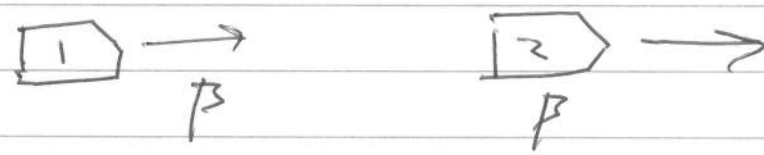
IN ALICE'S FRAME, HOWEVER, STRING CONTRACTS AND SO STRING BREAKS.

ANOTHER WAY TO SEE THIS IS TO LORENTZ TRANSFORM TO ONE OF THE ROCKETS AND ASK WHAT THE SEPARATION BTWN THE ROCKETS IS MEASURED TO BE.

Q3. CONT.

IN ALICE'S FRAME, BOTH ROCKETS HAVE SAME β, EVEN IF $\beta \neq$ CONST.

A:



L-T TO INSTANTANEOUS REST FRAME OF ROCKETS.

$$x_1' = -\beta\gamma ct + \gamma x_1$$

x_1': ROCKET FRAME

x_1: MEASURED IN A FRAME

$$x_2' = -\beta\gamma ct + \gamma x_2$$

$$x_2' - x_1' = \gamma \underbrace{(x_2 - x_1)}_{1\,m}$$

DISTANCE BTWN 2 SHIPS IN ROCKET FRAME

AS $\beta \uparrow$ (A'S FRAME), $\gamma \uparrow$

SO $\Delta x'$ GROWS, EVENTUALLY <u>BREAKING</u> <u>STRING</u>.