G 2.4 (b)

FOR $\Gamma \to 0$

$$\boxed{x(t') = A \cos \omega_d t' + C \cos(\omega_0 t' - \theta)}$$

$\text{w/ } t' = t + \pi/2\omega_d$

W/ A AS DEFINED ABOVE AND C & θ TO BE DETERMINED BY INITIAL CONDITIONS. IF YOU WANTED, YOU COULD USE

$$x(t=0) = 0 = x(\pi/2\omega_d)$$

$$\dot{x}(t=0) = 0 = \dot{x}(\pi/2\omega_d)$$

AND SOLVE FOR C & θ. THIS IS MESSY AND ADDS NOTHING TO THE PHYSICS SO WE WILL SKIP IT.

G 1.5

$$\dddot{f} + f = 0$$

DIFF EQ IS LINEAR & SATISFIES TIME INVARIANCE (ONLY EXPLICIT MENTION OF t IS IN DERIVATIVES)
Hence SOL'N IS of form $f = e^{\alpha t}$
AS ALWAYS "COMPLEXIFY" EQN

$$\dddot{z} + z = 0 \qquad w/\ z = e^{\alpha t}$$

$$\alpha^3 + 1 = 0$$
$$\alpha^3 = -1 = e^{i\pi}$$

3 ROOTS FOR α:

$$\alpha_1 = e^{i\pi/3}$$
$$\alpha_2 = e^{i\left(\frac{\pi}{3} + 2\pi/3\right)} = e^{i\pi}$$
$$\alpha_3 = e^{i\left(\frac{\pi}{3} + \frac{4\pi}{3}\right)} = e^{i\,5\pi/3}$$

$\alpha_1 = \cos \pi/3 + i \sin \pi/3 = \frac{1}{2} + i\frac{\sqrt{3}}{2}$
$\alpha_2 = -1$
$\alpha_3 = \cos 5\pi/3 + i \sin 5\pi/3 = \frac{1}{2} - i\sqrt{3}/2$

$z_1 = e^{t/2}\, e^{i\sqrt{3}\,t/2}$
$z_2 = e^{-t}$
$z_3 = e^{t/2}\, e^{-i\sqrt{3}\,t/2} = z_1^*$

G 1.5

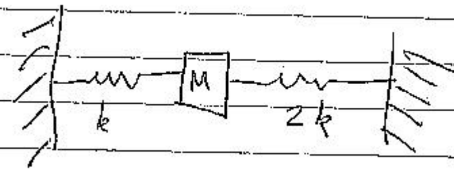
FINALLY,

$$f_1 = \mathrm{Re}\, Z = e^{t/2} \cos \frac{\sqrt{3}}{2} t$$

$$f_2 = e^{-t}$$

$$f_3 = e^{t/2} \sin \frac{\sqrt{3}}{2} t$$

G 1.6.



$$m\ddot{x} = -kx - 2kx = -3kx$$

$$\ddot{x} + (3k/m)x = 0$$

$$\Rightarrow \boxed{\omega_0^2 = 3k/m}$$

$$x(t) = A\cos\omega t + B\sin\omega t$$

$$\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$$

$$\dot{x} = v \quad @\ x=0 \qquad (B.C.)$$

I AM FREE TO START MY WATCH @ ANY TIME — THE SYSTEM IS TIME INVARIANT — SO I ALSO SET $t=0$ WHEN MASS IS @ $x=0$

$$v = B\omega$$

$$\boxed{B = v/\omega}$$

$$\Rightarrow \boxed{x(t) = (v/\omega)\cos\omega t}$$

G 2.4 (a)

SIMILAR TO SEISMOMETER.

CHANGE DEFINITION OF TIME TO MAKE DRIVER LOOK LIKE COS $\omega_d t$ SOURCE. AT END OF CALCULATION, SWITCH BACK

$$\sin \omega_d t = \cos\left(-\omega_d t + \pi/2\right)$$
$$= \cos\left(-\omega_d\left(t + \pi/2\omega_d\right)\right)$$
$$= \cos\left(-\omega_d t'\right) \quad w/\ t' = t + \frac{\pi}{2\omega_d}$$
$$= \cos \omega_d t'$$

ALSO NOTE $\frac{d}{dt} = \frac{d}{dt'}$

WE HAVE EQN 2.39 NOW.

GEN'L SOL'N IS $z(t') = z_c(t') + z_p(t')$

w/ $z_c(t')$ SATISFYING HOMOGENEOUS EQN

$$\ddot{z} + \Gamma \dot{z} + \omega_0^2 z = 0 .$$

$$\boxed{x_p(t') = A\cos\omega_d t' + B\sin\omega_d t' \quad (G2.25)}$$

$$w/\ A = \frac{\left(\omega_0^2 - \omega_d^2\right)\omega_0^2 d}{\left(\omega_0^2 - \omega_d^2\right)^2 + \Gamma^2\omega_d^2}$$

G 2.4 (a)

$$B = \frac{(\Gamma \omega_d)\, \omega_0^2 d}{(\omega_0^2 - \omega_d^2)^2 + \Gamma^2 \omega_d^2}$$

$$\boxed{x_c(t') = C e^{-\Gamma t'/2} \cos(\omega_0 t' - \theta)}$$

C & θ DETERMINED BY INITIAL CONDITIONS:

$$\dot{x}(t'=0)$$

&

$$x(t'=0)$$

OR

$$x(t' = \pi/2\omega_d)$$

$$\dot{x}(t' = \pi/2\omega_d)$$

EQN 2.29

$$\langle P \rangle = \frac{1}{2} F_0 \, \omega_d \, B$$

w/ $B = \dfrac{\Gamma \omega_d \; F_0/m}{(\omega_0^2 - \omega_d^2)^2 + \Gamma^2 \omega_d^2}$

B is max @ $\omega_0^2 = \omega_d^2$

$B_{max} = F_0/m \,/\, \Gamma \omega_d$

$$\Rightarrow \boxed{\langle P \rangle_{max} = \frac{1}{2} \, F_0^2/m\Gamma}$$

EQN 2.28 SHOW $\langle P \rangle = \frac{1}{2} F_0 \omega_d B$

FROM DISCUSSION IN GEORGI

$$\langle P \rangle = \frac{1}{\pi/\omega_d} \int_0^{\pi/\omega_d} F_0 \omega_d B \cos^2 \omega_d t$$

WHERE WE AVERAGE OVER 1/2-cycle
INTEGRAND IS NEVER NEGATIVE.
CHANGE VARIABLES

$$X \equiv \omega_d t$$

$$\Rightarrow \quad \langle P \rangle = \frac{1}{\pi} \int_0^{\pi} F_0 \omega_d B \cos^2 x \, dx$$

$$= \frac{1}{\pi} \left[\frac{X}{2} + \frac{1}{4} \sin 2X \right] \Big|_0^{\pi} \quad F_0 \omega_d B$$

$$\boxed{\langle P \rangle = \frac{1}{2} F_0 \omega_d B}$$