36. If kinetic energy of rotation were extracted at a constant rate (power $P = 10^{13}$ W) for a time t, until the period of rotation increased by 1 minute, then $\frac{1}{2}I_E(\omega_0^2 - \omega_f^2) = Pt$, where $\omega_0 = 2\pi/T_0$, and $\omega_f = 2\pi/(T_0 + 1\ \text{min})$. Since $T_0 = 1\ \text{d} \gg 1\ \text{min}$, we can approximate the difference of angular velocities squared as

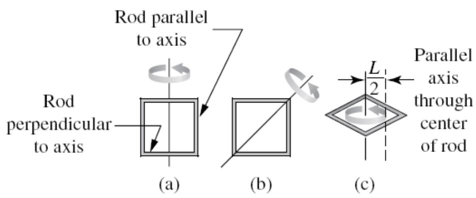
$$\frac{1}{2}(\omega_0^2 - \omega_f^2) = \frac{1}{2}\left[\left(\frac{2\pi}{T_0}\right)^2 - \left(\frac{2\pi}{T_0 + 1\ \text{min}}\right)^2\right] = \frac{1}{2}\left(\frac{2\pi}{T_0}\right)^2\left[1 - \left(1 + \frac{1\,\text{min}}{T_0}\right)^{-2}\right] \approx \left(\frac{2\pi}{T_0}\right)^2\left(\frac{1\ \text{min}}{T_0}\right)$$

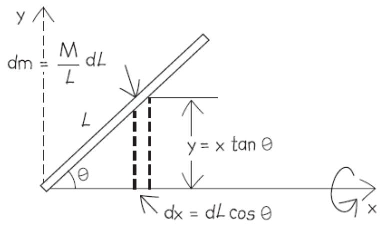
and moreover, take I_E from Exercise 33(**a**).
Then $t \approx (2\pi)^2 I_E(1\ \text{min})/PT_0^3 = (2\pi)^2(9.69\times10^{37}\ \text{kg}\cdot\text{m}^2)(60\ \text{s})/(86{,}400\ \text{s})^3(10^{13}\ \text{W}) = 3.56\times10^{13}\ \text{s}$ or about 1.13 My.

50. (**a**) The rotational inertia of each of the rods perpendicular to the axis is $\frac{1}{12}mL^2$, and that of each rod parallel to the axis is $m(\frac{1}{2}L)^2 = \frac{1}{4}mL^2$. The total is $I_a = [2(\frac{1}{12}) + 2(\frac{1}{4})]mL^2 = \frac{2}{3}mL^2$. (**b**) The rotational inertia of a uniform thin rod, about an axis through one end, making an angle θ with the length of the rod, is

$$I = \int y^2 dm = \int_0^{L\cos\theta}(x\tan\theta)^2\frac{Mdx}{L\cos\theta} = \frac{M\tan^2\theta}{L\cos\theta}\left.\frac{x^3}{3}\right|_0^{L\cos\theta} = \frac{1}{3}ML^2\sin^2\theta$$

Since there are four rods, all making the same angle, $\theta = 45°$, with the axis, the total I_b is $4(\frac{1}{3}mL^2\sin^2 45°) = \frac{2}{3}mL^2$. (**c**) The rotational inertia of one rod (from the parallel axis theorem) is $I_1 = \frac{1}{12}mL^2 + m(\frac{1}{2}L)^2 = \frac{1}{3}mL^2$, and there are four rods, so $I_c = \frac{4}{3}mL^2$.





56. Choose positive torques in the direction the drum turns, and positive forces on the weight upward. In the absence of friction, the net torque on the drum is the (vector) sum of the torque applied by the motor and the torque due to the tension in the rope, $\tau_{\text{net}} = \tau_{\text{app}} - RT = I\alpha$, since the rope is perpendicular to the radius of the drum (see Example 10.9 and Figure 10.19). The net force on the weight is $F_{\text{net}} = T - mg = ma$, and if the rope does not slip, the upward acceleration of the weight equals the tangential acceleration of the drum, $a = \alpha R$. Therefore, $\tau_{\text{app}} = I\alpha + RT = (\frac{1}{2}MR^2)(a = R) + R(mg + ma) = (1.2\ \text{m}/2)[\frac{1}{2}(51\ \text{kg})(1.1\ \text{m/s}^2) + (38\ \text{kg})(9.8 + 1.1)\ \text{m/s}^2] =$ 265 N · m.

