PHYS 3345

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G 3.3 (i)

FROM THE DIAGRAM IN THE BOOK

$k_{11} = 2200$
$k_{22} = 171$
$k_{33} = 1782$

$k_{12} = -90 = k_{21}$
$k_{13} = k_{31} = 0$

$k_{23} = k_{32} = -81$

$$m^{-1}k = \begin{pmatrix} \frac{1}{100} & 0 & 0 \\ 0 & 1/9 & 0 \\ 0 & 0 & 1/81 \end{pmatrix} \begin{pmatrix} 2200 & -90 & 0 \\ -90 & 171 & -81 \\ 0 & -81 & 1782 \end{pmatrix}$$

$$= \begin{pmatrix} 22 & -9/10 & 0 \\ -10 & 19 & -9 \\ 0 & -1 & 22 \end{pmatrix}$$

G 3.3.9

~~NORMA~~ USE THE RELATION THAT FOR NORMAL MODES:

$$M^{-1}K A = \omega^2 A$$

SO,

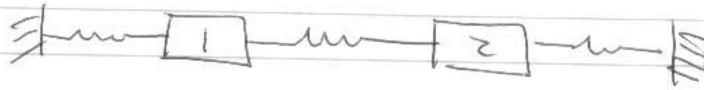
$$m^{-1}K \begin{pmatrix} 9 \\ -30 \\ 10 \end{pmatrix} = \underset{\omega^2}{25} \underbrace{\begin{pmatrix} 9 \\ -30 \\ 10 \end{pmatrix}}_{\text{NORMAL MODE}}$$

$\omega = 5$ RAD/SEC FOR $A = \begin{pmatrix} 9 \\ -30 \\ 10 \end{pmatrix}$

$$m^{-1}k \begin{pmatrix} 9 \\ 60 \\ 10 \end{pmatrix} = 16 \begin{pmatrix} 9 \\ 60 \\ 10 \end{pmatrix}$$

$\omega = 4$ RAD/SEC FOR $A = \begin{pmatrix} 9 \\ 60 \\ 10 \end{pmatrix}$

G 3-5 (a)



$$m^{-1} = \begin{pmatrix} \frac{1}{15} & 0 \\ 0 & \frac{1}{10} \end{pmatrix}$$

$$k = \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{pmatrix} = \begin{pmatrix} 105 & -90 \\ -90 & 100 \end{pmatrix}$$

$$\boxed{m^{-1} k = \begin{pmatrix} 7 & -6 \\ -9 & 10 \end{pmatrix}}$$

(b)

NORMAL MODES SATISFY

$$m^{-1} k A = \omega^2 A$$

$$\left[m^{-1} k - \omega^2 \mathbb{I} \right] A = 0$$

$$\Rightarrow \left| m^{-1} k - \omega^2 I \right| = 0$$

$$m^{-1}k \begin{pmatrix} 9 \\ \emptyset \\ -1\emptyset \end{pmatrix} = 22 \begin{pmatrix} 9 \\ \emptyset \\ -10 \end{pmatrix}$$

$$\omega = \sqrt{22} \text{ RAD/SEC} \quad \text{FOR} \quad A = \begin{pmatrix} 9 \\ 0 \\ -10 \end{pmatrix}$$

OTHER VECTORS IN PROBLEM ARE <u>NOT</u> NORMAL MODES BECAUSE THEY ARE NOT EIGENVECTORS OF $m^{-1}k$.

G 3.3.5

NEED TO SOLVE

$$X(t) = \sum_{\alpha=1}^{3} b_\alpha A^\alpha \cos \omega_\alpha t \quad + c_\alpha A^\alpha \sin \omega_\alpha t$$

FROM INITIAL CONDITIONS, $c_\alpha = \phi \ \forall \alpha$
AND

$$X(0) = \begin{pmatrix} 9 \\ 0 \\ 10 \end{pmatrix} = b_1 A^1 \quad + b_2 A^2 \quad + b_3 A^3$$

$$= b_1 \begin{pmatrix} 9 \\ 60 \\ 10 \end{pmatrix} + b_2 \begin{pmatrix} 9 \\ -30 \\ 10 \end{pmatrix} + b_3 \begin{pmatrix} 9 \\ 0 \\ -10 \end{pmatrix}$$

ANYWAY WE GO:

(1) $\quad 9 = 9b_1 + 9b_2 + 9b_3$

(2) $\quad 0 = 60b_1 - 30b_2 \quad \Rightarrow \boxed{b_1 = \frac{1}{2} b_2}$

(3) $\quad 10 = 10b_1 + 10b_2 - 10b_3$

(1) & (2) $\Rightarrow 18 = 27b_2 + 18b_3$

$$\Rightarrow \boxed{b_2 = \frac{2}{3} - \frac{2}{3} b_3}$$

6 3.3.6

FROM (3):

$$10 = 10b_1 + 10b_2 - 10b_3$$
$$= 5b_2 + 10b_2 - 10b_3$$
$$= 10 - 10b_3 - 10b_3$$

$$\boxed{b_3 = \phi}$$

$$\Rightarrow \boxed{\begin{aligned} b_2 &= \tfrac{2}{3} \\ b_1 &= 1/3 \end{aligned}}$$

HENCE

$$\boxed{\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 9 \\ 60 \\ 10 \end{pmatrix}\cos 4t + \frac{2}{3}\begin{pmatrix} 9 \\ -30 \\ 10 \end{pmatrix}\cos 5t}$$

TO FIND TIME WHEN

$X(t) = \begin{pmatrix} 9 \\ 0 \\ 10 \end{pmatrix}$, FIND TIME WHEN

$\cos 4t = \cos 5t = 1$

THIS OCCURS @ $t=0$ AND $t = 2\pi$ sec.

