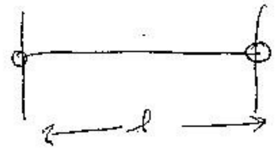
G 6.3.



2 FREE ENDS: $\left.\frac{\partial \psi(x,t)}{\partial x}\right|_{x=0} = 0$ & $\left.\frac{\partial \psi(x,t)}{\partial x}\right|_{x=l} = 0$

BC @ $x=0 \Rightarrow A_n(x) \sim \cos k_n x$

BC @ $x=l \Rightarrow k_n l = n\pi, \quad n = 0, 1, 2, 3, \ldots$

$$\boxed{A_n(x) = \cos \frac{n\pi x}{l}, \quad n = 0, 1, 2, \ldots}$$

G. 7.3.

BC @ $x=0$: $\frac{\partial\psi}{\partial x}\Big|_{x=0} = 0$

$\Rightarrow \psi(x) \sim \cos kx$

BC @ $x=\ell$:

$\psi(x=\ell, t) = A\cos k\ell \cos\omega t = \varepsilon \cos\omega t$

$\Rightarrow A = \frac{\varepsilon}{\cos k\ell}$

So, $\boxed{Z(t) = \frac{\varepsilon}{\cos k\ell} \cos\omega t \qquad \omega/\omega^2 = \frac{k\ell^2}{m} k^2}$

EQ 6.9

Prove $\frac{2}{\ell}\int_0^\ell dx \sin n\pi x/\ell \sin n'\pi x/\ell = \delta_{nn'}$

LET $A = n\pi x/\ell$

$B = n'\pi x/\ell$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\Rightarrow \sin A \sin B = (\cos(A-B) - \cos(A+B))/2$

So,

$$\frac{2}{\ell}\int_0^\ell dx \sin\frac{n\pi x}{\ell}\sin\frac{n'\pi x}{\ell} = \frac{1}{\ell}\int_0^\ell dx \left[\cos\frac{n-n'}{\ell}\pi x - \cos\frac{n+n'}{\ell}\pi x\right]$$

$$= \frac{1}{\ell}\left[\frac{\ell}{(n-n')\pi}\sin\frac{n-n'}{\ell}\pi x\Big|_0^\ell - \frac{\ell}{(n+n')\pi}\sin\frac{n+n'}{\ell}\pi x\Big|_0^\ell\right] \quad n\neq n'$$

$= 0, \quad n \neq n'$

For $n = n'$,

$$RHS = \frac{1}{\ell}\left[\int_0^\ell dx - \int_0^\ell \cos\frac{2n}{\ell}\pi x\, dx\right]$$

$$= \frac{1}{\ell}\left[x\Big|_0^{\ell} - \frac{\ell}{2n\pi} \sin \frac{2n\pi x}{\ell}\Big|_0^{\ell} \right]$$

$= 1$ for $n = n'$