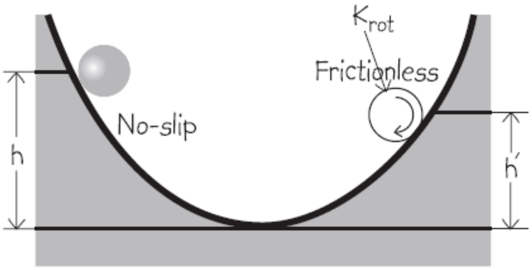
Note: there is a mistake in this solution (Prob.56), see correction in the last page.

64. Take gravitational potential energy to be zero at the bottom. The ball starts on the left side with purely potential energy of magnitude Mgh. At the bottom, the ball's energy is purely kinetic with magnitude $\frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv_{cm}^2$. Going up the right side, the ball's translational kinetic energy will be converted to potential energy, but because the right side is frictionless, the rotational kinetic energy will be unchanged.

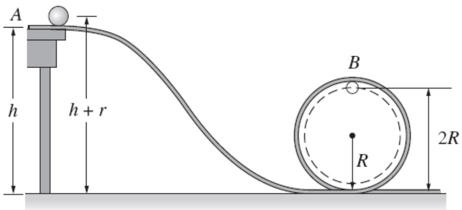
Energy conservation between the top and bottom gives:

$$Mgh = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv_{cm}^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)(v_{cm}/R)^2 + \frac{1}{2}Mv_{cm}^2 = \frac{7}{10}Mv_{cm}^2$$

So at the bottom, $v_{cm}^2 = \frac{10}{7}gh$, and the translational kinetic energy is $\frac{1}{2}Mv_{cm}^2 = \frac{5}{7}Mgh$. This translational kinetic energy is converted entirely to potential energy at the height h'. Therefore we have $Mgh' = \frac{1}{2}Mv_{cm}^2 = \frac{5}{7}Mgh$, or $h' = \frac{5}{7}h$. At the height of $h' = \frac{5}{7}h$ on the right side, the ball still has the same rotational kinetic energy of $\frac{2}{7}Mgh$ that it had at the bottom.



68. The CM of the marble travels in a circle of radius $R - r$ inside the loop-the-loop, so at the top, $mg + N = mv^2/(R - r)$. To remain in contact with the track, $n \geq 0$, or $v^2 \geq g(R - r)$. If we assume that energy is conserved between points A (the start) and B (the top of the loop), and use $K_A = 0$ and $K_B = (1 + \frac{2}{5})\frac{1}{2}mv_B^2$, we find $U_A + K_A = mg(h + r) = U_B + K_B = mg(2R - r) + \frac{7}{10}mv_B^2$, or $h = 2(R - r) + \frac{7}{10}(v_B^2/g) \geq 2(R - r) + \frac{7}{10}(R - r) = 2.7(R - r)$.



78. **Interpret** We are checking whether marketing figures about flywheel energy storage and power availability are correct. We will use angular kinetic energy, and the definition of power.

Develop The flywheel is ring-shaped, so we'll use $I = MR^2$, with $M = 48$ kg and $R = 0.411$ m. Rotational kinetic energy is $K = \frac{1}{2}I\omega^2$, and the initial angular velocity is $\omega = 30{,}000\ \frac{\text{rev}}{\text{min}} \times \frac{1\ \text{min}}{3600\ \text{s}} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} = 52.4$ rad/s.

Evaluate The energy stored in this flywheel is $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(MR^2)\omega^2 = 11.1$ kJ. This is almost a thousand times less than the claimed energy storage. The power $P = \frac{\Delta K}{t}$ tells us that if power were taken from this flywheel at a rate of 25 kW, the time until it stopped would be $t = \frac{\Delta K}{P} = 0.44$ s, or if we needed power for $t = 400$ s, the power it could supply for that time would be only $P = \frac{\Delta K}{t} = 27.8$ W.

Assess This flywheel might be able to run the taillight of the car for 400 seconds, but other than that it's nearly useless. The specs are wildly incorrect!

Note: there is a mistake in this solution (Prob.78), see correction in the last page.

Corrections: (CH 10)

56: The correct algebra is:

$$\tau_{app} = I\alpha + RT = (\tfrac{1}{2}MR^2)(a/R) + R(mg + ma)$$

78 The calculation of ω should be:

$$\omega = 30{,}000 \frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 3142\text{ rad/s},$$

and $K = \frac{1}{2}I\omega^2 \approx 10\text{ MJ}$. So the specs are correct.