$\boxed{t = 2\pi \text{ sec}}$

E 3.6.6

$(3) + (4) \Rightarrow \quad 2k_1 + 3k_3 = 60 \qquad (5)$

$(3) - (4) \Rightarrow \quad 2k_1 - 2k_2 - 3k_3 = 0 \qquad (6)$

$(5) + (6) \Rightarrow \quad 4k_1 - 2k_2 = 60$

$\boxed{2k_1 - k_2 = 30} \qquad (7)$

$(5) - (6) \Rightarrow \quad 6k_3 + 2k_2 = 60$

$\boxed{k_2 + 3k_3 = 30} \qquad (8)$

WORK w/ A^2 MORE NOW.

$$M^{-1}K \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \frac{k_1 + k_2}{15} + \frac{k_2}{15} = 4$$

$$\Rightarrow \boxed{k_1 + 2k_2 = 60}$$

w/ EQ (7), $k_2 = 2k_1 - 30$

$$\Rightarrow k_1 + 4k_1 - 60 = 60$$

$\boxed{k_1 = 24} \qquad (9)$

G 3.5 (b)

So

$$(7-\omega^2)(10-\omega^2) - 54 = 0$$

$$(\omega^4 - 17\omega^2 + 16) = 0$$

$$(\omega^2-1)(\omega^2-16) = 0$$

$$\omega^2 = 1, 16$$

$$\Rightarrow \boxed{\omega = 1, 4 \text{ RAD/SEC}}$$

Now,

$$[m^{-1}k - \omega^2 I]A = 0$$

$$\left[\begin{pmatrix} 7 & -6 \\ -9 & 10 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} 6 & -6 \\ -9 & 9 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \Rightarrow a_1 = a_2$$

So, SET $a_1 = 1$ AND

$$\boxed{A^1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{w/} \quad \omega = 1 \text{ RAD/S}}$$

6 3.5(b)

$$\left[\begin{pmatrix} 7 & -6 \\ -9 & 10 \end{pmatrix} \quad \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix}\right]\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0$$

$$-9a_1 - 6a_2 = 0 \quad \Rightarrow \quad a_1 = -\frac{2}{3}a_2$$

$$-9a_1 - 6a_2 = 0$$

SET $a_2 = 3 \Rightarrow a_1 = 2$

$$\boxed{A^2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \text{w/} \quad \omega = 4 \text{ RAD/s}}$$

G 3.6

$$\chi(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = b_1 A^1 \cos\omega_1 t + c_1 A^1 \sin\omega_1 t + b_2 A^2 \cos\omega_2 t + c_2 A^2 \sin\omega_2 t$$

$$\chi(0) = \begin{pmatrix} 5 \\ 0 \end{pmatrix} = b_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} + b_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$5 = 2b_1 + b_2$$
$$0 = 3b_1 - b_2$$

$$\Rightarrow \boxed{\begin{matrix} b_1 = 1 \\ b_2 = 3 \end{matrix}}$$

$$\dot{\chi}(0) = 0 \Rightarrow \boxed{C_1, C_2 = 0}$$

$$\chi(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cos t + 3 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cos 2t$$

@ $t = \pi$ sec, $x_2(t) = -3\text{ cm} - 3\text{ cm} = -6\text{ cm}$

BLOCK 2 MOVES TO LEFT 6 CM

G 3.6. (b)

USE RELATION $m^{-1} K A = \omega^2 A$

$$m^{-1} = \begin{pmatrix} \frac{1}{15} & 0 \\ 0 & 1/10 \end{pmatrix}$$

$$K = \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{pmatrix}$$

$$m^{-1} K A^1 = \omega_1^2 A^1$$

$$\begin{pmatrix} \frac{k_1 + k_2}{15} & \frac{-k_2}{15} \\ \frac{-k_2}{10} & \frac{k_2 + k_3}{10} \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 1^2 \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\frac{2}{15}(k_1 + k_2) - \frac{3}{15} k_2 = 2 \qquad (1)$$

$$-\frac{2}{10} k_2 + \frac{3}{10}(k_2 + k_3) = 3 \qquad (2)$$

$$(2) \Rightarrow \quad -2k_2 + 3(k_2 + k_3) = 30 \qquad (4)$$

$$(1) \Rightarrow \quad 2(k_1 + k_2) - 3k_2 = 30 \qquad (3)$$

G 3.6.b

FROM EQ (7), $\boxed{k_2 = 18}$

NOW USE EQ (8)

$\Rightarrow \boxed{k_3 = 4}$

SO,

$$\boxed{\begin{array}{l} k_1 = 24 \text{ DYNE/cm} \\ k_2 = 18 \text{ DYNE/cm} \\ k_3 = 4 \text{ DYNE/cm} \end{array}}$$