ch. 13, pp. 221–222

42. **(a)** While the mass is in contact with the spring, the net horizontal force on it is just the spring force, so it undergoes half a cycle of simple harmonic motion before leaving the spring with speed v_0 to the left. This takes time equal to half a period $\frac{1}{2}T = \pi/\omega = \pi\sqrt{m/k}$. **(b)** v_0 is the maximum speed, which is related to the maximum compression of the spring (the amplitude) by $v_0 = \omega A$. Thus $A = v_0/\omega = v_0\sqrt{m/k}$.

50. The frequencies before and after the steel workers jump, $f_2 = 0.8f_1$, can be related to the rotational inertias of the beam, with and without them, through Equation 13.2, $f_2 = f_1 = \sqrt{I_1/I_2}$, since the torsional constant κ is not changed. Before, $I_1 = \frac{1}{12}ML^2$ (Table 10.2), and after, $I_2 = I_1 + 2m(L/2)^2 = \frac{1}{12}L^2(M + 6m)$, where $m = 75$ kg is each steelworker's mass. Therefore $I_1 = (f_2/f_1)^2 I_2 = \frac{1}{12}ML^2 = \frac{1}{12}(0.8)^2L^2(M + 6m)$, or $M = 6(75\text{ kg})(0.64)/(1 - 0.64) = 800$ kg.

56. At equilibrium, both springs are either extended or compressed, since their forces must be in opposite directions. If the mass is moved by an amount Δx to the right of the equilibrium position, the force of the first spring increases by $k_1\Delta x$ to the left, and the force of the second spring decreases by $k_2\Delta x$ to the right (which is also an increase to the left). Thus, the net force is $(k_1 + k_2)\Delta x$ to the left, which represents a restoring force (opposite to the displacement Δx to the right). The effective spring constant is $k_1 + k_2$, and angular frequency $\omega = \sqrt{(k_1 + k_2)/m}$ for oscillations about the equilibrium position.

68. If the mass and total energy of two mass-spring systems is the same, then $\omega_1 A_1 = \omega_2 A_2$. Therefore, $A_1 = 2A_2$ implies **(a)** $\omega_1 = \frac{1}{2}\omega_2$ and **(b)** $a_{1,\max} = \omega_1^2 A_1 = (\frac{1}{2}\omega_2)^2(2A_2) = \frac{1}{2}a_{2,\max}$.

78. **Interpret** We use torsional oscillations of a bird feeder to determine the mass of the birds on the feeder. We will assume that the birds are point masses for this problem: knowing the rotational inertia of the feeder, the period, and the spring constant, we find the initial rotational inertia contributed by the birds, and thus the birds' masses.

Develop We use the torsional constant $\kappa = 5.00$ Nm/rad of the suspension wire, and $\omega = \sqrt{\frac{\kappa}{I_{total}}}$. The period of oscillation is $f = \frac{\omega}{2\pi} = 2.6$ Hz. The rotational inertia of the disk is $I = \frac{1}{2}MR^2$, where $M = 0.340$ kg and $R = 0.25$ m. The rotational inertia of the birds, assuming they are approximately point masses, is $I_b = 2mR^2$. We will solve $\omega = \sqrt{\frac{\kappa}{I_{total}}}$ for I_b, and thus find m.

Evaluate

$$\omega = \sqrt{\frac{\kappa}{I_b + I}} = \sqrt{\frac{\kappa}{2mR^2 + \frac{1}{2}MR^2}} \rightarrow 2mR^2 + \frac{1}{2}MR^2 = \frac{\kappa}{\omega^2}$$

$$\rightarrow 2m + \frac{1}{2}M = \frac{\kappa}{R^2\omega^2} \rightarrow m = \frac{\kappa}{2R^2\omega^2} - \frac{M}{4} = \frac{\kappa}{2R^2 4\pi^2 f^2} - \frac{M}{4}$$

$$\rightarrow m = 0.065\text{ kg} = 65\text{ g}$$

Assess This is a reasonable mass for a songbird.