4.2 (a) IN ANALOGY w/ THE PROBLEM IN THE TEXT,

$$S = \begin{pmatrix} 0 & 1 & & & & 0 \\ & 0 & 1 & & & \\ & & 0 & 1 & & \\ & & & 0 & 1 & \\ & & & & 0 & 1 \\ 1 & & & & & 0 \end{pmatrix}$$

$$A_j^k = (\beta_k)^{j-1}, \quad \beta_k = e^{2\pi i k/6}, \quad k=0,\ldots 5$$

DUE TO "NEAREST NEIGHBOR" INTERACTION,

$C = D = 0$ IN K-MATRIX (SEE 4.43)

AND

$E = 2B$. SO

$$K = \begin{pmatrix} E & -E/2 & 0 & 0 & 0 & -E/2 \\ -\frac{E}{2} & E & -E/2 & 0 & 0 & 0 \\ 0 & -\frac{E}{2} & E & -E/2 & 0 & 0 \\ 0 & 0 & -\frac{E}{2} & E & -E/2 & 0 \\ 0 & 0 & 0 & -\frac{E}{2} & E & -E/2 \\ -E/2 & 0 & 0 & 0 & -E/2 & E \end{pmatrix}$$

4.2

THE DISPERSION RELATION IS

$$\omega_K^2 = \frac{E}{m} - \frac{E}{m} \cos \frac{K\pi}{3} \qquad \text{(SEE 4.53)}$$

NOTE THAT FOR $K=0$, $\omega=0$. THIS MODE CORRESPONDS TO A GLOBAL ROTATION OF ALL THE MASSES, EITHER CW OR CCW.

PS 8

G 5.1

(a)

$$M^{-1}K = \begin{pmatrix} 2B & -C & 0 & 0 \\ -C & 2B & -C & 0 \\ 0 & -C & 2B & -C \\ 0 & 0 & -C & B' \end{pmatrix}$$

$$2B = g/l + 2k/m$$

$$B' = g/l + k/m$$

$$C = k/m$$

NOTE THAT MASS 4 HAS ONLY 1 SPRING ATTACHED TO IT.

(b) (i), (ii) AND (iii) ALL TRUE

NORMAL MODES ARE

$$A_j^n = \sin\left[j\frac{(2n-1)\pi}{9}\right], \quad n = 1,2,3,4$$

RHS B.C.: $A^n(x = \frac{9}{2}a) = \text{max/min}$

G 5.1

(c)

$$\omega^2 = 2B - 2C \cos ka$$

$$\text{w/ } B = \frac{g}{2l} + k/m$$

$$C = k/m$$

$$k = \frac{(2n-1)\pi}{9a} \qquad n = 1, 2, 3, 4$$

$$\Rightarrow \omega_1^2 = 2B - 2C \cos \pi/9$$

$$\omega_2^2 = 2B - 2C \cos \pi/3$$

$$\omega_3^2 = 2B - 2C \cos 5\pi/9$$

$$\omega_4^2 = 2B - 2C \cos 7\pi/9$$

5.3

BY ANALOGY w/ EXAMPLE 5.11 IN TEXT,

$$A_j = \sin[ka(j-\tfrac{1}{2})] \quad w/\ k = n\pi/L$$

$$L = 5a$$

$$\Rightarrow \boxed{A_j = \sin\left[\frac{n\pi(j-\frac{1}{2})}{5}\right], \quad n = 1, \ldots 5}$$

NOTE THAT $A_0 = -\sin\frac{n\pi}{10}$

$A_1 = +\sin n\pi/10$

$A_0 = -A_1$

(a) $A_5 = -A_6$

(b) SEE ABOVE FOR NORMAL MODES

$$\boxed{\omega^2 = \frac{4T}{ma}\sin^2\frac{ka}{2}, \quad k = \frac{n\pi}{5a} \quad n = 1 \to 5}$$