6.3.1 DRILL TYPE PROBLEMS TO VERIFY WE ARE SPEAKING THE SAME LANGUAGE:

$A = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ $B = (3 \ -2 \ 1)$

$C = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 2 & 2 & 0 \end{pmatrix}$

$BA = (3 \ -2 \ 1)\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = 0 - 4 + 1 = -3$

$BC = (3 \ -2 \ 1)\begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 2 & 2 & 0 \end{pmatrix} = (3+2 \quad 3+4+2 \quad 3-2)$

$= (5 \ 9 \ 1)$

$AB = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}(3 \ -2 \ 1) = \begin{pmatrix} 0 & 0 & 0 \\ 6 & -4 & 2 \\ 3 & -2 & 1 \end{pmatrix}$

S8.3.4

(9) $MS = b$

w/ $M = \begin{pmatrix} 3 & -1 & -1 \\ 1 & -2 & -3 \\ 4 & 1 & 2 \end{pmatrix}$ $\quad S = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$b = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$

FIND S: $\quad S = M^{-1}b$

FIND M^{-1}: $\quad M^{-1} = \dfrac{M_c^T}{\|M\|}$

$\|M\| = 3(-4+3) - (-1)(2+12) - 1(1--8)$

$= 2$

$M_c = \begin{bmatrix} +(-1) & -1(14) & +9 \\ -(-1) & +(10) & -(7) \\ +(1) & -(-8) & +(-5) \end{bmatrix} = \begin{bmatrix} -1 & -14 & 9 \\ +1 & 10 & -7 \\ 1 & 8 & -5 \end{bmatrix}$

S8.3.4

$$M_c^T = \begin{pmatrix} -1 & 1 & 1 \\ -14 & 10 & 8 \\ 9 & -7 & -5 \end{pmatrix}$$

$$s = \frac{M_c^T b}{\|M\|} = \frac{\begin{pmatrix} -1 & 1 & 1 \\ -14 & 10 & 8 \\ 9 & -7 & -5 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}}{2} = \frac{1}{2}\begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$$

$$s = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$x = 1$
$y = 2$
$z = -1$

S8.3.4 (b)

$$3x + y + 2z = 3$$
$$2x - 3y - z = -2$$
$$x + y + z = 1$$

$Ms = b$

$s = M^{-1}b$ w/ $M = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

w/ $s = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ w/ $b = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

$$M^{-1} = \frac{M_c^T}{\|M\|}$$

$\|M\| = 3(-2) - 1(3) + 2(5) = +1$

$M_c = \begin{bmatrix} +(-2) & -(3) & +(5) \\ -(-1) & +(1) & -(2) \\ +(5) & -(-7) & +(-11) \end{bmatrix}$

S8.3.4 (b)

$$M_C^T = \begin{pmatrix} -2 & 1 & 5 \\ -3 & 1 & 7 \\ 5 & -2 & -11 \end{pmatrix}$$

$$s = \frac{M_C^T b}{1} = \begin{pmatrix} -6 & -2 & +5 \\ -9 & -2 & +7 \\ 15 & +4 & -11 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ +8 \end{pmatrix}$$

$\Rightarrow$ $x = -3$

$y = -4$

$z = +8$